"Math is Cool" Championships – 2006-07

Sponsored by: Western Polymer Corporation

Pre-Calculus & Calculus – November 1, 2006

Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by <u>all</u> involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
  - *For problems dealing with money, a decimal answer should be given.*
  - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of π where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

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Pre-Calculus & Calculus – November 1, 2006

Individual Contest

1	Evaluate as a mixed number: $5\frac{2}{3}-2\frac{3}{4}$
2	How many minutes are there in one day?
3	Evaluate: $(9+8\cdot 7-6\cdot 5\div 4-2)\div 3$
4	In how many points do the graphs of $y = e^x + 1$ and $y = 2\sin(2\pi x) + 5$ intersect?
5	Evaluate $437_8 - 174_8$ as a base eight number.
6	What value(s) of *m* satisfy $m^2 + 12m - 28 = 0$?
7	At what point, in the form (x, y), does the line $y = 4x + 15$ intersect the line $-2x + 3y = 15$?
8	If u, v, and w are all positive integers, the average of u and v is 47 and the average of v and w is 38, what is the smallest possible value of u?
9	Jan drove 700 km at 140 kmph, then 200 km at 80 kmph. What was her average speed, in kmph, for the entire trip?
10	*C* is 67 less than D. If *C* is 44, what is the sum of *C* and D?
11	What is the period of the function $n(o) = 3\tan(5o) + 7$?
12	What are the coordinates, in (*x*, *y*) form, of the vertex of the parabola $y = x^2 + 4x - 9$?
13	If $s(r) = 3e^{2r} + 1$, determine $s^{-1}(r)$.
14	What is the area, in square centimeters, of a triangle with sides measuring 5, 8, and 9 cm?
15	What is the smaller angle, in degrees, between the hands of a standard twelve-hour clock at 5:50?
16	What is the secant, in cm, of the smallest angle in a right triangle with legs measuring 5 and 12 cm?
17	Express $41°3'9''$ as a decimal number of degrees.
18	A pentagon has interior angles measuring 80, 90, 100, and 110 degrees. What is the measure, in degrees, of the fifth interior angle?
19	What is the area, in square centimeters, of a 40 degree sector of a circle with a radius of 6 cm?
20	In triangle ABC, $m\angle A = 30°$ and $m\angle B = 105°$. If $\overline{AB} = 4$ cm, what is the length of $\overline{AC}$, in centimeters?
21	A triangle has sides measuring 6, 8, and 9 cm, while a similar triangle has two sides measuring 12 and 9 cm. What is the length, in centimeters, of the third side of the larger triangle?

22	A triangle has sides measuring 6, 7, and 8 cm. An angle bisector is drawn to the 6 cm side. What is the length, in centimeters, of the smaller of the two segments into which the 6 cm side is divided?
23	If Z is the set of all positive multiples of four less than 100, and Y is the set of all multiples of three greater than 50, how many elements are in the set $Z \cap Y'$ (Y' means Y complement)?
24	Evaluate: $i^{147}\left(i^{222}\right)-\frac{i^{213}}{i^{344}}$, where $i=\sqrt{-1}$
25	Evaluate: $\log_4 128$
26	Jessica will arrive at the bookstore at a random time between 3 PM and 4 PM, browse for 15 minutes, and then leave. Doug will arrive at the bookstore at a random time between 3 PM and 4 PM, browse for 20 minutes, and then leave. What is the probability that Jessica and Doug will be in the bookstore at the same time?
27	What is the coefficient of the q^5 term in the expansion of $(2q-1)^8$ after like terms are combined?
28	What value(s) of g satisfy $2^{2g+1}-5\cdot 2^g=3$?
29	What is the largest area, in square meters, that can be enclosed in the corner of a large square room using two four-meter-wide screens?

Challenge Questions

30	What is the area of the ellipse represented by $2x^2-28x+3y^2+24y=-140$?
31	What is the length of a latus rectum of the hyperbola represented by $9y^2-54y-16x^2+64x=127$?
32	Two circles with radii of 9 and 12 cm have their centers separated by 29 cm. What is the length, in centimeters, of one of their common internal tangents?
33	What is the sum of the squares of the roots of $5k^2+3k-1=0$
34	How many positive five digit palindromes are multiples of four?
35	Evaluate: $\cfrac{3}{2+\cfrac{3}{2+\cdots}}$
36	Evaluate: $1324\cdot 1276$
37	My favorite number is a positive four-digit integer. My second favorite number is also a positive four-digit integer, and uses the same four digits. When I subtract the two, I get a third four-digit number, three of whose digits are 2, 3, and 9. What is the fourth digit of the difference?
38	Two circles with radii of 24 and 54 are tangent to one another, and one of their common external tangents is drawn. What is the radius of a third circle inscribed tangent to both original circles and their common external tangent?
39	How many integers are in the range of the function $y(x)=\frac{4x^2+75}{2x^2+3}$?
40	In the cryptarithm shown below, each instance of a particular letter represents the same digit (0-9), and no two different letters represent the same digit. What is the maximum possible value of the five-digit number ABCDE? $\begin{array}{r} ABCD \\ -\,BDEA \\ \hline CBA \end{array}$

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11th & 12th Grade – November 1, 2006

Individual Multiple Choice Contest

NOTA = None of the Above

1 Given that $\log_{17}(\log_{19}(\log_{25} x)) = 12$, what are the prime factors of x?

A) 5, 17 and 19 B) 5 C) 2 and 3 D) 17 E) NOTA

2 If a circle of radius 2 rolls (like a gear) around a circle of radius 80, how many revolutions will it make before returning to the place it started?

A) $4\sqrt{5}$ B) 160 C) 41 D) 40 E) NOTA

3 Evaluate: $\sqrt{9+\sqrt{9+\sqrt{9+\cdots}}}$

A) 3 B) $3+\frac{\sqrt{37}}{9}$ C) $\frac{1+\sqrt{37}}{2}$ D) $1+\sqrt{37}$ E) NOTA

4 When 200^6 is converted to base 4, how many zeroes does it have at the end?

A) 3 B) 12 C) 6 D) 9 E) NOTA

5 A certain trapezoid can be decomposed into a unit square and two similar, but incongruent, triangles. One side of one of the triangles has length $\frac{1}{2}$. What is the trapezoid's perimeter?

A) $3+\sqrt{5}$ B) $\frac{9+3\sqrt{5}}{2}$ C) $\frac{13+3\sqrt{5}}{2}$ D) $3\frac{1}{2}$ E) NOTA

6 Simplify: $\sin^2(x\cdot y)+\cos^2\left(\frac{1}{4}\left((x+y)^2-(x-y)^2\right)\right)$.

A) $\sin x \bullet \cos y$ B) $\tan^2(x\cdot y)+\sec^2(x+y)$

C) $y\sin(\cos x)$ D) 1 E) NOTA

7 Points N, M, and L are on circle K such that $m\angle NML = 150°$. Point J is outside circle K such that $\overline{NJ}$ and $\overline{LJ}$ are tangent to circle K. What is the measure, in degrees, of $m\angle NJL$?

A) 60 B) 90

C) 120 D) 150 E) NOTA

8 Below is a board from the game of Minemop. Numbers signify the number of adjacent grid squares, including diagonals that have mines in them. Squares with numbers in them never have mines in them. How many mines are adjacent to **x**?

1	2	2	2	1
3		X		
				1
3			1	1

A) 0 B) 1 or 2 C) 3 or 4 D) 5 or 6 E) NOTA

9 Eustace and Elizabeth will each show up for a date at a random instant during an hour and wait fifteen minutes for the other, what is the probability that they will meet?

A) $\frac{7}{16}$ B) $\frac{1}{2}$ C) $\frac{7}{8}$ D) $\frac{5}{16}$ E) NOTA

"Math is Cool" Championships – 2006-07

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11th & 12th Grade – November 1, 2006

Team Contest

1	Express 45,629 in scientific notation rounded to two significant figures.
2	Tom's age is nine years less than the sum of Cherie and Katie's ages, while Cherie's age, in years, is eight more than the average of Tom and Katie's ages. If Tom's age minus Cherie's age is half of Katie's age, what is Tom's age, in years?
3	Lori likes extra-strength fruit punch that is 120% the strength of normal fruit punch. Tom likes weak fruit punch that is 75% the strength of normal fruit punch. How many **milliliters** of water should Tom add to three liters of Lori's punch to produce his desired punch strength?
4	A triangle has sides measuring 4, 5, and 6 cm. What is the length, in centimeters, of the altitude to the 4 cm side?
5	When five cards are drawn from a standard 52-card deck, what is the probability that none of the five cards have the same rank?
6	How many rectangles of any size are there in a five-by-six grid of unit squares?
7	The sum of the first five terms of an arithmetic sequence is 2280, while the sum of the first fifteen terms of the sequence is 6240. What is the sum of the first twenty terms of the sequence?
8	How many terms are there in the expansion of $(2n+3o-4)^5$ after like terms are combined?
9	What is the sum of all values of m between 0 and 2π inclusive satisfying $\sin^2 m + \sin m = \cos^2 m$?
10	What is the product, in base seven, of the base seven numbers 23_7 and 34_7?

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11th & 12th Grade – November 1, 2006

Pressure Round Contest

1	Machine A takes an input number and adds 4. Machine B takes an input number and doubles it. Machine C takes an input number and subtracts 2. Machine D takes an input number and divides by 3. The output number from one machine becomes the input number for the next machine. If the four machines are arranged in random order and the number 6 is input into the first machine, what is the probability that 6 will be the output number from the fourth machine?
2	If $i^{451} - \frac{i^{-8}}{i^6} + i^x = 1$ and $i = \sqrt{-1}$, list all of the following that x could be: 2007, 4, 0, -1, -2007
3	Let *a*, *b*, and *c* be positive integers, not necessarily different. There exists exactly 1 ordered triple (*a*,*b*,*c*) such that the product *abc* = 1 [namely, (1,1,1)] and there exist exactly 3 ordered triples such that *abc* = 3 [namely, (1,1,3), (1,3,1), and (3,1,1)]. What is the next larger value of *n* for which there exist exactly *n* ordered triples (*a*,*b*,*c*) such that *abc* = *n*?
4	In the equation TAP + PAT + APT = XYXX, each different letter stands for a different digit and a given letter stands for the same digit wherever it occurs. If P is odd, what digit does Y stand for?
5	When $(ax + by)^2$ is expanded, the expansion is $a^2x^2 + 2abxy + b^2y^2$, and the sum of the coefficients of this expansion is found by adding $a^2 + 2ab + b^2$. What is the sum of the coefficients of the expansion of $(3x - 4y)^5$?

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11th & 12th Grade – November 1, 2006

Mental Math Contest

PERSON 1		
1.1	When the quantity "a" plus two "b" is raised to the fifth power and expanded, what is the sum of the coefficients of the resulting polynomial?	243
1.2	How many possible integer lengths are there for the third side of a triangle whose other two sides are 8 and 11?	15
1.3	If x equals 30 degrees, simplify sine of x times cosine of x times tangent of x times cotangent of x times secant of x times cosecant of x?	1
1.4	What is the probability of getting at least 3 heads on 5 flips of a fair coin?	1/2
PERSON 2		
2.1	What is the sum of the solutions to the equation: 2 x squared plus 9 x equals 13.	-9/2
2.2	For what value of x is the log base 4 of x equal to negative one-half?	1/2
2.3	For how many values, x, between 0 and 2 pi is it true that sine squared x equals cosine squared x?	4
2.4	What is the lateral surface area of a right cone with base radius equals 6 and height equal 8?	60π
PERSON 3		
3.1	How many terms will be in the simplified expansion of the quantity a plus b plus c, squared?	6
3.2	When I square a positive integer between 10 and 20 and then divide by 5, what are the possible remainders?	0, 1, and 4
3.3	The number of diagonals that can be drawn in a certain regular polygon is 35, how many sides does the polygon have?	10
3.4	John travels 40 miles per hour to visit his aunt and 60 miles per hour home, what was his average speed?	48 [mph]
PERSON 4		
4.1	A square is inscribed in a circle of radius 2, what is the area inside the circle but outside the square?	$4\pi - 8$
4.2	What is the domain of the function, f of x equals the square root of the log of x?	x >= 1 or $[1,\infty)$
4.3	What is the sum of the positive integers from 1 to 40 minus the sum of the positive integers from 1 to 20?	610
4.4	A bag has 6 blue marbles and 4 red marbles, what is the probability of drawing two marbles of the same color without replacement?	7/15

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11th & 12th Grade – November 1, 2006

COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	What is the sum of the reciprocals of the solutions to 2 x squared minus 12 x plus 3 equals 0?	4
2	A triangle is formed by connecting the origin and the points 2 comma 4 and 1 comma -1. What are the coordinates of the centroid?	(1,1)
3	If 2 to the quantity x plus 2 equals 8 to the quantity x minus 2, what is x?	4
4	If the common log of 2 is "a", what is the common log of 50 in terms of "a"?	2 – a
5	Given only 7-cent and 11-cent coins, what is the largest value, in cents, for which we cannot make exact change?	59 [cents]
6	Tom and Joe are playing cards with a standard 52-card deck. Joe picks two cards and gets a pair of tens. Tom then picks two cards. What is the probability that he gets a higher pair than Joe (note: aces are high)?	24/1225
7	If I write down all the ways that the letters in the word M-A-T-H can be ordered and put them in alphabetical order, what place will be the word MATH?	14 [th]
	Extra Problem – Only if Needed	
8	What is the range of the cotangent function?	All reals

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COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	What is two-one-two base 4 in base 16?	26 [16]
2	For what values of r, does the series 1 plus 1 over r plus 1 over r squared and so on, converge?	r < -1 or r > 1
3	What is the positive difference between the mean and median of the set of numbers: 4, 10, 8, 9, 3, 5, and 10?	1
4	What are the coordinates of the midpoint of the line segment connecting the x-intercepts of the equation: y = 3 x squared minus 2 x minus 1?	(1/3, 0)
5	A spherical triangle on a sphere of radius 1 has angles that measure pi over 2, pi over 2, and pi over 3. What is its area?	Pi / 3
6	What is one-half plus two-fourths plus three-eighths plus four-sixteenths and so on?	2
7	Josh rolls two fair n-sided dice, what is the probability that of the numbers shown, one is bigger than the other?	(n-1)/n
	Extra Problem – Only if Needed	
8	Find the area enclosed by f of x equals the absolute value of x and g of x equals 4 minus the absolute value of x.	8

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COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	One solution to the equation "x" cubed minus 2 "x" squared plus 4 "x" minus 8 equals 0 is 2i, what is the real solution?	2
2	The graph of the equation y = x squared minus 4 x plus 1 passes through all quadrants but one. Which one?	III or 3
3	If r equals one-half, what is 1 plus r squared plus r to the 4th power plus r to the sixth power and so on?	4/3
4	What is the tangent of 15 degrees plus the cotangent of 15 degrees?	4
5	What is the area inside the graph of 9 x squared plus 4 y squared equals 36?	6π
6	For what value of c does the equation 3 x squared plus 4 x plus c equal 0, have exactly one solution?	4/3
7	What is the unit vector in the same direction as 6 **i** minus 8 **j**?	3/5 **i** – 4/5**j**
	Extra Problem – Only if Needed	
8	What is the x-coordinate of the local maximum of the graph of minus x cubed plus 6 x squared plus 9 x plus 7 equals 0?	3

"Math is Cool" Championships – 2006-07

Pre-Calculus & Calculus – November 1, 2006

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

KEY

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	$2\frac{11}{12}$		
2	1440 [min]		
3	$\frac{37}{2}$		
4	3 [points]		
5	$243_{[8]}$		
6	-14, 2 [need both]		
7	(-3, 3)		
8	19		
9	120 [kmph]		
10	155		
11	$\frac{\pi}{5}$		
12	(-2, -13)		
13	$\frac{1}{2}\ln\left(\frac{r-1}{3}\right)$		
14	$6\sqrt{11}$ [cm^2]		
15	125 [°]		
16	13/12 [cm]		
17	41.0525 [°]		
18	160 [°]		
19	4π [cm^2]		
20	$2+2\sqrt{3}$ [cm]		

	Answer	1 or 0	1 or 0
21	$\frac{27}{2}$ [cm]		
22	$\frac{14}{5}$ [cm]		
23	20 [elements]		
24	0		
25	$\frac{7}{2}$		
26	$\frac{143}{288}$		
27	-1792		
28	$\log_2 3$		
29	$8+8\sqrt{2}$ [m^2]		
30	$\pi\sqrt{6}$ [un^2]		
31	9/2 [un]		
32	20 [cm]		
33	$\frac{19}{25}$		
34	200		
35	1		
36	1689424		
37	4		
38	$\frac{216}{25}$		
39	23		
40	43582		

First Score
KEY

"Math is Cool" Championships – 2006-07

11th & 12th Grade – November 1, 2006

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

INDIVIDUAL MULTIPLE CHOICE - 15 minutes

This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. ***Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet.*** *No talking during the test.*

DO NOT WRITE IN SHADED REGIONS

Answer		-1, 0 or 2	-1, 0 or 2
1	B		
2	D		
3	C		
4	D		
5	B		
6	D		
7	C		
8	C		
9	A		

First Score

KEY

"Math is Cool" Championships – 2006-07

11th & 12th Grade – November 1, 2006

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Team Contest – Score Sheet

TEAM TEST - 15 minutes

*When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as **2 or 0**. Record all answers on the colored answer sheet.*

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	4.6×10^4		
2	52 [yrs]		
3	1800 [mm]		
4	$\frac{15\sqrt{7}}{8}$ [cm]		
5	$\frac{2112}{4165}$		
6	315 [rec]		
7	7920		
8	21		
9	$\frac{5\pi}{2}$		
10	$1145_{[7]}$		

"Math is Cool" Championships – 2006-07

11th & 12th Grade – November 1, 2006

First Score

KEY

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

PRESSURE ROUND - 10 minutes

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

Pressure Round Answers

	Answer
1	$\frac{5}{24}$
2	-2007
3	[n=] 18
4	6
5	-1