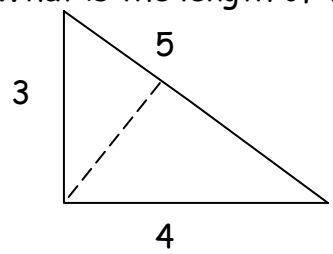
"Math is Cool" Masters – 2005-06

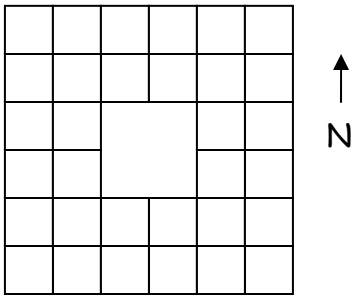
11th & 12th Grade – November 19, 2005
Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

1	Evaluate: $(3+4)\cdot 3+4\cdot(3+4)$.
2	What is the slope of a line perpendicular to $y=6x+12$?
3	Evaluate the function $r(t)=\frac{1}{t^2}$ at $t=7$.
4	If a square is inscribed in a circle that is inscribed in a square with area 4, what is the area of the inner square?
5	Evaluate and write in standard form: (3-2*i*)(3+2*i*)where $i=\sqrt{-1}$.
6	What is the perimeter of a dodecagon if the arithmetic mean of its edge lengths is two?
7	If I pull two cards at once from a standard 52-card deck, what is the probability that they are of the same rank (e.g. two 5's)?
8	Simplify as much as possible, i^{175} where $i=\sqrt{-1}$.
9	What is the sum of x and y in the solution to the following system of equations? 3x – 3y = 13 x + y = 6
10	Factor completely as the product of a linear factor and a quadratic factor with integer coefficients: $x^3 - 27$
11	What is the length of the altitude shown as a dotted line below? 
12	Samson agrees that if his students raise $800 in a fundraiser they can cut his hair. They can sell chocolate-scented paper for $5 a roll and peanut-butter-scented paper for $8 a roll, but they only have 50 peanut-butter rolls. If they sell all of their peanut-butter rolls, how many chocolate rolls must they sell to reach their goal?

13	If $f(x)$ is an even function, $g(x)$ is an odd function, and $h(x) = \|x\|$, is $h(f(x)) \cdot g(h(x))$ necessarily odd, necessarily even, or neither of these?
14	What is the sum of the roots of $x^2 + 8x + 15 = 0$?
15	What is the sum of the infinite geometric series $401 + \frac{1604}{5} + \frac{6416}{25} + \cdots$?
16	Skip and Chuck stop by a river. When Chuck skips a rock, it hits the water 20 feet away, skips half that far and then half of that skip. How many feet farther does the rock that Skip chucks 40 feet (which stops right away) go than the rock that Chuck skipped?
17	Paul has four nickels, two dimes, and two one-dollar coins. How many different quantities of money can he make by combining one or more of them?
18	What is the sum of the elements in the top four rows of Pascal's triangle?
19	On a full tank, the plane can fly to a point 750 miles away, turn around, and fly back to where it started. If Jon has a sixth of a tank left, how many miles can he go to land?
20	Find the sum of the solutions for x: $\sqrt{2x-7} = 2 + \sqrt{x-7}$
21	Determine k so that $kx^2 - 12x + 4 = 0$ has one rational solution.
22	Evaluate: $e^{(4 \ln 4)}$
23	Alan wants to learn how many of his town's 1000 pigeons are albino. He watches until he has seen one-hundred distinct pigeons, of which twenty were albino, so he estimates that there are 200 albinos in all. What is the largest possible positive difference between his estimate and the true number of albino pigeons?
24	Find the sum of all real solutions to the following equation: $\log_3 x + \log_3 (x-6) = 3$
25	Evaluate the determinant: $\begin{vmatrix} 1 & 3 & 27 & 7 \\ 2 & 9 & 81 & 9 \\ 0 & 0 & 0 & 2 \\ 7 & 0 & 2 & \pi \end{vmatrix}$
26	At what value of x does $y = \frac{x^3 + 4x^2 + 4x}{x^3 - 4x}$ have a vertical asymptote?
27	What is the sum of the solutions to the following equation where $0 \le x \le 2\pi$: $1 + 4 \sin x = 4 \cos^2 x$
28	Evaluate: $\lambda^4 + 7.5\lambda^3 - 3\lambda^2 - 0.5\lambda + 20$ when $\lambda = 2$.
29	Evaluate and express in base eight (octal): $00110010011110001 1_2$ $+\underline{10101100011110001 0_2}$

Challenge Questions

30	What is the units digit when x^{16} is expressed in base 16, if x is 21_{10}?
31	Pick a point *D* on the interior of a regular hecagon (100-sided polygon) so that more diagonals pass through it than through any other point. How many diagonals pass through *D*?
32	What is the sum of the coefficients in the expansion of $(x-y)^{12}$?
33	A group of students decides to pool their money to buy a large box of candy. If there had been one fewer person in the group, each member would have paid seven cents more, while if there had been two fewer people in the group each member would have paid 15 cents more. How much did the box of candy cost, in **dollars?** [Express answer as a decimal rounded to the nearest hundredth (cent).]
34	If $a \oplus b = a\sqrt{b\sqrt{a \oplus b}}$, evaluate $2 \oplus 3$.
35	Daniel wants to walk from the northwestern most corner to the southeastern most corner. If he never walks west and never walks north and can only follow the gridlines shown, how many different routes can he take? 
36	Sarah, on a motorcycle, starts at the back of a 2 km train as its front enters a 4 km tunnel. Both travel at constant speed and she exits the tunnel just as the train is entirely in the tunnel. When the front of the train emerges from the tunnel, Sarah turns (instantly) and heads back toward the train. How many **meters** from the tunnel does Sarah meet the front of the train?
37	Robert has three fair coins and Dunkirke has two. Each round, they both flip all of their coins and give the coins showing heads to the other player to flip on the next turn. If either player gets all five coins, he wins. What are the odds in favor of Robert's eventual victory? [Express answer in the form of a:b.]
38	Jim arrives at the library at a random moment between 1:00 and 1:30, stays for half an hour, leaves the library for a random time between thirty and sixty minutes, then returns to the library for an hour. If Jill is at the library from 1:45 to 2:15, what is the probability that they are never in the library at the same moment?
39	Evaluate, in degrees, in the interval [-90°, 90°]: $\sin^{-1}(\cos 110^\circ)$
40	What is the shortest word equivalent to *GAABG*, if letters cannot be rearranged but can be replaced according to the rules *AB=Y, AY=G, YG=B* ? (example: *GG=AYG=AB=Y*)

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11th & 12th Grade – November 19, 2005

Individual Multiple Choice Contest

Record only a letter as your answer on the colored sheet.

1	If $x - y = 3$ and $x^3 - y^3 = \frac{1}{3}$, what is the value of $x^2 + y^2$? A) 1 B) $\frac{83}{27}$ C) $-\frac{9}{4}$ D) $\frac{9}{8}$ E) -1
2	How many values of r, $0 \le r \le 5\pi$, satisfy $\frac{2\sec^2 r}{\csc r} - \tan^2 r = 1$? A) 3 B) 4 C) 5 D) 6 E) 7
3	What is the total surface area, in square centimeters, of a cylinder with a base diameter measuring 16 cm and a height measuring 9 cm? A) 272π B) 800π C) 584π D) 416π E) 200π
4	If $\log(2) \approx .301$, which of the following is closest to $\log(5000)$? A) 3.010 B) 3.301 C) 3.602 D) 3.699 E) 3.903
5	What is the product of the base five numbers 43_5 and 234_5, expressed in base five? A) 4332_5 B) 22322_5 C) 4444_5 D) 21012_5 E) 4312_5
6	When four marbles are drawn from a bag containing three red and three green marbles, what is the probability that you draw two of each color? A) $\frac{13}{15}$ B) $\frac{4}{5}$ C) $\frac{11}{15}$ D) $\frac{2}{3}$ E) $\frac{3}{5}$

7	If $a_{10} = 10$ and $a_n = \frac{a_{n-1}}{2} + 4$, what is the value of a_7? A) $\frac{257}{32}$ B) $\frac{33}{4}$ C) 10 D) 24 E) 64
8	My coin collection contains only half-dollars, quarters, and dimes, and has a total value of \$6.00. If there are 30 coins and the value of my dimes is half the total value of my quarters and half-dollars, how many quarters do I have? A) 8 B) 7 C) 6 D) 5 E) 4
9	What is the length, in centimeters, of the median to the shortest side of a triangle with sides measuring 8 cm, 10 cm, and 16 cm? A) $9\sqrt{2}$ B) $\frac{15\sqrt{3}}{2}$ C) 12 D) $8\sqrt{3}$ E) $\frac{27}{2}$

"Math is Cool" Masters – 2005-06

11th & 12th Grade – November 19, 2005
Team Contest

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1	If $\pi \le x \le \frac{3\pi}{2}$, $\sin x = -\frac{1}{2}$, $\frac{3\pi}{2} \le y \le 2\pi$, and $\cos y = \frac{1}{7}$, evaluate cos(x + y).
2	For how many real values of x will $\left(x^2 - 5x + 4\right)^{\left(x^2 - 9x - 20\right)} = 145$?
3	How many distinguishable ways can each face of a cube be painted one of six different colors if no two faces may be painted the same color?
4	The foci of the graph of $7x^2 - 5y^2 + 84x + 40y = -67$ are (a, b) and (c, d). Evaluate $a + b + c + d$.
5	When the digits of a positive three-digit base eight number are reversed to make an equivalent base five number, what is the sum of its digits?
6	What is the difference between the largest and smallest real values of r that satisfy $\left(r^2 + 6r - 18\right)^2 + 25\left(r^2 + 6r - 18\right) + 46 = 0$?
7	The four lines 3x + y = -2, ax – 2y = 59, 13x + by = -3, and 10x + 3y = -1 all intersect at the point (c, d). Evaluate a + b + c + d.
8	If $\log_a 336 = b$ and $\log_a 21 = c$, express $\log_4 a$ in terms of b and c.
9	What is the sum of all positive two-digit integers that contain no digits other than 2, 3, 5, and 7?
10	In triangle ABC with a = 4, b = 5, and c = 6, points D and E are chosen on $\overline{AB}$ so that $\overline{CD}$ and $\overline{CE}$ each divide angle C into two angles, one of which is three times the other. What is the length of $\overline{DE}$?

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Pressure Round Contest

1	Find the sum of the reciprocals of the solutions of the following equation: $2x^2 - 14x + 21 = 0$.
2	The first 10 positive integers are separated into two groups. The numbers in the first group are multiplied to give the product P_1 and the numbers in the second group are multiplied to give the product P_2. Name the letters of all the following fractions that are possible values for $\frac{P_1}{P_2}$: (A) $\frac{27}{28}$; (B) $\frac{63}{64}$; (C) $\frac{7}{9}$; (D) $\frac{25}{28}$; (E) $\frac{5}{7}$; (F) $\frac{7}{8}$
3	Two cards are drawn from a standard deck without replacement. What is the probability that at least one is a king and at least one is a heart?
4	A cube has p distinct pairs of parallel faces and q distinct pairs of parallel edges. Find p + q.
5	The 2-digit positive integer n has units digit $\underline{u}$ and tens digit $\underline{t}$. Find the sum of all values of n such that the 2-digit positive integer $\underline{ut}$ (with units digit $\underline{t}$ and tens digit $\underline{u}$) is equal to 2n – 1.

"Math is Cool" Masters – 2005-06

11th & 12th Grade – November 19, 2005
Mental Math Contest

Express all answers as reduced fractions in terms of radicals and π where applicable, unless stated otherwise.

PERSON 1		
1.1	What is the product of thirty-three and thirty-seven?	1221
1.2	What is the area, in square centimeters, of the largest square that can be placed in a circle with an area of four pi square centimeters?	8 [cm^2]
1.3	What is the probability that I get fewer than four heads when I toss a fair coin four times?	15/16
1.4	What is the remainder when x cubed plus x squared plus x plus one is divided by the quantity x minus one?	4
PERSON 2		
2.1	How many positive integers are factors of sixty?	12
2.2	If a diagonal is drawn in a regular pentagon, what is the measure, in degrees, of the acute angles in the triangle that is created?	36 [°]
2.3	How many ways are there to place two distinguishable math books and three distinguishable poetry books on a shelf if each subject must stay together?	24 [ways]
2.4	If the tangent of angle R is five-twelfths and angle R is in the third quadrant, what is the cosine of R?	-12/13
PERSON 3		
3.1	What is the sum of the positive integer factors of two-hundred fifty-six?	511
3.2	What is the surface area, in square centimeters, of a regular tetrahedron with edges measuring four centimeters?	$16\sqrt{3}$ [cm^3]
3.3	If I am dealt two cards from a standard fifty-two card deck, what is the probability that I get two cards of the same rank, for example two kings?	1/17
3.4	If the cosine of g is three-fifths, what is the cosine of two g?	-7/25
PERSON 4		
4.1	In a magic square, each row, column and diagonal adds up to the same sum. In a five by five magic square using the numbers one to twenty-five, what is that sum?	65
4.2	Which of these is NOT a congruence theorem for triangles: angle-side-angle, side-angle-side, side-side-angle, or side-angle-angle?	side-side-angle [or SSA or ASS]
4.3	If B is a subset of A, the probability of A is two-thirds, and the probability of B is one-fifth, what is the probability of the union of A complement and B?	8/15
4.4	What is the period of the function f of x equals three times the cosine of the quantity four x plus two?	$\pi/2$

"Math is Cool" Masters – 2005-06

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COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	What is the sum of the three largest prime numbers less than fifty?	131
2	What values of x satisfy x squared plus fourteen x plus forty-eight equals zero?	-6, -8
3	If the common log of four is x, what is the common log of five?	1-(x/2) [or 1 – (1/2)x]
4	Express the base ten number five-four in base eight.	$66_{[8]}$
5	When two fair, six-sided dice are rolled, what is the probability that the product of the numbers shown is twelve?	1/9
6	What is the sum of the first four terms of a geometric sequence with first term six and common ratio three?	240
7	In a proposal to modify US currency, four "skronks" would equal three "leegs", and two "skronks" plus one "leeg" would equal one dollar and ninety cents. How many cents would a "leeg" be worth under this new system?	76 [cents]
	Extra Problem – Only if Needed	
8	A rectangle is inscribed in a circle with an area of one-hundred pi. If the ratio of the lengths of the sides of the rectangle is four to three, what is the area of the rectangle?	192 [un^2]

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COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	What is the square of half the sum of thirty-two and ten?	441
2	The line y equals two x plus q intersects the line five x plus three y equals seven at the point negative ten comma nineteen. What is the value of q?	39
3	What is the length, in centimeters, of an edge of the base of a right square pyramid with a height measuring six centimeters and a volume measuring ninety-eight cubic centimeters?	7 [cm]
4	What is the logarithm in base twenty-seven of the quantity one over two-hundred forty-three?	-5/3
5	What is the greatest common factor of three-hundred fifteen and five-hundred twenty-five?	105
6	When three cards are drawn from a standard fifty-two card deck, what is the probability that they contain exactly two cards of the same rank, for example two jacks?	72/425
7	If eighteen, f, and one-hundred twenty-eight form a geometric sequence, what is the largest possible value of f?	48
	Extra Problem – Only if Needed	
8	What is the surface area, in square centimeters, of a right rectangular prism with edges measuring eight centimeters, four centimeters, and twelve centimeters?	352 [cm^2]

"Math is Cool" Masters – 2005-06

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COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	What is the sum of the reciprocals of the two largest single digit positive integers?	17/72
2	If m is an angle in the interval of three halves pi to two pi and the cosine of m is four-fifths, what is the sine of two m?	-24/25
3	Consider a red triangle with sides measuring eight, eight, and eleven. If all medians, angle bisectors, and altitudes are drawn in blue, how many distinct blue line segments with endpoints on the triangle have been drawn inside the triangle?	7 [line seg]
4	What is the area, in square centimeters, of an ellipse with a major axis measuring ten centimeters and having foci six centimeters apart?	20π [cm^2]
5	How many positive integers less than fifteen-thousand eight-hundred forty-two leave a remainder of nineteen when divided by five-thousand three-hundred forty-eight?	3 [int]
6	In a certain game, a turn consists of flipping a coin and then rolling a die. If the coin shows heads, you roll a standard six-sided die, while if the coin shows tails, you roll a fair four-sided die numbered one to four. Your score for the turn is the number shown on the die you roll. What is the expected value of a player's score for one turn?	3
7	What is the sum of the perfect squares less than one-hundred fifty?	650
	Extra Problem – Only if Needed	
8	What value of u satisfies the equation four u minus nineteen equals forty-nine?	17

"Math is Cool" Masters – 2005-06

11th & 12th Grade – November 19, 2005

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score: KEY

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	49		
2	-1/6		
3	1/49		
4	2 [un^2]		
5	13		
6	24 [un]		
7	1/17		
8	$-i$		
9	6		
10	$(x-3)(x^2 + 3x + 9)$		
11	12/5		
12	80 [rolls]		
13	Even		
14	-8		
15	2005		
16	5 [ft]		
17	26 [quant]		
18	15		
19	250 [miles]		
20	24		

	Answer	1 or 0	1 or 0
21	9		
22	256		
23	720		
24	9		
25	12		
26	2		
27	π		
28	83		
29	$675705_{[8]}$		
30	1		
31	50 [diag]		
32	0		
33	[$]16.80		
34	$2\sqrt[3]{18}$		
35	524 [routes]		
36	1500 [meters]		
37	1:1		
38	3/8		
39	-20 [°]		
40	B		

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11th & 12th Grade – November 19, 2005

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

(out of 18)

STUDENT NAME_____

Individual Multiple Choice Contest – Score Sheet

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	B		
2	D		
3	A		
4	D		
5	B		
6	E		
7	D		
8	E		
9	A		

"Math is Cool" Masters – 2005-06

11th & 12th Grade – November 19, 2005

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

(out of 20)

STUDENT NAME_____

Team Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	$-\frac{5\sqrt{3}}{14}$		
2	4		
3	30 [ways]		
4	-4		
5	7		
6	10		
7	-3		
8	$\frac{2}{b-c}$		
9	748		
10	28/11		

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School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

STUDENT NAME_____

Pressure Round Answers

	Answer
1	2/3
2	B,C,D
3	29/442
4	21
5	37