"Math is Cool" Masters – 2006-07

Sponsored by: Wilbert Precast Inc.

Algebra II, Pre-Calculus, Calculus – January 13, 2007

Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by <u>all</u> involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
    - *For problems dealing with money, a decimal answer should be given rounded to the nearest cent.*
    - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of π where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

"Math is Cool" Masters – 2006-07

Algebra II, Pre-Calculus, Calculus – January 13, 2007
Individual Contest

1	34 is what percent of 85?
2	What is the product of the number of days in a week, the number of vertices on a triangle, and the number of centimeters in a meter?
3	Express in simplest radical form: $\sqrt[3]{3024}$
4	Simplify by rationalizing the denominator: $\frac{12}{3-\sqrt{5}}$
5	What value(s) of y satisfy $3(2y+1)+5(y-3)=65$?
6	What value(s) of n satisfy $2n^2+3n+1=0$?
7	What value(s) of s satisfies $3s^2+4s+5=0$?
8	If five engineers can design three circuits in six days, how many engineers would be needed to design twenty-eight circuits in eight days?
9	What is the slope of a line perpendicular to the line y = 8x + 4?
10	What is the distance between the points (-3, -2) and (5, 2)?
11	What is the distance from the point (-3, 8) to the line 3x - 4y = 19?
12	Beau's rectangular lawn is fenced on all sides and measures 40 meters by 50 meters. When Beau mows his lawn, he mows everything that is not within two meters of the fence. What is the area, in square meters, of the region that Beau **does not** mow?
13	Express the base thirteen number 234_{13} as a base ten number.
14	Sam's bug collection consists of spiders (8 legs each) and beetles (6 legs each). If Sam has a total of 47 bugs with a total of 350 legs, how many beetles does he have?
15	What is the length, in centimeters, of the hypotenuse of a right triangle with legs measuring 12 and 18 cm?

16	An isosceles triangle has a base angle of 32 degrees. What is the measure, in degrees, of its vertex angle?
17	What is the perimeter, in centimeters, of a rectangle with an area of 200 cm^2 and a side measuring 5 cm?
18	What is the area, in square centimeters, of a rhombus with sides measuring 18 cm and an interior angle measuring 60 degrees?
19	A regular n-gon has interior angles measuring 160 degrees. How many diagonals can be drawn in this n-gon?
20	What is the height, in centimeters, of a right circular cylinder with a base radius of 8 cm and a surface area of 168π cm^2?
21	When three standard six-sided dice are rolled, what is the probability that the product of the numbers shown is six?
22	How many edges does a regular octahedron have?
23	In how many ways can a committee of four women and one man be chosen from a group of ten women and five men?
24	What is the fifth term of an arithmetic sequence with a first term of 18 and a common difference of 31?
25	If $C = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & -3 \\ 2 & 3 & -4 \end{bmatrix}$, determine $\left\|C^{-1}\right\|$.
26	If $w(v) = 3v^2 - 12v + 5$, evaluate $w^{-1}(-7)$.
27	Chuck invests $2500 in an account that receives 24% annual interest compounded monthly. How much money will be in the account after three months?
28	What is the value of the constant term in the expansion of $\left(p - \frac{1}{p^2}\right)^6$ after like terms are combined?
29	How many positive integer factors does 208 have?

Challenge Questions

30	If two "zoinks" are worth three "yikes", four "xylos" are worth five "womps", and six "yikes" are worth seven "womps", how many "zoinks" are twenty-one "xylos" worth?
31	If A is the set of all positive multiples of five less than 1000, B is the set of all composite numbers between 120 and 140 inclusive, and C is the set of all multiples of two between 150 and 350, how many elements does the set $(B \cup C) \cap A$ contain?
32	Two right rectangular pyramids are similar to one another, having volumes of 8 and 64 cm^3. If the larger pyramid has a surface area of 100 cm^2, what is the surface area, in square centimeters, of the smaller pyramid?
33	In the cryptarithm below, in which each letter represents a distinct digit and all instances of the same letter represent the same digit, what is the smallest possible value of the four-digit number ABCD? $\begin{array}{r} ABC \\ -BCA \\ \hline BDA \end{array}$
34	A coin is flipped five times, and you are told (truthfully) that there were at least two tails. What is the probability that there were exactly four tails?
35	A triangle has sides measuring 5, 6, and 7 cm. What is the length, in centimeters, of the median to the 6 cm side?
36	A circle is inscribed in a square with sides measuring 24 cm. What is the radius, in centimeters, of the largest circle that can be inscribed in the corner of the square between the circle and the square?
37	When two real numbers are selected between 5 and 8, what is the probability that they differ by less than one?
38	A teacher administers a make-up test to five students, each of whom earns an integer score between 0 and 100 inclusive. If the mean of their scores is 61, their unique mode is 42, and their median is 70, what is the maximum value of the range of the scores?
39	What real value(s) of h satisfy $e^{2h} - 2e^{h} = 3$?
40	How many positive three-digit even multiples of three are comprised of three distinct digits?

"Math is Cool" Masters – 2006-07

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11th & 12th Grade – January 13, 2007

Individual Multiple Choice Contest

1	Let $f(x)$ be an even function and $g(x)$ be on odd function, both defined on all integers. If $f(2)=5$ and $g(-5)=2$. What is the sum $g(f(-2))+f(g(-f(2)))$? A) 3 B) 5 C) 12 D) -7 E) Answer not given
2	Consider an isosceles trapezoid ABCD with minor base AB of length 6 and major base CD of length 14, and area 30. What is the value of the sine of the angle at A? A) $\frac{21}{4\sqrt{13}}$ B) $\frac{3}{5}$ C) $\frac{4}{5}$ D) $\frac{42}{5\sqrt{109}}$ E) Answer not given
3	How many 3-digit numbers are there such that the digits are in increasing order such as 138 not 495 not 366? A) 450 B) 84 C) 150 D) 120 E) Answer not given
4	A solid sphere has radius r and surface area SA. A smaller concentric sphere is hollowed out, with radius $r/2$. The larger sphere is then bisected. What is the ratio between the surface area of one of the new hemispheres and the original surface area SA? A) $7/8$ B) $3/2$ C) $5/7$ D) $13/16$ E) Answer not given
5	Let a, g, and h correspond respectively to the arithmetic, geometric and harmonics means of two positive numbers. What below corresponds to the correct ordering of a,g and h? A) $g \le h \le a$ B) $g < a \le h$ C) $h \le g \le a$ D) $a \le h \le g$ E) Answer not given
6	What is the remainder when the polynomial $x^5 - 16x^3 + 5x - 4$ is divided by $(x+2)$? A) 82 B) -4 C) 0 D) 116 E) Answer not given
7	What is the amplitude of the equation $y = 4\sin(3x) - 6\cos\left(2x + \pi/4\right)$? A) $\sqrt{17}$ B) 10 C) $2\sqrt{13}$ D) 2 E) Answer not given
8	Evaluate the sum: $\sum_{i=11}^{100} \left[\log\left(\log_{i-1} i\right)\right]$. A) log10 B) $\frac{100!}{10!}$ C) 10 D) log2 E) Answer not given
9	What is the area of the largest circle that can be inscribed in a regular hexagon with side length s? A) πs^2 B) $\pi s^2(\sqrt{3}-1)$ C) $\pi s^2\left(\frac{2}{3}\right)$ D) $\pi s^2\left(\frac{3}{4}\right)$ E) Answer not given

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11th & 12th Grade – January 13, 2007

Team Contest

1	Evaluate: $108^3 + 92^3$
2	An isosceles right triangle has legs measuring eight centimeters each. What is the area, in square centimeters, of the region within the triangle that is at least one centimeter from each side?
3	A group of thirsty hikers split the cost of a Huge Glug soft drink, splitting the cost evenly. If there had been four fewer people in the group, each person would have paid three more cents, while if there had been two more people in the group, each person would have paid one cent less. What was the total cost of the Huge Glub, in dollars rounded to the nearest hundredth (cent)?
4	A circle is inscribed in a square with sides measuring 12 cm. What is the area, in square centimeters, of the largest square that can be inscribed in the corner between the original circle and square?
5	What is the missing term of the sequence 1, 4, 15, 40, ___, 156, 259?
6	The probability that Sam drinks a soda on Wednesday is $\frac{1}{4}$, the probability that he passes his calculus test on Wednesday is $\frac{1}{3}$, and the probability that both occur is $\frac{1}{5}$. If Sam does not drink a soda on Wednesday, what is the probability he passes his calculus test that day?
7	What value(s) of y satisfy $\log_4 y^{\log_2 y} = 2 - 3\log_4 y$?
8	How many positive integers less than 100 have exactly four positive integer factors?
9	If $\cos Q = \frac{5}{13}$, $\pi \le Q < 2\pi$, $\tan P = \frac{4}{3}$, and $\pi \le P < 2\pi$, what is the value of $\sin(Q + P)$?
10	What are the coordinates, in the form (x, y), of the uppermost focus of the ellipse represented by $25x^2 + 150x + 16y^2 - 64y = 111$?

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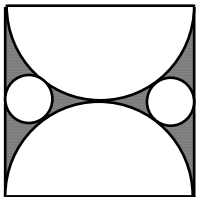
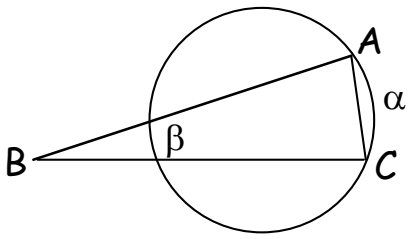
11th & 12th Grade – January 13, 2007

Pressure Round Contest

1	Angles A, B, C, and D are each between 0° and 360°, and terminate in quadrants I, II, III, and IV respectively. Find the largest possible sum of $\lfloor A \rfloor + \lfloor B \rfloor + \lfloor C \rfloor + \lfloor D \rfloor$, when the angles are measured in radians. (Each angle has its vertex at the origin and its initial ray coincident with the positive x axis. The symbol $\lfloor x \rfloor$ indicates the greatest integer less than or equal to x.)
2	My calculator sometimes thinks backwards. When I multiply two numbers ($m \cdot n$), it reverses the digits of **at least one** of the three numbers (m, n, and the product). If it reverses the digits of m and/or n, it multiplies the reversed numbers. When I repeatedly multiply the same two positive integers (one with 3 digits and the other with 2 digits), I get the following different products displayed: 33125, 446, 61682, 5747, 6440, 7475, and 51233. What are the two numbers I entered?
3	Two congruent semicircles of radius 2 are inscribed in a square. The square also contains two congruent circles tangent to the semicircles and to the square, as shown. Find the the area of the shaded region. 
4	If angle ABC = 14°, arc β = 17°, and the radius of the circle = 5 cm, find the area (in square cm) of the the region between arc α and chord AC. 
5	Find the largest integer that divides the following three numbers with the same remainder in each case: 54437, 20093, and 52997.

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11th & 12th Grade – January 13, 2007

Mental Math Contest

PERSON 1		
1.1	What is the remainder when x squared minus 5 x plus 7 is divided by x minus 1?	3
1.2	How many positive even factors does the number 60 have?	8 [fac]
1.3	Alice, Bob, Carol and David ran a race and there were no ties. If David did not get second, in how many ways could they have finished?	18 [ways]
1.4	In radians between 0 and 2 pi, what angle in the fourth quadrant has a cosine of 1/2?	$\frac{5\pi}{3}$
PERSON 2		
2.1	If x is in the second quadrant and the tangent of x equals negative 12 fifths, what is the sine of x?	12/13
2.2	What is the largest four-digit base 4 number when expressed in base 10?	$255_{[10]}$
2.3	Trevor rolls two fair six-sided dice. What is the probability that the product of the two numbers showing will be prime?	1/6
2.4	The middle third of a line segment of length 1 is discarded. In the second series of cuts, the middle third of the two remaining segments is removed. After the fourth series of cuts, what is the total length of the remaining pieces?	16/81 [un]
PERSON 3		
3.1	What is the highest number of non-overlapping equilateral triangles of side length 1 that can be placed inside an equilateral triangle of side length 3?	9 [tri]
3.2	Suppose a coin is weighted so that the probability of a head is two-thirds. What is the probability of getting exactly 2 heads out of 3 coin flips?	4/9
3.3	What is the geometric mean of the two missing values in the geometric sequence: 8, blank, blank, 1?	$2\sqrt{2}$
3.4	What is the smallest positive integer that has exactly six positive integer factors?	12 [int]
PERSON 4		
4.1	What is the log of 2 divided by the log of 8?	1/3
4.2	Mary has a fair six-sided die and Barbara has a fair eight-sided die. If they both roll their die, what is the probability that Mary will get a higher number that Barbara?	5/16
4.3	In a 3 by 4 by 5 prism, what is the distance between the two farthest corners?	$5\sqrt{2}$
4.4	How many two digit numbers contain at least one 2?	18

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COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	A right triangle has sides 5 and 12. What is the sum of all possible lengths for the third side?	$13 + \sqrt{119}$ [un]
2	What is the sum of the factors of 36?	91
3	The cube root of the square root of x is 3. What is the sum of the digits of x?	18
4	Three fair four-sided dice with faces numbered one through four are rolled. What is the probability that the product of the numbers shown is two?	3/64
5	f of x equals x squared plus 6 x plus 8. g of x equals two x plus 4. For what value(s) of x does f of x equal g of x?	-2
6	The angles of a convex pentagon form an arithmetic sequence with a difference of one. What is the smallest of these angles in degrees?	106 [°]
7	a squared plus b squared is equal to 24. The quantity a plus b squared is equal to 8. What is the product of a and b?	-8
	Extra Problem – Only if Needed	
8	2007 in base 8 is what number expressed in base 10?	$1031_{[10]}$

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COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	When 2 x plus the quantity 1 over x squared is raised to the sixth power, what is the constant term?	240
2	What fraction of the numbers, from 20 to 40 inclusiv, have digits whose product is even?	16/21
3	What is the largest area, in square meters, that can be enclosed using 20 meters of fencing?	$100/\pi$ [m^2]
4	What is the sum of 1 over 11 plus 1 over 121 plus 1 over 1331 plus and so on?	1/10
5	What is the product of the two largest prime numbers less than 40?	1147
6	This year is 2007. There is no polygon whose interior angles sum to 2007 degrees. How many years ago was the year equal to the sum, in degrees, of the interior angles of a convex polygon?	27 [yrs]
7	Rebecca has an infinitely large supply of 2 kinds of stamps, 17-cent stamps and stamps worth x cents. She cannot make postage worth exactly $2.71, but can make any amount greater than $2.71. What is x?	18
	Extra Problem – Only if Needed	
8	What is the surface area of the shape determined by the equation: x squared plus y squared plus z squared equals 4?	64π [un^2]

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11th & 12th Grade – January 13, 2007

COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	What is the limit as x approaches infinity of the quantity 3 x squared plus x minus 10 divided by the quantity 2 x squared minus 4 x minus 1?	3/2
2	7 factorial times 8 factorial is equal to c times n factorial, where c is a positive integer and n is as large as possible. What is the sum of c and n?	66
3	A circle is inscribed in an equilateral triangle and a square is inscribed in that circle. What is the ratio of the area of the square to the triangle expressed as a fraction?	$\frac{2\sqrt{3}}{9}$ [or 2 root 3 over 9]
4	What is the sum of the positive real solutions of 2 x to the fourth minus 5 x squared plus 2 equals 0?	$\frac{3\sqrt{2}}{2}$ [or 3 (root 2)/2]
5	What is the quantity i plus 1 to the fourth power, where i is the square root of -1?	-4
6	The rectangular coordinates 3 comma root 3 can be rewritten as what polar coordinates using radians?	$\left(2\sqrt{3}, \frac{\pi}{6}\right)$
7	Consider the graph of x squared plus y squared minus 4 x plus 6 y equals 3. What is the slope of the tangent line at the point 2 comma 1?	0
	Extra Problem – Only if Needed	
8	What is the sum of 5 plus 10 plus 15 and so on until 100?	1050

"Math is Cool" Masters – 2006-07

Algebra II, PreCalculus & Calculus – January 13, 2007

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

KEY

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	40 [%]		
2	2100		
3	$6\sqrt[3]{14}$		
4	$9+3\sqrt{5}$		
5	7		
6	-1/2, -1 [in any order]		
7	$(-2\pm i\sqrt{11})/3$		
8	35 [engin]		
9	-1/8		
10	$4\sqrt{5}$ [un]		
11	12 [un]		
12	344 [m^2]		
13	$381_{[10]}$		
14	13 [beetles]		
15	$6\sqrt{13}$ [cm]		
16	116 [°]		
17	90 [cm]		
18	$162\sqrt{3}$ [cm^2]		
19	135 [diag]		
20	5/2 [cm]		

	Answer	1 or 0	1 or 0
21	1/24		
22	12 [edges]		
23	1050 [ways]		
24	142		
25	-1/40		
26	2		
27	[$] 2653.02		
28	15		
29	10 [factors]		
30	15 [zoinks]		
31	26 [elements]		
32	25 [cm^2]		
33	3165		
34	5/26		
35	$2\sqrt{7}$ [cm]		
36	$36 - 24\sqrt{2}$ [cm]		
37	5/9		
38	38		
39	ln3		
40	115 [numbers]		

"Math is Cool" Masters – 2006-07

11th & 12th Grade – January 13, 2007

First Score
(out of 18)

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

INDIVIDUAL MULTIPLE CHOICE - 15 minutes

This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. ***Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet.*** *No talking during the test.*

DO NOT WRITE IN SHADED REGIONS

Answer		-1, 0 or 2	-1, 0 or 2
1	A		
2	B		
3	B		
4	D		
5	C		
6	A		
7	C		
8	D		
9	D		

"Math is Cool" Masters – 2006-07

11th & 12th Grade – January 13, 2007

First Score
(out of 20)

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Team Contest – Score Sheet

TEAM TEST - 15 minutes

*When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as **2 or 0**. Record all answers on the colored answer sheet.*

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	2038400		
2	$19-6\sqrt{2}$ [cm^2]		
3	[$] 1.44		
4	$54-36\sqrt{2}$ [cm^2]		
5	85		
6	$\frac{8}{45}$		
7	$2, \frac{1}{16}$		
8	32 [int]		
9	$\frac{16}{65}$		
10	(-3, 5)		

First Score

"Math is Cool" Masters – 2006-07

11th & 12th Grade – January 13, 2007

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

PRESSURE ROUND - 10 minutes

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

Pressure Round Answers

Answer		
1	14	[radians]
2	511 & 56	[either order]
3	$16-\frac{9\pi}{2}$	[un^2] or equiv.
4	$\frac{25\pi}{8}-\frac{25\sqrt{2}}{4}$	[cm^2] or equiv.
5	72	