"Math is Cool" Championships – 2006-07

Sponsored by: Wenatchee Valley Medical Center PreAlgebra & Algebra I - November 17, 2006 Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- Good sportsmanship is expected throughout the competition by <u>all</u> involved.
 Bad sportsmanship may result in disqualification.
- Calculators or any other aids may not be used on any portion of this contest.
- Unless stated otherwise:
 For problems dealing with money, a decimal answer should be given.
 Express all rational, non-integer answers as reduced common fractions.
- All radicals must be simplified and all denominators must be rationalized.
- Units are not necessary unless it is a problem that deals with time and in that
- case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.
 - Leave all answers in terms of π where applicable.
 - Do not round any answers unless stated otherwise.
 - Record all answers on the colored cover sheets in the answer column only.
 - Make sure all answer sheets have all the information at the top of the sheet filled out.
 - Tests will be scored as a 0 if answers are not recorded on the answer sheets.
 - Blank answer sheets and answer sheets with no name will also be scored as a 0.

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

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1	Evaluate: 2.371 × 1.4 [Round to the nearest hundredth.]
2	Evaluate: $\frac{3}{7} \div \frac{5}{3}$
3	Evaluate: 2 ³ + 8 x (-4) ÷ .5
4	Evaluate: 4! x 5 ³
5	What is the name of the polygon with twice as many edges as a pentagon?
6	Express 2187 as a power of 3.
7	Solve for y: 21y + 14 = -98
8	How many postive factors does the number 343 have?
9	$(x - h)^2 + (y - k)^2 = r^2$ is the equation for a circle with center at (h,k) and a radius of r. What is the sum of the coordinates of the center of the circle given by the equation $(x - 5)^2 + (y + 3)^2 = 36$?
10	What is the mean of the following seven numbers? 1,234,567 3,456,712 5,671,234 7,123,456 2,345,671 4,567,123 and 6,712,345
11	What is the sum of all multiples of 6 between 20 and 50?
12	If 2a = b and 3b = c and 4c = 3d, what is the ratio of a:d?
13	Expand: - 5(2m + 3)
14	What is the probability that a randomly chosen card from a standard deck is a red 7 or a jack?
15	Four hundred middle school students are on a boat cruise with access to unlimited soda. If each student drinks an average of four 8-oz cups, how many total gallons of soda will be consumed?
16	Evaluate: 5607 -21013 [Answer with a base ten number.]
17	The coordinates of three vertices of a rectangle are (-4, 1), (5, 1) and (5, 12). What is the sum of the coordinates of the fourth vertex?
18	What is the area, in square units, of the rectangle in #17?

19	What is the smallest integer that is a perfect cube and has two distinct prime factors?
20	How many millimeters are in the length of the longest leg of a triangle with hypotenuse 15 mm and short leg 9 mm?
21	Evaluate: 2 ⁴ - 3 ⁴
22	If 6 jars and 4 cans cost \$28 and 4 jars and 2 cans cost \$19, how much does a jar cost, in dollars?
23	How many three-digit positive integers have no 0's and no 5's?
24	Evaluate: $1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{3}}}}$
25	Two dimes and two pennies are randomly placed in a row. What is the probability that the first and last coins are both pennies?
26	A 52° B Given the triange ABC, how many centimeters are in the length of AB? [Diagram not necessarily drawn to scale!]
27	The sum of two numbers is 48 and their difference is 18. What is the product of the two numbers?
28	Evaluate: $(3.6 \times 10^5) \times (2.4 \times 10^7) \div (1.2 \times 10^{-2})$ [Answer in scientific notation.]
29	A set of five distinct whole numbers has a median of 7 and a mean of 20. One of the numbers is 11. What is the largest possible number in the set?
	Challenge Questions
30	If you can only travel down or to the right along a line segment in the following diagram, how many ways can you get from A to B?
31	Let t(g) = -3g° + g° - 14g + 9. Evaluate t(1).

32	A tennis ball has a volume of 4.5π cubic inches. How many square inches are in the surface area of the cylindrical can, which contains three tennis balls, assuming the radius of the can is the same as that of the balls and the height of the can is the same as the combined diameters of the three balls? Answer as a common fraction in terms of π .
33	If the bases of a trapezoid are represented by the binomial expressions $r - 6$ and $r - 8$ respectively, and its area is represented by the expression $2r^2 - 11r - 21$, what binomial expression would represent the height of the trapezoid?
34	What is the probability that a randomly chosen three-digit positive integer has either two digits that are the same (such as 252) or three digits that are the same (such as 222)?
35	Through how many degrees of a circle does the hour hand of a clock move from 2:11 pm to 5:33 pm?
36	Three different digits from 1 to 9 are selected to form six different three-digit numbers. What is the largest number that must be a factor of the sum of the six three-digit numbers?
37	At Newkirk Middle School students can take classes in Band, Spanish and/or Technology. There are 49 students who take band and 71 students who take Spanish and 53 students who take Technology. If every student must take at least one of these classes, what is the positive difference between the least and the greatest number of students who may attend the school?
38	What is the equation in slope-intercept form of the perpendicular bisector of the line segment with endpoints (2,-4) and (-8,1)?
39	It takes Bridget one hour and 40 minutes to paint a room. With her brother helping, it takes them one hour and 15 minutes. How many hours would it take her brother to paint the room by himself?
40	How many cubic feet are in the volume of the largest cone that fits inside a hemisphere with diameter 24 feet?

"Math is Cool" Championships – 2006–07 Sponsored by: Wenatchee Valley Medical Center 7th Grade – November 17, 2006 Individual Multiple Choice Contest

The Peterson family is going on a road trip to Canada. They plan to leave Seattle and visit Vancouver, Edmonton, Saskatoon, and Winnipeg, and then return along the same route. Table 1 shows the trip data, including the distance from a given point along the route to Seattle and the time the family plans to spend visiting each city:

Table 1:		
Destination	Distance from Seattle	Time spent at each destination, each
		way
Seattle	-	-
Vancouver	240 km	3 hours
Edmonton	1600 km	24 hours
Saskatoon	2100 km	48 hours
Winnipeg	3000 km	5 days (total)

The Peterson family also needs to decide which of their 4 vehicles they wish to drive on the trip. Table 2 provides the data for each of the vehicles including gas mileage, frequency of rest stops in given vehicles, the duration of each stop, and the cost of each stop for snacks and other purchases.

Table 2:

Vehicle	Km per gallon	Time in between	Duration of each	Cost of each stop
		stops	stop	
Hummer	20	No stops	-	-
Jeep Wrangler	30	7 hours	45 minutes	\$25
Honda Accord	48	4 hours	40 minutes	\$20
Honda Civic	60	3 hours	15 minutes	\$25

1	What is the total distance traveled on the round trip?					
	A) 3470 km	B) 13880km	C) 5200 km	D) 6000km	E)6940 km	
2	 If the Petersons average 100km per hour while on the road, and ignoring time spent at rest stops, how long would the round trip vacation take? A)13 d, 18 hrs B)12d, 19hrs C)14d, 7 hrs D)13 d, 15 hrs E)13 d, 8hrs 					
3	Again, ignoring time spent in rest stops, what is the ratio of the time the Peterson's spend at their destination compared to the total time of the trip?A) 3/4B) 6/7C) 7/9D) 5/7E) 9/11					

4	How much longer would the family have to spend in Winnipeg to raise the percent of their vacation time spent at their destinations (again ignoring rest stops) to 90%?				
	A) 10d, 6 hrs	B) 11d, 8hrs	C) 12d, 2hrs	D) 11d, 6hrs	E) 10d, 8hrs
5	If a gallon of g Petersons' dro	gas costs \$3.00 we the Wrangle), how much wou r?	ld it cost to pay	for the gas for the trip if the
	A) \$585	B) \$600	C) \$609	D) \$618	E) \$636
6	The frequency of stops gives the amount of time it takes for the given vehicle to become unbearable, forcing the family to pull over and take a break. However, if the family reaches a destination before the frequency is reached, the time is reset. How many rest stops would be needed for the round trip if the Petersons drove the Honda Accord?				
	A) 11	B) 12	<i>C</i>) 13	D) 14	E) 15
7	Given the cost the family dro	of each stop in the Civic?	n each vehicle, f	ind the cost of	all the rest stops for the round trip if
	A) \$325	B) \$300	C) \$350	D) \$400	E) \$375
8	Accounting on for the family	ly for gas mone to drive.	y and money spe	ent at rest stop	s, which would be the cheapest vehicle
	A) Wrangler	B) Hummer	C) Accord	D) Civic	
9	If the family believed their vacation time was worth \$50 an hour, the rest stops would seem to cost significantly more. Which vehicle spends the most time at rest stops?				
	A) Wrangler	B) Hummer	C) Accord	D) Civic	

"Math is Cool" Championships – 2006-07 Sponsored by: Wenatchee Valley Medical Center 7th Grade – November 17, 2006 Team Contest

1	You wake up in the darkness. Your drawer is contains 20 blue socks and 20 green socks. You don't care whether you wear a green pair or a blue pair, but you want to wear two of the same color. What is the minimum number you must pull out and carry into a lighted room to be sure you have a matching pair?
2	If the average (arithmetic mean) of three distinct positive integers is 70, what is the greatest possible value of one of the integers?
3	Katie's coin collection consists of 21 coins with a total value of 92 cents. If the collection contains only pennies, nickels, and dimes, and the number of nickels is one more than the combined number of dimes and pennies, how many nickels are there?
4	How many times a day do the hands of a standard twelve-hour clock make an angle of eighteen degrees?
5	How many three-digit numbers have no 3s or 5s and at least one 2?
6	Suppose that as we come to each Δ in the expression that follows, we replace each Δ with a plus sign or a minus sign, chosen at random. What is the probability that the result is positive? 11 Δ 10 Δ 8 Δ 6 Δ 2
7	In a survey, 50 people liked artichokes, 56 liked broccoli, 56 liked carrots, and 18 liked none of the above. If 11 liked all three, 25 liked both artichokes and broccoli, 27 liked both broccoli and carrots, and 23 liked both artichokes and carrots, how many people were surveyed?
8	What is the largest number less than 1000 that leaves a remainder of 17 when divided by 23?
9	The measures of two angles of a triangle are in a ratio of 3:2. The third angle is ten more than the sum of the other two. What is the number of degrees in the smallest angle of the triangle?
10	The first digit of a string of 2007digits is a 2. Any two-digit number formed by consecutive digits within this string is divisible by 17 or 23. What is final digit?

"Math is Cool" Championships – 2006–07 Sponsored by: Wenatchee Valley Medical Center 7th Grade – November 17, 2006 Pressure Round Contest

1	What is the smallest positive integer <i>n</i> for which 126/n is a perfect square?
2	In the figure, the semicircles are all congruent with radius r, and are tangent to adjacent semicircles, to the inscribed circle (which also has radius r), and to the circumscribed circle. Find the shaded area in square units if $r = 3$.
3	Four jump-ropes are lying in a tangle on the gym floor. Three people run up and each person grabs a different end of a rope. What is the probability (as a reduced fraction) that two people grab the same rope?
4	The price of a certain calculator has increased by 12% in the past year. If the price last year was \$9 less than the price this year, what is this year's price, in dollars?
5	Curiously, the only printed numerals (digits) that remain the same when rotated a half-turn or 180° are 1, 8, and 0. We will define an upsy number as a positive integer that remains the same when rotated 180°. How many upsy numbers are there less than 100,000? (The first or leading digit of an integer cannot be 0.)

"Math is Cool" Championships – 2006-07 Sponsored by: Wenatchee Valley Medical Center 7th Grade – November 17, 2006 Mental Math Contest

PERSO	DN 1			
1.1	What power of 2 is 256?	8		
1.2	What is the sum of the measures of three interior angles of a regular	324 ^[o]		
	pentagon?			
1.3	What is the probability that a randomly chosen integer from 1 to 11,	5/11		
	inclusive, is prime?			
1.4	What is the positive difference between 5 to the third power and 2 to the	3		
	seventh power.			
PERSO	DN 2			
2.1	Name the sixteenth positive odd number.	31		
2.2	How many zeros does the product of the first twenty even numbers end in?	4		
2.3	On a coordinate plane, what is the slope of the line passing through the	0		
	points negative three comma two and twelve comma two?			
2.4	What is the length of the hypotenuse of a right triangle with legs of length	25 [f†]		
	seven feet and twenty-four feet?			
PERSO	DN 3			
3.1	When two standard dice are rolled, what is the probability that their sum is	1/9		
	10 or 12?			
3.2	Solve for x: x over 72 equals 60 over 96.	45		
3.3	In how many different ways can the letters in the word cha-cha, C-H-A-C-H-	90		
	A, be arranged?			
3.4	What is the measure of one exterior angle of a regular octagon?	45 ^[o]		
PERSON 4				
4.1	What is the perimeter of a square with an area of 169 square inches?	52 [in]		
4.2	If five x equals two x minus thirty-three, what is x?	-11		
4.3	How many multiples of three, that are not multiples of eighteen, are between	28		
	one and one hundred?			
4.4	What is the remainder when you divide 14,283 by 7?	3		

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COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	What is the greatest common factor of the number of months in	6
	a leap year and the number of days in a leap year?	
2	Justin lives in a rectangular shed that is 10 feet wide, 20 feet	12.5
	long, and 8 feet tall. To sleep in, Justin gets a rectangular	[percent]
	cardboard box that is 4 feet tall, 6.25 feet wide, and 8 feet long.	
	What percent of the volume of Justin's shed is occupied by the	
	box? Give your answer as a decimal.	
3	How many square centimeters are in two-and-one-fourth square	22,500 [sq cm]
	meters?	
4	162 people were watching the NHL Finals on a big-screen TV and	19 [people]
	everybody admired the jerseys. 87 people liked Edmonton's	
	jerseys and 94 people liked Carolina's jerseys. How many people	
	liked the jerseys of both teams?	
5	What is 8 factorial divided by 10 factorial?	1/90
6	A paper equilateral triangle of side length 4 units is divided into	48 [units]
	equilateral triangles of side length 1 unit, and these little	
	triangles are cut apart. What is the sum of the number of units	
	in the perimeters of all these little triangles?	
7	Jenny is careless. On Tuesday she lost one-third of all her toys.	54 [toys]
	Then, on Wednesday she lost half of the toys that were left. On	
	Thursday Jenny lost twelve more toys and was left with six toys.	
	How many toys did Jenny start with?	
	Extra Drahlam Only if Needed	
	Extra Prodiem - Univ it ineeded	
8	Find the sum of the 3 largest prime numbers less than 100.	269

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COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	Find the sum of the numerator and denominator when the	69
	sum of six-fifths and five-fourths is reduced.	
2	Aaron has played 7 games of Tetris per day for the past 12	91
	days. If the games lasted an average of 13 minutes each,	1440
	what fraction of his time did Aaron spend playing Tetris?	
3	A farmer got 300 bales of hay from a 10-acre field and	28 [bales, or
	400 bales from a 15-acre field. What was the average	bales per acre]
	number of bales per acre?	
4	Vampires and werewolves are playing tag in London.	4 [vampires]
	Vampires have 22 teeth while werewolves have 38. If I	
	spot 12 heads and 392 teeth, how many vampires do I see?	
5	24 people are standing equally spaced in a circle, numbered	19
	1, 2, 3, and so on. What number is on the person directly	
	opposite person #7?	
6	Barry Bonds takes 6 pills a day: 1 orange, 1 white, 2 yellow,	16 [pills]
	and 2 red. Barry puts exactly three days' worth of pills in	
	a jar. If Barry were to randomly pull out pills from the jar,	
	what would be the least number he would have to take out	
	to ensure he had all the pills he needed for 1 day?	
7	A divided by B equals C divided by D, and A equals 2 times	1
	B Then by what number would you have to multiply C to	2
	find D2	
	Extra Problem - Only if Needed	
8	What is the measure in degrees of each exterior angle of a regular	45 [degrees]
	octagon?	

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COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	My 3-digit number is divisible by nine. The units digit is half the	864
	hundreds digit. What is the largest my number could be?	
2	A bag of popcorn has 336 pieces. If Levi eats 3 and one-third	10 [pieces]
	bags of popcorn while he watches a movie that is 112 minutes	
	long, how many pieces of popcorn did he eat each minute?	
3	What is half the sum of one-eighth and one-sixth?	7/48
4	Steve is unlucky. On Friday the 13 th of October he was hit by	3 [percent]
	lightning, attacked by African Killer Bees, and knocked off a 20-	
	foot tall scaffold by a gust of wind. The probability of surviving	
	a lightning strike is 50%, an attack by African Killer Bees 20%,	
	and a fall from 20 feet is 30%. What is the percent chance	
	that Steve survived this difficult day?	
5	The average of A and B is C, and the average of C and 10 is 12.	24
	If A is 4, what is B?	
6	Leah can write 240 digits with one stick of Cheapo Lead. She	5 [sticks]
	needs to number the pages of her new 430-page notebook. How	
	many sticks of Cheapo Lead will Leah need to buy?	
7	In Sharon's Seashore Shop, she sells 77 seashells each 60	4620
	seconds. Seek the sum of the seashells sold by Sharon from 6	
	to 7 at sunset on Sunday the second of September.	
	Extra Problem - Only it Needed	
8	What is the area, in square inches, of a circle inscribed in a	25 pi [sq inches]
	square with area 100 square inches?	

"Math is Cool" Championships – 2006–07

PreAlgebra & Algebra I - November 17, 2006

Final Score:

KEY

School Name_____Team #_____ Proctor Name_____

Room #

First Score

STUDENT NAME_____

Individual Contest – Score Sheet DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0		Answer	1 or 0	1 or 0
1	3.32			21	-65		
2	9/35			22	[\$] 5.00		
3	-56			23	512 [integers]		
4	3000			24	58/41	· ·	
5	Decagon			25	1/6		
6	3 ⁷			26	7 [cm]		
7	[y =] - 16/3			27	495		
8	4 [factors]			28	7.2 × 10 ¹⁰		
9	2			29	81		
10	4,444,444			30	38 [ways]		
11	180			31	-7		
12	1/8 or 1:8			32	$\frac{63}{2}\pi$ [sq in]	·	
13	-10m -15 {or} -10m + (-15)			33	2r + 3		
14	$\frac{3}{26}$			34	7/25		
15	100 [gallons]			35	101[•]		
16	223[10]			36	222		
17	8			37	102		
18	99 [un²]			38	y = 2x + 9/2		
19	216			39	5 [hours]		
20	12 [mm]			40	576π [ft³]		

"Math is Cool" Championships – 2006–07 7th Grade – November 17, 2006	First Score (out of 18)	
School NameTeam # Proctor NameRoom #		
STUDENT NAME		

INDIVIDUAL MULTIPLE CHOICE - 15 minutes

This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet. No talking during the test.

	Answer	-1, 0 or 2	-1, 0 or 2
1	D		
2	A		
3	E		
4	D		
5	В		
6	В		
7	С		
8	С		
9	С		

DO NOT WRITE IN SHADED REGIONS

"Math is Cool" Championships – 2	006-07	First Score
7th Grade – November 17, 2006		(out of 20)
School Name	Team #	
Proctor Name	Room #	
STUDENT NAME		_

Team Contest - Score Sheet

TEAM TEST - 15 minutes

When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as **2 or 0**. Record all answers on the colored answer sheet.

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	3		
2	207		
3	11		
4	44		
5	154		
6	13/16		
7	116		
8	983		
9	34 ^[o]		
10	3		

"Math is Cool" Championships 7th Grade - November 17,	First Score	
School Name	Team #	
Proctor Name	Room #	

PRESSURE ROUND - 10 minutes

STUDENT NAME

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

Answer		
1	14	
2	9π	
3	3/7	
4	[\$] 84	
5	34 [numbers]	

Pressure Round Answers