

"Math is Cool" Masters - 2005-06

Sponsored by: Inflation System Inc

9th & 10th Grade - November 19, 2005

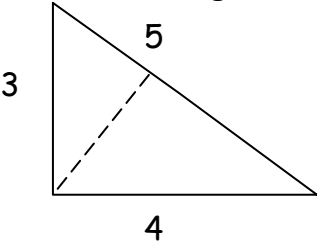
Individual Contest

Express all answers as reduced fractions unless stated otherwise.

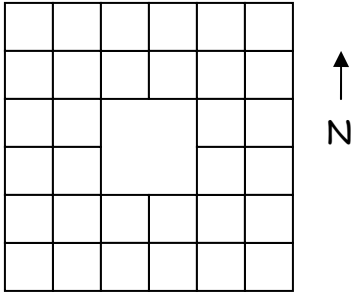
Leave answers in terms of π where applicable.

Do not round any answers unless stated otherwise.

Record all answers on the colored cover sheet.

1	Evaluate: $(3 + 4) \cdot 3 + 4 \cdot (3 + 4)$.
2	Evaluate: $5^4 - (2^4 - 3^2)^3$
3	What is the slope of a line perpendicular to $y = 6x + 12$?
4	In how many distinguishable ways can the letters in "rabbit" be arranged?
5	Evaluate the function $r(t) = \frac{1}{t^2}$ at $t = 7$.
6	What is the perimeter of a dodecagon if the arithmetic mean of its edge lengths is two?
7	If I pull two cards at once from a standard 52-card deck, what is the probability that they are of the same rank (e.g. two 5's)?
8	What is the sum of x and y in the solution to the following system of equations? $3x - 3y = 13$ $x + y = 6$
9	What is the slope of the line containing the y -intercept of the line $3x + 2y = 24$ and the x -intercept of the line $y = 3x - 12$?
10	What is the greatest common factor of $x^2 - x - 6$ and $x^2 + x - 6$?
11	What is the length of the altitude shown as a dotted line below? 
12	Samson agrees that if his students raise \$800 in a fundraiser they can cut his hair. They can sell chocolate-scented paper for \$5 a roll and peanut-butter-scented paper for \$8 a roll, but they only have 50 peanut-butter rolls. If they sell all of their peanut-butter rolls, how many chocolate rolls must they sell to reach their goal?

13	Factor completely as the product of a linear factor and a quadratic factor with integer coefficients: $x^3 - 27$
14	What is the base ten numeral equivalent to the base six numeral 2354_6 ?
15	If a square is inscribed in a circle that is inscribed in a square with area 4, what is the area of the inner square?
16	What is the next term of the following sequence? 31, 18, 9, 4, 3, 6, 13, _____
17	What is the sum of the roots of $x^2 + 8x + 15 = 0$?
18	Evaluate and write in standard form: $(3-2i)(3+2i)$ where $i = \sqrt{-1}$.
19	Simplify as much as possible, i^{175} where $i = \sqrt{-1}$.
20	What is the sum of the infinite geometric series $401 + \frac{1604}{5} + \frac{6416}{25} + \dots$?
21	Skip and Chuck stop by a river. When Chuck skips a rock, it hits the water 20 feet away, skips half that far and then half of that skip. How many feet farther does the rock that Skip chucks 40 feet (which stops right away) go than the rock that Chuck skipped?
22	What is the area, in square centimeters, of a triangle with sides measuring 14, 20, and 14 cm?
23	What are the coordinates, in the form (x,y) , of the vertex of the parabola with equation $y = 4x^2 - 28x + 81$?
24	Paul has four nickels, two dimes, and two one-dollar coins. How many different quantities of money can he make by combining one or more of them?
25	What is the sum of the elements in the top four rows of Pascal's triangle?
26	On a full tank, the plane can fly to a point 750 miles away, turn around, and fly back to where it started. If Jon has a sixth of a tank left, how many miles can he go to land?
27	Find the sum of the solutions for x : $\sqrt{2x - 7} = 2 + \sqrt{x - 7}$
28	Determine k so that $kx^2 - 12x + 4 = 0$ has one rational solution.
29	Alan wants to learn how many of his town's 1000 pigeons are albino. He watches until he has seen one-hundred distinct pigeons, of which twenty were albino, so he estimates that there are 200 albinos in all. What is the largest possible positive difference between his estimate and the true number of albino pigeons?
<h2>Challenge Questions</h2>	
30	Jovan is four years older than Azja, twice Azja's age is one less than the sum of Jovan and Seoni's ages, and in three years twice Jovan's age will be equal to three times Seoni's age. What is the sum of their current ages?

31	A group of students decides to pool their money to buy a large box of candy. If there had been one fewer person in the group, each member would have paid seven cents more, while if there had been two fewer people in the group each member would have paid 15 cents more. How much did the box of candy cost, in dollars ? [Express answer as a decimal rounded to the nearest hundredth (cent).]																
32	What is the sum of the coefficients in the expansion of $(x-y)^{12}$?																
33	Pick a point D on the interior of a regular hecagon (100-sided polygon) so that more diagonals pass through it than through any other point. How many diagonals pass through D ?																
34	Sarah, on a motorcycle, starts at the back of a 2 km train as its front enters a 4 km tunnel. Both travel at constant speed and she exits the tunnel just as the train is entirely in the tunnel. When the front of the train emerges from the tunnel, Sarah turns (instantly) and heads back toward the train. How many meters from the tunnel does Sarah meet the front of the train?																
35	Evaluate the determinant: <table style="display: inline-table; vertical-align: middle;"> <tr><td style="border-right: 1px solid black; padding: 0 5px;">1</td><td style="padding: 0 5px;">3</td><td style="padding: 0 5px;">27</td><td style="padding: 0 5px;">7</td></tr> <tr><td style="border-right: 1px solid black; padding: 0 5px;">2</td><td style="padding: 0 5px;">9</td><td style="padding: 0 5px;">81</td><td style="padding: 0 5px;">9</td></tr> <tr><td style="border-right: 1px solid black; padding: 0 5px;">0</td><td style="padding: 0 5px;">0</td><td style="padding: 0 5px;">0</td><td style="padding: 0 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 0 5px;">7</td><td style="padding: 0 5px;">0</td><td style="padding: 0 5px;">2</td><td style="padding: 0 5px;">π</td></tr> </table>	1	3	27	7	2	9	81	9	0	0	0	2	7	0	2	π
1	3	27	7														
2	9	81	9														
0	0	0	2														
7	0	2	π														
36	Tell how many ordered pairs of integers (x,y) are solutions of the system of inequalities: $ x + 3 < 2$ $ y < x + 3 + 2$																
37	Daniel wants to walk from the northwestern most corner to the southeastern most corner. If he never walks west and never walks north and can only follow the gridlines shown, how many different routes can he take? <div style="text-align: center; margin-top: 10px;">  </div>																
38	If $a \oplus b = a\sqrt{b}\sqrt{a \oplus b}$, evaluate $2 \oplus 3$.																
39	Robert has three fair coins and Dunkirke has two. Each round, they both flip all of their coins and give the coins showing heads to the other player to flip on the next turn. If either player gets all five coins, he wins. What are the odds in favor of Robert's eventual victory? [Express answer in the form of a:b.]																
40	What is the shortest word equivalent to GAABG, if letters cannot be rearranged but can be replaced according to the rules $AB=Y$, $AY=G$, $YG=B$? (example: $GG=AYG=AB=Y$)																

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Individual Multiple Choice Contest

Record only a letter as your answer on the colored sheet.

1	Evaluate: $23^3 - 19^3$ A) 5204 B) 5296 C) 5308 D) 5428 E) 5566
2	Which of the following is an equation of the line perpendicular to $y = 3x - 7$ and passing through the point $(4, -2)$? A) $x - 3y = 10$ B) $y = \frac{1}{3}x + \frac{14}{3}$ C) $y = -\frac{1}{3}x + \frac{14}{3}$ D) $x + 3y = -2$ E) $y = -\frac{1}{3}x + \frac{10}{3}$
3	What is the total surface area, in square centimeters, of a cylinder with a base diameter measuring 16 cm and a height measuring 9 cm? A) 272π B) 800π C) 584π D) 416π E) 200π
4	If $\log(2) \approx .301$, which of the following is closest to $\log(5000)$? A) 3.010 B) 3.301 C) 3.602 D) 3.699 E) 3.903
5	What is the product of the base five numbers 43_5 and 234_5 , expressed in base five? A) 4332_5 B) 22322_5 C) 4444_5 D) 21012_5 E) 4312_5

6	<p>When 16 coins are flipped, what is the probability that exactly one of them is heads?</p> <p>A) $\frac{1}{16^{16}}$ B) $\frac{1}{4096}$ C) $\frac{15}{256}$ D) $\frac{15}{2^{16}}$ E) $\frac{1}{16}$</p>
7	<p>If $a_{10} = 10$ and $a_n = \frac{a_{n-1}}{2} + 4$, what is the value of a_7?</p> <p>A) $\frac{257}{32}$ B) $\frac{33}{4}$ C) 10 D) 24 E) 64</p>
8	<p>My coin collection contains only half-dollars, quarters, and dimes, and has a total value of \$6.00. If there are 30 coins and the value of my dimes is half the total value of my quarters and half-dollars, how many quarters do I have?</p> <p>A) 8 B) 7 C) 6 D) 5 E) 4</p>
9	<p>What is the length, in centimeters, of the median to the shortest side of a triangle with sides measuring 8 cm, 10 cm, and 16 cm?</p> <p>A) $9\sqrt{2}$ B) $\frac{15\sqrt{3}}{2}$ C) 12 D) $8\sqrt{3}$ E) $\frac{27}{2}$</p>

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Team Contest

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1	What is the length, in centimeters, of a common internal tangent of two circles of radii 4 cm and 6 cm whose centers are separated by a distance of 20 cm?
2	A throat lozenge is formed by attaching the circular base of a hemisphere to each congruent circular base of a right circular cylinder. If the cylinder has a diameter of 4 mm and a length of 5 mm, what is the surface area of the lozenge, in square millimeters?
3	How many distinguishable ways can each face of a cube be painted one of six different colors if no two faces may be painted the same color?
4	The foci of the graph of $7x^2 - 5y^2 + 84x + 40y = -67$ are (a, b) and (c, d) . Evaluate $a + b + c + d$.
5	What is the probability that when four dice are rolled, the largest is a five (perhaps on more than one die)?
6	What is the difference between the largest and smallest real values of r that satisfy $(r^2 + 6r - 18)^2 + 25(r^2 + 6r - 18) + 46 = 0$?
7	The four lines $3x + y = -2$, $ax - 2y = 59$, $13x + by = -3$, and $10x + 3y = -1$ all intersect at the point (c, d) . Evaluate $a + b + c + d$.
8	If $\log_a 336 = b$ and $\log_a 21 = c$, express $\log_4 a$ in terms of b and c .
9	What is the sum of all positive two-digit integers that contain no digits other than 2, 3, 5, and 7?
10	If $a_1 = 1$ and $a_n = a_{n-1}^2 - 4a_{n-1}$ for $n > 1$, evaluate a_4 .

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Pressure Round Contest

1	In simplest radical form, find the geometric mean of the solutions of the following equation: $x^2 - 19x + 27 = 0$.
2	What is the smallest factor of $10!$ that is not also a factor of $8!$?
3	British coins in current usage include the 1p, 2p, 5p, 10p, and 20p coins. In how many ways can you make exactly 20p (pence) using these coins?
4	Two parallel lines are both tangent to the same circle. How many points are equidistant from both lines and from the circle?
5	How many positive integers less than or equal to 2005 are relatively prime to 2005?

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Mental Math Contest

Express all answers as reduced fractions in terms of radicals and π where applicable, unless stated otherwise.

PERSON 1		
1.1	What is the product of thirty-three and thirty-seven?	1221
1.2	What is the area, in square centimeters, of the largest square that can be placed in a circle with an area of four pi square centimeters?	8 [cm ²]
1.3	What is the probability that I get fewer than four heads when I toss a fair coin four times?	15/16
1.4	If two raised to the quantity two x plus one equals one-half, what is x?	-1
PERSON 2		
2.1	How many positive integers are factors of sixty?	12
2.2	If a diagonal is drawn in a regular pentagon, what is the measure, in degrees, of the acute angles in the triangle that is created?	36 [°]
2.3	How many ways are there to place two distinguishable math books and three distinguishable poetry books on a shelf if each subject must stay together?	24 [ways]
2.4	What is the largest integer n for which n squared minus four n minus twelve is less than zero?	5
PERSON 3		
3.1	What is the sum of the positive integer factors of two-hundred fifty-six?	511
3.2	What is the surface area, in square centimeters, of a regular tetrahedron with edges measuring four centimeters?	16 $\sqrt{3}$ [cm ²]
3.3	If I am dealt two cards from a standard fifty-two card deck, what is the probability that I get two cards of the same rank, for example two kings?	1/17
3.4	For how many integers x is it true that the absolute value of the quantity two x minus one is less than or equal to seven?	8
PERSON 4		
4.1	In a magic square, each row, column and diagonal adds up to the same sum. In a five by five magic square using the numbers one to twenty-five, what is that sum?	65
4.2	Which of these is NOT a congruence theorem for triangles: angle-side-angle, side-angle-side, side-side-angle, or side-angle-angle?	side-side-angle [or SSA or ASS]
4.3	If B is a subset of A, the probability of A is two-thirds, and the probability of B is one-fifth, what is the probability of the union of A complement and B?	8/15
4.4	What is the coefficient of the x-squared term when the fourth power of the quantity x plus two is expanded and like terms are combined?	24

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COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	What is the sum of the three largest prime numbers less than fifty?	131
2	What values of x satisfy x squared plus fourteen x plus forty-eight equals zero?	-6, -8
3	What is the surface area, in square centimeters, of the largest sphere that can be contained in a right rectangular prism with edges measuring fourteen centimeters, thirteen centimeters, and twelve centimeters?	144π [cm ²]
4	Express the base ten number five-four in base eight.	$66_{[8]}$
5	When two fair, six-sided dice are rolled, what is the probability that the product of the numbers shown is twelve?	$\frac{1}{9}$
6	What is the sum of the first four terms of a geometric sequence with first term six and common ratio three?	240
7	In a proposal to modify US currency, four "skronks" would equal three "leegs", and two "skronks" plus one "leeg" would equal one dollar and ninety cents. How many cents would a "leeg" be worth under this new system?	76 [cents]
	Extra Problem - Only if Needed	
8	A rectangle is inscribed in a circle with an area of one-hundred pi. If the ratio of the lengths of the sides of the rectangle is four to three, what is the area of the rectangle?	192 [un ²]

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COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	What is the square of half the sum of thirty-two and ten?	441
2	The line y equals two x plus q intersects the line five x plus three y equals seven at the point negative ten comma nineteen. What is the value of q ?	39
3	What is the length, in centimeters, of an edge of the base of a right square pyramid with a height measuring six centimeters and a volume measuring ninety-eight cubic centimeters?	7 [cm]
4	What is the logarithm in base twenty-seven of the quantity one over two-hundred forty-three?	$-5/3$
5	What is the greatest common factor of three-hundred fifteen and five-hundred twenty-five?	105
6	When three cards are drawn from a standard fifty-two card deck, what is the probability that they contain exactly two cards of the same rank, for example two jacks?	$72/425$
7	If eighteen, f , and one-hundred twenty-eight form a geometric sequence, what is the largest possible value of f ?	48
	Extra Problem - Only if Needed	
8	What is the surface area, in square centimeters, of a right rectangular prism with edges measuring eight centimeters, four centimeters, and twelve centimeters?	$352 \text{ [cm}^2\text{]}$

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COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	What is the sum of the reciprocals of the two largest single digit positive integers?	17/72
2	What is the sum of the coordinates, expressed as an ordered pair x comma y , of the point of intersection of the lines x minus three y equals seven and two x minus nine y equals two?	23
3	Consider a red triangle with sides measuring eight, eight, and eleven. If all medians, angle bisectors, and altitudes are drawn in blue, how many distinct blue line segments with endpoints on the triangle have been drawn inside the triangle?	7 [line seg]
4	What is the area, in square centimeters, of an ellipse with a major axis measuring ten centimeters and having foci six centimeters apart?	20π [cm ²]
5	How many positive integers less than fifteen-thousand eight-hundred forty-two leave a remainder of nineteen when divided by five-thousand three-hundred forty-eight?	3 [int]
6	In a certain game, a turn consists of flipping a coin and then rolling a die. If the coin shows heads, you roll a standard six-sided die, while if the coin shows tails, you roll a fair four-sided die numbered one to four. Your score for the turn is the number shown on the die you roll. What is the expected value of a player's score for one turn?	3
7	What is the sum of the perfect squares less than one-hundred fifty?	650
	Extra Problem - Only if Needed	
8	What value of u satisfies the equation four u minus nineteen equals forty-nine?	17

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School Name _____ Team # _____

Proctor Name _____ Room # _____

STUDENT NAME _____

Final Score:

KEY

First Score

(out of 18)

Individual Multiple Choice Contest - Score Sheet

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	C		
2	D		
3	A		
4	D		
5	B		
6	B		
7	D		
8	E		
9	A		

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Final Score:

KEY

First Score

(out of 20)

School Name _____ Team # _____

Proctor Name _____ Room # _____

STUDENT NAME _____

Team Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	$10\sqrt{3}$		
2	36π		
3	30 [ways]		
4	-4		
5	$41/144$		
6	10		
7	-3		
8	$\frac{2}{b-c}$		
9	748		
10	357		

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Final Score:

KEY

First Score

School Name _____ Team # _____

Proctor Name _____ Room # _____

STUDENT NAME _____

Pressure Round Answers

Answer	
1	$3\sqrt{3}$
2	25
3	41 [ways]
4	3 [points]
5	1600 [integers]