"Math is Cool" Championships – 2009-10

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October 23, 2009

High School Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by all involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
  - *For problems dealing with money, a decimal answer should be given.*
  - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of π where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

"Math is Cool" Championships – 2009-10

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High School Individual Contest

1	If forty-eight is three more than five times my magic number, what is my magic number?
2	What is the area, in square centimeters, of a circle with a radius measuring 14 cm?
3	Each pair of bunnies produces 4 (2 pairs) bunnies every two months. Mary was given a pair of bunnies for her birthday. How many bunnies will she have on her next birthday?
4	In publishing his book, Matt had to pay $1000 overall setup, a fixed cost of $2.00 per book and $0.02 per page per copy of the book. If his cost was $4470.00 for 500 copies, how many pages did his book have?
5	A right triangle has legs measuring 8 and 12 cm. What is the length of the hypotenuse in centimeters?
6	What is the length of the largest chord of a circle with radius of 8?
7	Raffle tickets are consecutively numbered. George sold the tickets numbered from 94 to 175 inclusive but kept every ticket ending in 0 since he considered them lucky. How many tickets did George sell?
8	What is $6\frac{1}{4}$% of 400?
9	How many edges does a regular octahedron have?
10	True or False? The perpendicular bisector of a chord in a circle is guaranteed to go through the center.
11	If a=3, b=4 and c=5, what is $\frac{a+b^2}{c}$?
12	In playing a standard game of tic-tac-toe with nine squares, 'X' makes a move followed by 'O'. How many distinct sets of moves are there if we do not count those that are symmetric repeats of others included?
13	What is the units digit of 7 to the power of 2009?
14	What is the result when the positive difference between twenty-three and sixteen is multiplied by twelve?
15	When two cards are drawn at random from a standard 52-card deck, what is the probability they are a pair of eights?
16	What is the sum of the first ten positive squares?

17	Evaluate: $3+2\sqrt{3+2\sqrt{3+\ldots}}$
18	Use the form ax + by = c, where a, b, and c are relatively prime integers and a>0. What is the equation of the line perpendicular to the line 3x-y=2 going through the point (-1, 2)?
19	I averaged 50 miles per hour driving to Spokane. How many miles per hour must I average on the drive home to average 60 miles per hour for the whole trip?
20	What is the smallest positive multiple of 24 that is a perfect square?
21	What is the area of the shape bounded by the curve: $4x^2-8x+y^2+4y=0$?
22	Evaluate: $\cfrac{2}{1+\cfrac{2}{1+\cfrac{2}{1+\ldots}}}$
23	With 5 colors to choose from, how many ways can a flag be made consisting of four vertical stripes if no stripes of the same color can be placed next to each other?
24	Fourteen students in a class received an A on the final exam, representing 40 percent of the class. How many students are in the class?
25	Three sides of a convex quadrilateral are 3, 5 and 9. How many possibilities are there for the length of the fourth side, if it is also an integer?
26	In an analog clock, what is the area, in square inches, swept out by a 6 inch minute hand in 25 minutes?
27	A combination lock has numbers from 0 to 29 on it. The three numbers to the combination all have different remainders when divided by 5. How many different combinations are possible?
28	For how many integers, m, is: $\lvert 2m-5\rvert<6$?
29	If 9 chickens can lay five eggs in two days, how many chickens would be needed to lay twenty eggs in four days?
30	For what value of x are the two vectors: <x+1,5> and <2,x-3> perpendicular?

Challenge Questions

31	What is the larger value of x so that: $2^{7x} \bullet 8^{x^2} = 64$?
32	What is the determinant of the product: $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 6 & 3 \end{bmatrix}$?
33	Three fair six-sided dice are rolled and it is given that there are no fours, what is the probability that the sum is six?
34	For what values of z is $\frac{2z+5}{2} < \frac{2}{2-z}$? Give your answer in interval notation.
35	Consider the function: $f(x) = \frac{x^2+x-6}{2x^2+7x+3}$. If h is the number of horizontal, v the number of vertical, and s the number of slant asymptotes, what is the ordered triple (h,v,s)?
36	The first three terms of a geometric sequence are: $\sqrt{3}, \sqrt[3]{3}, \sqrt[6]{3}$; what is the next term?
37	Betsy has 5 blue marbles and 7 red marbles in a bag. If she randomly draws 5 marbles without replacement, what is the probability she gets 2 blue marbles and 3 red marbles?
38	Of the 8 chairs in a row, students occupy three of them. If none of them sit next to each other, in how many ways can this be done?
39	Given positive integers a,b; $a < b$ and $\sqrt{4+\sqrt{49+12\sqrt{10}}} = \sqrt{a}+\sqrt{b}$, what is the value of a?
40	When Cassie miscopies the value of b in a problem of the form $ax^2+bx+c=0$, she gets roots of 4 and 12. When Darian miscopies the value of c in then same problem, she gets roots of 4 and 10. What are the correct roots of the problem?

"Math is Cool" Championships – 2009-10

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11th & 12th Grade – October 23, 2009

Individual Multiple Choice Contest

1	If Stacey sold 1/5 of her land, she would have twice as much land in acres as if she sold only 3 acres. How many acres of land does Stacey have? A) 5 B) 10 C) 15 D) 20 E) Answer Not Given
2	Simplify: $\frac{1}{1(2)}+\frac{1}{2(3)}+\frac{1}{3(4)}+\cdots+\frac{1}{19(20)}$. A) 1 B) $\frac{9}{10}$ C) $\frac{19}{20}$ D) $\frac{11}{30}$ E) Answer Not Given
3	What is the probability of obtaining a sum of 7 when I roll 3 six-sided dice? A) $\frac{1}{8}$ B) $\frac{5}{72}$ C) $\frac{35}{216}$ D) $\frac{7}{36}$ E) Answer Not Given
4	What is the period of the function $f(x)=3\sin(2x)+4\cos(3x)$? A) $\frac{2}{3}\pi$ B) $\frac{3}{2}\pi$ C) 2π D) 3π E) Answer Not Given
5	What is the sum of all natural numbers that divide 780? A) 780 B) 1541 C) 1561 D) 2352 E) Answer Not Given
6	Given three non-collinear points in space, what does the set of all points equidistant from all three points form, i.e. the locus? A) Sphere B) Ellipsoid C) Parabola D) Plane E) Line
7	Simplify: $\frac{1}{\log_2(20!)}+\frac{1}{\log_3(20!)}+\frac{1}{\log_4(20!)}+\cdots+\frac{1}{\log_{20}(20!)}$ A) $\frac{1}{2}$ B) 1 C) 20 D) $1+\log_{20}19!$ E) Answer Not Given
8	What is the determinant of the matrix: $\begin{bmatrix} 8 & 3 & 2 \\ 0 & 4 & 1 \\ 1 & 6 & 9 \end{bmatrix}$? A) 0 B) 112 C) 173 D) 235 E) Answer Not Given
9	Wednesday, Thursday and Friday are Spirit Days at Bertha's high school where every day is a different theme of dress. But scatter-brained as she is, she mixes up the order of the themes. If Bertha dresses up every day, what is the probability that she doesn't dress correctly on any day that week? A) $\frac{1}{6}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) Answer Not Given

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11th & 12th Grade – October 23, 2009

Team Contest

1	What is the measure, in degrees, of the smaller angle between the hour and minute hands of a standard 12-hour analog clock at 2:50 PM?
2	If the number of hippos in an ecosystem is four less than three times the number of ibex, and the number of ibex is half the number twenty-nine less than the number of hippos, how many hippos are there?
3	Set A contains 5 elements and set B has 7 elements. How many functions f, from set A to set B, exist so that f-inverse also exists? The functions must map each element of A.
4	What is the product of two numbers if their sum is 19 and the sum of their squares is 185?
5	What is $9^{\log_3 5}$?
6	Two circles have radii of 7 and 13 cm and have centers 10 cm apart. What is the length, in centimeters, of one of their common external tangents?
7	What is the 37th item in an alphabetical list of all the permutations of the word T H E T A?
8	Express the base 7 number 426511_7 in base 6.
9	A parallelogram has two sides of length 5 and 10; what is the sum of the squares of the lengths of diagonals?
10	The sum of the first nine elements of an arithmetic sequence is m and the seventh element is n. What is an expression for the first element in terms of m and n?

"Math is Cool" Championships – 2009-10

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$9^{th}/10^{th}$ & $11^{th}/12^{th}$ Grade – October 23, 2009

Pressure Round Contest

1	If $f(x) = 3x + 1$ and $g(f(x)) = \frac{2x^3 + x + 1}{x^2 + 1}$, what is g(7)?	
2	Evaluate: (91,314)(91,324) – (91,310)(91,328)	
3	If $14x^2 + 2x - 5 = (ax + b)(cx + d)$, what is $ac + bd$?	
4	In its first 20 games, the Maple Avenue Mathematicians won 6 games. They finished the season winning 50% of their games. What is the minimum number of games in the season?	
5	After finishing lunch, Tom and Suzy often walk from their houses to meet each other and go for a walk together. In walking toward each other, they each walk at a consistent speed although each may walk at different consistent speeds from each other. When they each leave at noon, they meet at 12:08; if Tom leaves at noon but Suzy doesn't leave until 12:05, they meet at 12:10. How many minutes would it take Tom to walk all the way to Suzy's house?	

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Mental Math Contest

PERSON 1		
1.1	What is the slope of the line perpendicular to the line 3x+4y=12	4/3
1.2	If f of x equals 3 "x" minus 8, what is f inverse of 7?	5
1.3	Ten billion to the third power divided by one hundred thousand squared is one followed by how many zeroes?	20
1.4	I roll two dice. What is the probability the sum is at least 11 given that at least one of them is a 6.	3/11
PERSON 2		
2.1	All Snuffs are Puffs. All Puffs like Tuffs. If I meet a Snuff, can I be sure she will like Tuffs?	Yes or True
2.2	How many terms are there in the arithmetic sequence: 8 comma 15 comma 22 and so on until 106?	15
2.3	Shini is making a personal puzzle cube using 125 unit cubes. If she paints the outside of the puzzle cube, then how many unit cubes are not painted?	27
2.4	How many pairs of prime numbers have a sum of 30?	3
PERSON 3		
3.1	If f of x equals 4 "x"-squared plus 17, what is the positive difference between f of 5 and f of 6?	44
3.2	What is the quantity 6 minus one-sixth divided by the quantity 6 plus one-sixth?	35/37
3.3	What is the area of a square whose side length is the hypotenuse of an isosceles right triangle with leg 2?	8
3.4	Given that the square root of 3 is approximately 1 point 7, 3, 2, [PAUSE], what is the square root of one-third expressed as a decimal, to the nearest hundredth?	.58
PERSON 4		
4.1	What is the area of a square with a diagonal of length 6?	18
4.2	Express as a mixed number: the square root of three and one-sixteenth.	$1\frac{3}{4}$ [or] One and three fourths
4.3	The ratio of boys to girls on a field trip is 4 to 3, if 36 boys were on the trip, how many students total participated?	63
4.4	Form all permutations of the digits, 1, 2 and 3. What is the sum of all of these permutations?	1332

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COLLEGE KNOWLEDGE BOWL ROUND #1 - SET A

#	Problem	Answer
1	Evaluate as a decimal: twenty-eight point three minus two point eight three.	25.47
2	What is the name for the point at which the medians of a triangle intersect?	Centroid
3	What is the area, in square centimeters, of a right triangle with legs measuring 4 and 9 cm?	18 [cm^2]
4	If a plus b equals 2 and a squared plus b squared is 3, what is the value of a times b?	1/2
5	What is the prime factorization of 120 in exponential form?	2^3*3*5 or 2 cubed times 3 times 5
6	In how many ways can the letters in the word TENNESSEE be arranged?	3780
7	What is the mean of the data set {1, 4, 5, 5, 25}?	8
	Extra Problem – Only if Needed	
8	Twenty-two more than my code number is six less than fifty-seven. What is my code number?	29

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COLLEGE KNOWLEDGE BOWL ROUND #2- SET A

#	Problem	Answer
1	Evaluate as a mixed number: four and one-sixth times four and one-fifth.	17 and 1/2
2	What is the final result when ninety-six is divided by four and this result is then reduced by nine?	15
3	What is the area, in square centimeters, of a 150 degree sector of a circle with a radius measuring 6 centimeters?	15 pi [cm^2]
4	What is the coefficient of the c to the fourth in the expansion of the quantity 3 minus 2 c raised to the sixth power when like terms are combined?	2160
5	Express the base 10 number 9 8 7 in base 5.	$12422_{[5]}$
6	A conference table at the center of a round room has eight seats uniformly arranged around it. In how many distinguishable ways can three of the chairs be painted blue?	7
7	What is the sum of the terms of an infinite geometric sequence with a first term of eighteen and a common ratio of one-third?	27
	Extra Problem – Only if Needed	
8	How many diagonals can be drawn in a regular 15-gon?	90 [diagonals]

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COLLEGE KNOWLEDGE BOWL ROUND #3- SET A

#	Problem	Answer
1	What percent of 150 is 96?	64 [%]
2	What is the smallest positive integer divisible by the integers from 1 to 10?	2520
3	What is the surface area, in square centimeters, of a square pyramid with congruent faces and each edge measuring 2 cm?	4 + 4 root 3 [cm^2]
4	If I invest $30,000 in an account with 3% annual interest compounded annually, how much money will be in the account after two years?	[$] 31,827
5	How many positive integer factors does 350 have?	12
6	In how many ways can four different CDs and two different DVDs be arranged next to one another on a shelf if the CDs must all be together but the DVDs do not need to be together?	144
7	What is the eighth term of the recursive sequence defined by z sub 1 equals 3 and z sub n equals 2 times z sub n minus 1 [PAUSE] minus 2?	130
	Extra Problem – Only if Needed	
8	Evaluate: the quantity 1 plus i raised to the eighth power.	16

#3

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COLLEGE KNOWLEDGE BOWL ROUND #1 - SET B

#	Problem	Answer
1	How many minutes are there in three weeks?	30240 [min]
2	At Unusual Farm, they raise ostriches and rattlesnakes in the same pen. If there are 32 heads and 38 legs in the pen, how many rattlesnakes are there?	13 [rattlesnakes]
3	What is the total surface area, in square centimeters, of a right circular cone with a height of 5 cm and a base radius of 12 cm?	300pi [cm^2]
4	If u is inversely proportional to v and u equals 144 when v is 18, what value of v will correspond to u equals 96?	27
5	What is the least common multiple of 30 and 48?	240
6	In a survey of 528 math competitions, 418 were multiple choice and 356 covered some probability. If there were 71 competitions that met neither of these criteria, how many met both?	317
7	How many positive three-digit multiples of 37 contain at least two different digits?	16
	Extra Problem – Only if Needed	
8	A trusted friend flips five coins behind a screen and says that there is at least one head and at least one tail. What is the probability that there are exactly three heads?	1/3

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COLLEGE KNOWLEDGE BOWL ROUND #2- SET B

#	Problem	Answer
1	Simplify by rationalizing the denominator: 39 divided by the quantity 4 minus the square root of 3.	12+3 root3
2	What is the smallest positive three-digit number that is 495 less than the positive three-digit number formed when its digits are reversed?	106
3	What is the perimeter, in centimeters, of a right triangle with legs measuring 12 and 30 cm?	42+6root29 [cm]
4	If f of g equals 3 g squared minus 5 times 2 to the g power, evaluate f of 3.	-13
5	How many positive five-digit palindromes are there?	900
6	When four fair coins are flipped, what is the probability that exactly three show heads?	1/4
7	What is the sum of the positive multiples of 5 less than or equal to 100?	1050
	Extra Problem – Only if Needed	
8	What is the remainder when 849 is divided by 17?	16

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COLLEGE KNOWLEDGE BOWL ROUND #3- SET B

#	Problem	Answer
1	What is the sum of the number of sides in a heptagon, the number of faces on a tetrahedron, and the number of months in a year?	23
2	How many kilometers will an airplane travel in six hours at a constant speed of three-hundred kilometers per hour?	1800 [km]
3	What is the shortest possible integer number of centimeters in the length of the third side of a triangle with two sides each measuring 9 and 24 centimeters?	16 [cm]
4	What are the coordinates, in the form x comma y, of the center of the circle with equation x squared plus y squared minus 4 x plus 8 y equals 48?	(2,-4)
5	Express the sum of the base 8 numbers 374_8 and 613_8 in base 8.	$1207_{[8]}$
6	A ball rebounds to $\frac{1}{3}$ of the height of its drop. If this ball is dropped from the roof of a 24 foot building, what is the total distance traveled by the ball when it comes to a rest?	48 [feet]
7	What is the sum of the positive integer factors of 72?	195
	Extra Problem – Only if Needed	
8	In a triangle with sides measuring 4, 5, and 7 centimeters, how many centimeters long is the altitude to the 4 centimeter side?	2 root 6 [cm]

"Math is Cool" Championships – 2009-10

October 23, 2009

Final Score: KEY

First Score

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

High School Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0		Answer	1 or 0	1 or 0
1	9			21	4π or 4 pi		
2	196π [cm^2]			22	1		
3	1458 [bunnies]			23	320 [ways]		
4	247 [pages]			24	35		
5	$4\sqrt{13}$ [cm]			25	15		
6	16			26	15π		
7	74 [tickets]			27	12,960		
8	25			28	6		
9	12 [edges]			29	18 [chickens]		
10	True			30	13/7		
11	19/5			31	2/3		
12	12			32	60		
13	7			33	7/125		
14	84			34	$(-\infty,-2)\cup\left(\frac{3}{2},2\right)$		
15	1/221			35	(1,1,0)		
16	385			36	1		
17	9			37	175/396		
18	x + 3y = 5			38	20 [ways]		
19	75 [mph]			39	2		
20	144			40	6,8 [in any order]		

"Math is Cool" Championships – 2009-10

11th & 12th Grade – October 23, 2009

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Final Score:

KEY

First Score

(out of 18)

INDIVIDUAL MULTIPLE CHOICE - 15 minutes

This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. ***Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet.*** *No talking during the test.*

DO NOT WRITE IN SHADED REGIONS

Answer		-1, 0 or 2	-1, 0 or 2
1	A		
2	C		
3	B		
4	C		
5	D		
6	E		
7	B		
8	D		
9	B		

"Math is Cool" Championships – 2009-10

11th & 12th Grade – October 23, 2009

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Final Score:

KEY

First Score

(out of 20)

Team Contest – Score Sheet

TEAM TEST - 15 minutes

When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as ***2 or 0****. Record all answers on the colored answer sheet.*

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	145 [degrees]		
2	95 [hippos]		
3	2520		
4	88		
5	25		
6	8 [cm]		
7	TAEHT		
8	$1332101_{[6]}$		
9	250		
10	$m/3 - 2n$ or $\frac{m-6n}{3}$		

"Math is Cool" Championships – 2009-10

9th/10th & 11th/12th Grade – October 23, 2009

Final Score:
KEY

First Score

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

PRESSURE ROUND - 10 minutes

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

Pressure Round Answers

Answer	
1	19/5
2	56
3	9
4	28 [games]
5	40/3 [min]