

# "Math is Cool" Championships - 2007-08

Sponsored by:

Pre-Calculus & Calculus - November 14, 2007

Individual Contest

**Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.**

## **GENERAL INSTRUCTIONS applying to all tests:**

- *Good sportsmanship is expected throughout the competition by all involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
  - *For problems dealing with money, a decimal answer should be given.*
  - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of  $\pi$  where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

## **INDIVIDUAL TEST - 35 minutes**

*When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.*

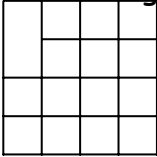
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Individual Contest

1	Evaluate: $4147 + 3284$
2	What is 40% of the number 25% larger than 180?
3	Evaluate as a mixed number: $5\frac{1}{4} \div 3\frac{1}{2}$
4	For what value of a does $6a + 4 = 46$ ?
5	What are the coordinates, in the form (x,y), of the reflection of the point (7,-8) across the line $y = 22$ ?
6	Sam walks 3 kilometers to school in forty minutes. What was Sam's average speed in kilometers per hour?
7	For what value(s) of d does $2d^2 - 3d - 1 = 0$ ?
8	What are the coordinates, in the form (x,y), of the point of intersection of the lines $y = 3x+4$ and $2x + 3y = -21$ ?
9	If you can buy K kilograms of flour with N nickels, how many dimes would be necessary to buy G grams of flour?
10	A right triangle has a hypotenuse measuring 18 cm and a leg measuring 8 cm. What is the length of the other leg, in centimeters?
11	What is the volume, in cubic centimeters, of a right rectangular pyramid with a height measuring 8 cm and a base measuring 6 cm by 10 cm?
12	What is the measure of an interior angle, in degrees, in a regular 24-gon?
13	What is the area, in square centimeters, of a $30^\circ$ sector of a circle with a radius of 6 cm?
14	What is the surface area, in square centimeters, of an octahedron with edges measuring 4 cm?
15	What is the smallest number of regions into which four distinct lines can divide a plane?
16	Simplify, where $i = \sqrt{-1}$ : $(3 - 4i) - (-1 + 2i)(-5 - 6i)$
17	What are the coordinates, in the form (x,y), of the vertex of the given conic section that is farthest from the origin? $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{36} = 1$
18	What is the sum of all values of k satisfying $3^{k+1} + 9^k = 4$ ?

19	What is the least common multiple of 48 and 50?
20	What is the third-largest six-digit palindrome that can be formed using the digits 2, 3, and 5 exactly twice each?
21	In an election with six candidates, how many ways can a Secretary and an Assistant Secretary be chosen?
22	In how many ways can the letters in the phrase "MAMA LLAMA" be arranged? (Ignore the space.)
23	What is the tenth term of an arithmetic sequence with a first term of 84 and a common difference of -9?
24	What is the sixth term of a recursively defined sequence with its first term defined as $r_1 = 8$ and subsequent terms defined as $r_n = (8 - r_{n-1})^2 + 7$ ?
25	What is the mean of the data set 1, 2, 2, 3, 3, 3, 4, 6, 6?
26	How many subsets of $\{1,2,3,4,5,6,7\}$ are supersets of $\{2,3,5\}$ ?
27	Evaluate: $\begin{bmatrix} 1 & 3 & -5 \\ 0 & -4 & -1 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$
28	Evaluate: $\frac{3}{1 + \frac{3}{1 + \dots}}$
29	How many squares of any size can be drawn along the lines of the grid of unit squares shown with one missing grid line? 

## Challenge Questions

<b>30</b>	In a right triangle with legs measuring 8 cm and 10 cm, what is the cosine of the smallest angle?
<b>31</b>	What is the sum of all values of $f$ between 0 and $2\pi$ inclusive that satisfy $\tan f + 1 = \sec^2 f$ ?
<b>32</b>	What is the minimum distance, in centimeters, an ant can crawl along the surface of a regular tetrahedron with edges measuring 8 cm if he is on an edge 1 cm from its midpoint and is trying to reach a point on the opposite edge 2 cm from its midpoint?
<b>33</b>	Which of the following is an eigenvector of $\begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$ ? A) $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ B) $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$ C) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ D) $\begin{bmatrix} -5 \\ 2 \end{bmatrix}$ E) $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$
<b>34</b>	In a twenty-element arithmetic sequence with first term 8 and common difference three, what is the largest element that can be part of a three-term geometric sequence using elements of the arithmetic sequence? E.g. if 1, 2, and 4 were part of the arithmetic sequence (none of them are), they could be chosen as a three-term geometric sequence.
<b>35</b>	Two people play a game in which they take turns rolling a single six-sided die. If someone rolls the number their opponent just rolled, or any multiple of that number, then they win; otherwise the game continues and their opponent takes a turn. What is the probability that the first player wins the game on their second turn?
<b>36</b>	What is the quotient, expressed in base 9, when $44684_9$ is divided by $234_9$ ?
<b>37</b>	If $\log_4 6 = p$ , express $\log_3 12$ in terms of $p$ .
<b>38</b>	A right triangular prism with two faces that are equilateral triangles is sized so that its inscribed sphere touches all five of its faces. A monolith (a right rectangular prism with edges in the ratio of 1:4:9) is inscribed in the sphere. What is the ratio, expressed as a fraction, of the surface area of the monolith to that of the right triangular prism?
<b>39</b>	In the system of equations below, what is the sum of $k$ and $m$ ? $\begin{aligned} 4k + 3l + 4m - n &= 13 \\ k - l + m + n &= -12 \\ -2k + l - 2m + 3n &= 11 \\ k + 5l + m + n &= 36 \end{aligned}$
<b>40</b>	Evaluate: $43^3 + 27^3$

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Individual Multiple Choice Contest

1	Evaluate: $111^2 - 89^2$ A) 3850    B) 4000    C) 4180    D) 4400    E) 4650
2	What ordered pair $(e, f)$ satisfies the equations $3e + 2f = -2$ and $2e - f = 15$ ? A) $(6, -10)$ B) $(4, -7)$ C) $(-2, 2)$ D) $(0, -15)$ E) $(-6, 27)$
3	Using a 50 m rope, a horse is tied to the inner corner of an L-shaped fence with two sections measuring 20 m and 40 m meeting at a right angle. What is the total area, in square meters, the horse can graze? A) $1150\pi$ B) $1200\pi$ C) $1250\pi$ D) $1500\pi$ E) $1675\pi$
4	Express the range of the function $h(j) = \sqrt{-2j^2 + 8j - 3}$ in interval notation. A) $[0, \sqrt{5}]$ B) $\left[\frac{4 - \sqrt{10}}{2}, \frac{4 + \sqrt{10}}{2}\right]$ C) $\left[0, \frac{4 + \sqrt{10}}{2}\right]$ D) $(0, 2)$ E) $\left(\frac{4 - \sqrt{10}}{2}, \frac{4 + \sqrt{10}}{2}\right)$
5	Which of the following numbers is prime? A) 148    B) 237    C) 355    D) 707    E) 1117
6	When one card is drawn from a standard 52-card deck, what is the probability that it is black or a face card (J, Q, or K)? A) $\frac{15}{26}$ B) $\frac{8}{13}$ C) $\frac{17}{26}$ D) $\frac{9}{13}$ E) $\frac{19}{26}$
7	In a set of 7 integers from 0 to 100 inclusive, the range is 33, the mean is 72, and the mode is 85. What is the largest possible value of the median? A) 72    B) 78    C) 85    D) 87    E) 88

8	<p>If <math>U</math> is the set of all powers of 2 between 1 and 1000 inclusive, <math>V</math> is the set of all multiples of 4 between 100 and 900 inclusive, and <math>W</math> is the set of all multiples of 6 between 800 and 1000 inclusive, how many elements are in <math>(V \cap W^c) \cup U</math>, where <math>W^c</math> is the complement of <math>W</math>?</p> <p>A) 203      B) 202      C) 201      D) 200      E) 199</p>
9	<p>What is the secant of the largest angle in a triangle with sides measuring 5, 6, and 9 cm?</p> <p>A) -3      B) <math>\frac{5}{6}</math>      C) 2      D) <math>-\frac{1}{2}</math>      E) <math>\frac{3}{2}</math></p>

# "Math is Cool" Championships - 2007-08

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Team Contest

1	Express in simplest radical form: $\sqrt[3]{6048}$
2	Cherie bikes downhill to work on the shortest route, going five kilometers at a speed of 30 kilometers per hour. She bikes home on a less steep route, going seven kilometers at 20 kilometers per hour. What is her average speed for the round trip, in kilometers per hour?
3	A log with a radius of 30 cm is rolled against a wall and chocked, after which a log with a radius of 20 cm is placed resting on the first log and touching the wall. How many cm above the ground is the top of the second log?
4	Simplify: $\frac{6g^5 + 9g^4 - 14g^3 + 11g^2 + 25g - 28}{2g^2 + 3g - 4}$
5	How many positive four-digit integers satisfy the conditions that their first digit is a multiple of 2, their first two digits can be read as a two-digit number divisible by 3, their first three digits can be read as a three-digit number divisible by 5, and the entire number is divisible by 6?
6	What is the next term of a harmonic sequence beginning with the terms 5 and 3?
7	What is the population standard deviation of the data set 4, 5, 7, 7, 7?
8	If $C = \begin{bmatrix} 1 & -3 \\ -5 & 7 \end{bmatrix}$ , evaluate $ C^{-1} $ .
9	In the following cryptarithm, each instance of a letter stands for the same digit (0-9), and different letters stand for different digits. E.g. if one D is a 1, then all D's are 1's and E cannot be 1. What is the value of the seven-digit number ABCDEFG? $\begin{array}{r} AB \\ \times CA \\ \hline DEF \\ GF \\ \hline DFEF \end{array}$
10	In a three-by-three array of desks in a classroom, A sits immediately in front of I, F sits immediately to C's left, B and D sit in the same row (somewhere to the left or right of one another, perhaps with a person in between them), D sits immediately behind E, E and I are immediately diagonal from one another, H does not sit near (is not immediately next to, in front of, in back of, or diagonal from) C or D, and G is not near F. Which letters might sit in the front left desk?

# "Math is Cool" Championships - 2007-08

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Pressure Round Contest

1	In a regular hexagon, three diagonals are drawn from the same vertex, dividing the hexagon into four triangular areas. What is the area of the largest triangle divided by the area of the smallest triangle?
2	What is the most specific name for the shape of the locus of all points that are four times as far from the point $(3,3)$ as they are from the origin?
3	How many integer values of $q$ greater than 1 satisfy $88 \equiv 160 \pmod{q}$ ?
4	Write an expression which evaluates to 14 using the numbers 1, 2, 3, and 4 exactly once each, the operators $+$ , $-$ , $\times$ , and $\div$ as much or as little as you like, and parentheses as much or as little as you like. E.g. if the numbers were 2, 3, 5, and 8, you could write $3 \times 8 - 2 \times 5$ .
5	If $\cos(d) = \frac{3}{4}$ , and $d$ is in the first quadrant, what is the value of $\sin(2d)$ ?



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Mental Math Contest

<b>PERSON 1 Name:</b>		
1.1	Express the number seventy-three point one eight five in scientific notation rounded to three significant figures.	$7.32 \times 10^1$ or $7.32 \times 10$
1.2	What are the coordinates, in the form X comma Y, of the x-intercept of the line two X minus three Y equals negative 18?	(-9,0)
1.3	What is the area, in square centimeters, of a circle with a perimeter of twenty-two pi root two centimeters?	$242\pi$ [cm <sup>2</sup> ]
1.4	Evaluate the sine of two-hundred ten degrees.	-1/2
<b>PERSON 2 Name:</b>		
2.1	Evaluate three [PAUSE] minus four times five [PAUSE] plus six squared.	19
2.2	What is the equation, in slope-intercept form (Y equals M X plus B), of the line through the points one comma one and two comma three?	$Y=2x-1$
2.3	What is the perimeter, in centimeters, of an equilateral triangle with an area of thirty-six root three square centimeters?	36 [cm]
2.4	What is the sum of the two smallest prime numbers greater than 80?	172
<b>PERSON 3 Name:</b>		
3.1	What is the sum of the number of seconds in a minute, the number of days in a week, and the number of months in a year?	79
3.2	In which quadrant does the point five comma negative three lie?	IV or 4th
3.3	A rhombus has sides and a diagonal all measuring fourteen centimeters. What is the length, in centimeters, of its other diagonal?	$14\sqrt{3}$
3.4	What is the sum of the terms of an infinite geometric sequence with a first term of twenty-seven and a common ratio of one-fourth?	36
<b>PERSON 4 Name:</b>		
4.1	The sum of two numbers is one-hundred eighty-four and their positive difference is eight. What is the larger of the two numbers?	96
4.2	If the vertex angle of an isosceles triangle measures sixty-six degrees, what is the measure of a base angle, in degrees?	57 [°]
4.3	Evaluate the logarithm base-three of seven-hundred twenty-nine.	6
4.4	When two fair six-sided dice are rolled, what is the probability that the sum of the numbers shown is ten?	$\frac{1}{12}$

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## COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	Urn A contains six red marbles, four yellow marbles and eleven blue marbles. Urn B contains four red marbles. If I draw one marble from Urn A and put it in Urn B, then draw a marble from Urn B, what is the probability that the marble drawn from Urn B is not red?	$\frac{1}{7}$
2	What is the result when the quantity one plus I times the square root of three [PAUSE] is raised to the eighth power? Express your answer in rectangular form A plus B I.	$-128 + 128i\sqrt{3}$
3	An ice cream cone is made up of a hemisphere resting on top of a cone. If the lateral surface area of the cone is $18\pi$ and the ratio of the radius to the height is 1 to the square root of 3, what is the volume of the hemisphere?	$18\pi \text{ [un}^3\text{]}$
4	What value or values of Z satisfy the equation four Z minus five equals thirty-one?	9
5	Find the sum of the infinite geometric sequence: two-thirds, one-half, three-eighths, and so on.	$\frac{8}{3}$
6	How many distinct ways can you arrange five keys on a keyring? Assume only the order of the keys matters, not their forward/backward orientation relative to one another.	12 [ways]
7	A rectangular prism with edges of length two, four and X has a space diagonal of length six. What is its surface area?	64
	<b>Extra Problem - Only if Needed</b>	
8	How many paths can I take to travel between the bottom left corner and upper right corner of a four by seven grid of unit squares if I can only travel up or right along the gridlines?	330 [ways]

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## COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	A six-foot tall man is standing near a twenty-seven-foot tall tree. If the man's head is seventy-eight inches from the tip of his shadow, how long is the tree's shadow, in feet and inches?	11 feet, 3 inches
2	Evaluate five cubed minus four squared.	109
3	Evaluate: log base five of eight divided by log base twenty-five of four.	3
4	How many diagonals can be drawn in a convex thirteen-sided figure?	65 [diagonals]
5	What is the area of the triangle defined by the points two comma three, five comma eight and four comma ten?	$11/2$ [ $\text{un}^2$ ]
6	What is the largest X that satisfies the equation: X cubed plus eight X squared plus nineteen X plus twelve equals zero?	-1
7	Find the sum of the terms in the arithmetic sequence with first term thirteen, last term one-hundred nine and common difference four.	1525
7.1	Extra Problem - Only if Needed	
8	Convert one, one, zero, one, zero, one, one base two to base four.	1223 [base 4]

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## COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	A cow is tied to the corner of a twenty foot by thirty foot barn. If its rope is forty feet long, how many square feet of land can the cow graze on?	$1325 \pi$ [ft <sup>2</sup> ]
2	On the polar coordinate plane, what is the distance between the points five root three comma pi over three [PAUSE] and four comma pi over six?	$\sqrt{31}$
3	I flipped seven fair coins. What is the probability that at least three of them were heads?	99/128
4	I drop a ball. Each time it bounces, it bounces to two-thirds of its previous height. If it bounced six feet high after the second bounce, how far will the ball have traveled from the time I dropped it until it comes to rest?	135/2 [ft]
5	[DO NOT PAUSE WHILE READING] Evaluate one plus two minus three times four.	-9
6	What is the sum of the integers from negative fifty to positive one-hundred inclusive?	3775
7	The cosine of an angle in the first quadrant is five-thirteenths. What is the tangent of twice that angle?	-120/119
	Extra Problem - Only if Needed	
8	An equilateral triangle is inscribed in a circle. If the triangle has sides of length four, what is the area of the circle?	$16/3 \pi$ [un <sup>2</sup> ]

# "Math is Cool" Championships - 2007-08

Pre-Calculus & Calculus - November 14, 2007

School Name \_\_\_\_\_ Team # \_\_\_\_\_

Proctor Name \_\_\_\_\_ Room # \_\_\_\_\_

**STUDENT NAME** \_\_\_\_\_

Final Score:

**KEY**

First Score

**DO NOT WRITE IN SHADED REGIONS**

	Answer	1 or 0	1 or 0
1	7431		
2	90		
3	1 1/2		
4	7		
5	(7,52)		
6	9/2 [kph]		
7	$\frac{3 \pm \sqrt{17}}{4}$		
8	(-3,-5)		
9	$\frac{GN}{2000K}$		
10	$2\sqrt{65}$ [cm]		
11	160 [cm <sup>3</sup> ]		
12	165 [°]		
13	$3\pi$ [cm <sup>2</sup> ]		
14	$32\sqrt{3}$ [cm <sup>2</sup> ]		
15	5 [regions]		
16	-14		
17	(-2,11)		
18	0		
19	1200		
20	352253		

	Answer	1 or 0	1 or 0
21	30 [ways]		
22	1260 [ways]		
23	3		
24	7		
25	10/3		
26	16		
27	$\begin{bmatrix} -7 \\ -1 \end{bmatrix}$		
28	$\frac{-1 + \sqrt{13}}{2}$		
29	26 [squares]		
30	$\frac{5\sqrt{41}}{41}$		
31	$9\pi/2$		
32	7 [cm]		
33	B		
34	50		
35	59/216		
36	181 <sub>[9]</sub>		
37	$\frac{2p+1}{2p-1}$		
38	$\frac{2\sqrt{3}}{27}$		
39	-3		
40	99190		

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School Name \_\_\_\_\_ Team # \_\_\_\_\_

Proctor Name \_\_\_\_\_ Room # \_\_\_\_\_

**STUDENT NAME** \_\_\_\_\_

Final Score:

**KEY**

First Score

(out of 18)

## INDIVIDUAL MULTIPLE CHOICE - 15 minutes

*This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. **Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet. No talking during the test.***

**DO NOT WRITE IN SHADED REGIONS**

	Answer	-1, 0 or 2	-1, 0 or 2
1	D		
2	B		
3	A		
4	A		
5	E		
6	B		
7	C		
8	E		
9	A		

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School Name \_\_\_\_\_ Team # \_\_\_\_\_

Proctor Name \_\_\_\_\_ Room # \_\_\_\_\_

**STUDENT NAME** \_\_\_\_\_

Final Score:

**KEY**

First Score

(out of 20)

## Team Contest - Score Sheet

**TEAM TEST - 15 minutes**

*When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as 2 or 0. Record all answers on the colored answer sheet.*

**DO NOT WRITE IN SHADED REGIONS**

	Answer	2 or 0	2 or 0
1	$6\sqrt[3]{28}$		
2	$\frac{720}{31}$		
3	$50 + 20\sqrt{6}$		
4	$3g^3 - g + 7$		
5	39		
6	$\frac{15}{7}$		
7	$\frac{2\sqrt{10}}{5}$		
8	$-\frac{1}{8}$		
9	4521809		
10	G		

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11th & 12th Grade - November 14, 2007

Final Score:

**KEY**

First Score

School Name \_\_\_\_\_ Team # \_\_\_\_\_

Proctor Name \_\_\_\_\_ Room # \_\_\_\_\_

**STUDENT NAME** \_\_\_\_\_

## PRESSURE ROUND - 10 minutes

*When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.*

## Pressure Round Answers

Answer	
1	2
2	Circle
3	11
4	$2 \times (3+4) \times 1$ or $3 \times 4 + 2 \times 1$
5	$\frac{3\sqrt{7}}{8}$