"Math is Cool" Championships – 2006-07

Sponsored by: IEEE - Central Washington Section

Geometry – November 17, 2006

Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by <u>all</u> involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
  - *For problems dealing with money, a decimal answer should be given.*
  - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of π where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

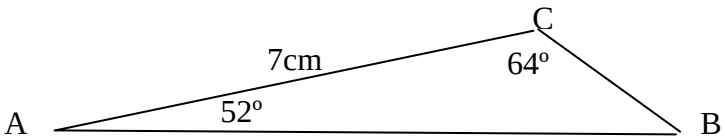
"Math is Cool" Championships – 2006-07

Sponsored by: IEEE - Central Washington Section

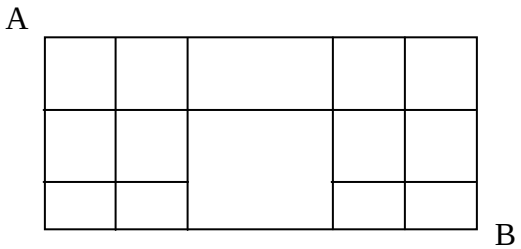
Geometry – November 17, 2006

Individual Contest

1	Evaluate: 2.371 x 1.4 [Round to the nearest hundredth.]
2	Evaluate: $\frac{3}{7} \div \frac{5}{3}$
3	Evaluate: $2^3 + 8 \times (-4) \div .5$
4	Evaluate: $4! \times 5^3$
5	What is the name of the polygon with twice as many edges as a pentagon?
6	Express 2187 as a power of 3.
7	Solve for y: 21y + 14 = -98
8	How many postive factors does the number 343 have?
9	The equation for a circle with its center at (3,-2) and a radius of 5 is $(x - 3)^2 + (y + 2)^2 = 25$. Two points on the circumference of the circle have an x-coordinate of 7. What is the sum of the y-coordinates of these two points?
10	What is the mean of the following seven numbers? 1,234,567 3,456,712 5,671,234 7,123,456 2,345,671 4,567,123 and 6,712,345
11	What is the sum of all multiples of 6 between 20 and 50?
12	If 2a = b and 3b = c and 4c = 3d, what is the ratio of a:d?
13	Expand: (3m - 5)(m + 12)
14	What is the probability that a randomly chosen card from a standard deck is a red 7 or a jack?
15	Four hundred middle school students are on a boat cruise with access to unlimited soda. If each student drinks an average of four 8-oz cups, how many total gallons of soda will be consumed?
16	Evaluate: $560_7 - 2101_3$ [Answer with a base ten number.]
17	The coordinates of three vertices of a rectangle are (-4, 1), (-7, 5) and (4, 7). What is the sum of the coordinates of the fourth vertex?
18	How long is the diagonal of the rectangle in #17?

19	What is the smallest integer that is a perfect cube and has two distinct prime factors?
20	What is the length of the longest leg of a triangle with hypotenuse 41 mm and short leg 9 mm?
21	Evaluate: $2^9 - 3^6$
22	If 6 jars and 4 cans cost \$28 and 5 jars and 2 cans cost \$19, how much does a jar cost, in dollars?
23	How many three-digit positive integers have no 0's and no 5's?
24	Evaluate: $1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \frac{1}{3}}}}$
25	Four dimes and four pennies are randomly placed in a row. What is the probability that the first and last coins are both pennies?
26	 Given the triange ABC, how many centimeters are in the length of AB? [Diagram not necessarily drawn to scale!]
27	The sum of two numbers is 48 and their product is 18. What is the sum of the reciprocals of the two numbers?
28	Evaluate: $(3.6 \times 10^5) \times (2.4 \times 10^7) \div (1.2 \times 10^{-2})$ [Answer in scientific notation.]
29	A set of five distinct whole numbers has a median of 7 and a mean of 20. One of the numbers is 11. What is the largest possible number in the set?

Challenge Questions

30	If you can only travel down or to the right along a line segment in the following diagram, how many ways can you get from A to B? 
31	Let $f(g) = -3g^3 + g^2 - 14g + 9$. Evaluate f(4).

32	A tennis ball has a volume of 4.5π cubic inches. How many square inches are in the surface area of the cylindrical can, which contains three tennis balls, assuming the radius of the can is the same as that of the balls and the height of the can is the same as the combined diameters of the three balls? Answer as a common fraction in terms of π.
33	If the bases of a trapezoid are represented by the binomial expressions $r - 6$ and $r - 8$ respectively, and its area is represented by the expression $2r^2 - 11r - 21$, what binomial expression would represent the height of the trapezoid?
34	What is the probability that a randomly chosen three-digit positive integer has either two digits that are the same (such as 252) or three digits that are the same (such as 222)?
35	Through how many degrees of a circle does the hour hand of a clock move from 2:11 pm to 5:33 pm?
36	Three different digits from 1 to 9 are selected to form six different three-digit numbers. What is the largest number that must be a factor of the sum of the six three-digit numbers?
37	At Newkirk Middle School students can take classes in Band, Spanish and/or Technology. There are 49 students who take band and 71 students who take Spanish and 53 students who take Technology. If every student must take at least one of these classes, what is the positive difference between the least and the greatest number of students who may attend the school?
38	What is the equation in slope-intercept form of the perpendicular bisector of the line segment with endpoints (2,-4) and (-8,1)?
39	It takes Bridget one hour and 40 minutes to paint a room. With her brother helping, it takes them one hour and 15 minutes. How many hours would it take her brother to paint the room by himself?
40	How many cubic feet are in the volume of the largest cone that fits inside a hemisphere with diameter 24 feet?

"Math is Cool" Championships – 2006-07

Sponsored by: IEEE - Central Washington Section

8th Grade – November 17, 2006

Individual Multiple Choice Contest

The MIC Cell Phone Company is the only cell phone service provider in MathLandia. They offer five plans to choose from based upon expected use in minutes each month. Below is a table describing each plan's fixed cost per month, number of free minutes, price per minute after the free minutes are used, market share relative to the other plans, and customer satisfaction with the plan. The following questions are based on the table (some data is intentionally blank):

Plan:	**A**	**B**	**C**	**D**	E
Cost / month:	0	20	50	100	150
Free Minutes:	0		30	75	unlimited
Cost / minute after					
free minutes used:	1.25	0.75	0.5		N/A
MIC Market Share:	10%	25%	25%	20%	20%
Customer Satisfaction:	70%	80%	60%	20%	25%

1	Fermat subscribes to plan C and had a bill for $80 last month. How many minutes did he talk for last month? A) 90 B) 60 C) 120 D) 80 E) 100
2	Euclid is subscribed to plan D. His bill for January was $107 after speaking for 95 minutes. What is the price per minute of plan D? A) $0.20 B) $0.25 C) $0.30 D) $0.35 E) $0.40
3	Descartes uses plan B. If he spoke for 30 minutes last month and was billed $35, how many minutes does he get free each month? A) 5 B) 10 C) 15 D) 20 E) 25
4	Which plan would be the least expensive for Albert, who plans to only talk for 55 minutes a month? A) A B) B C) C D) D E) E

5	After how many minutes does plan C cost the exact same per month as plan D? A) 80 B) 85 C) 90 D) 95 E) 100
6	Analyzing the five plans, Sampson, the chief accountant at MIC Phones, realizes one of the plans is never the most cost efficient plan, that is, there is always a better choice for a given number of minutes. Which plan did Sampson discover was never a good deal? A) E B) D C) C D) B E) A
7	MIC Phones is the middle of a customer survey where they randomly call subscribers to their services. What is the probability that of the first three people they call, two have plan D and one has plan A? A) 1/250 B) 1/125 C) 3/250 D) 4/125 E) 1/50
8	Tycho is looking to subscribe to a plan. He knows he will be talking some amount of minutes between 10 and 100 a month, but he has no idea the exact number of minutes. Which phone will be most likely to be a better deal than the others? A) A B) B C) C D) D E) E
9	After conducting their survey, MIC arrives at the Customer Satisfaction numbers seen above. WMIC, the local news station is planning to interview one random MIC cell phone user. What is the probability that random cell phone user is unsatisfied with their service? A) 52% B) 48% C) 50% D) 51% E) 49%

"Math is Cool" Championships – 2006-07

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8th Grade – November 17, 2006

Team Contest

1	The measures of two angles of a triangle are in a ratio of 3:2. The third angle is ten more than the sum of the other two. What is the number of degrees in the smallest angle of the triangle?
2	You wake up in the darkness. Your drawer is contains 20 blue socks and 20 green socks. You don't care whether you wear a green pair or a blue pair, but you want to wear two of the same color. What is the minimum number you must pull out and carry into a lighted room to be sure you have a matching pair?
3	Katie's coin collection consists of 21 coins with a total value of 92 cents. If the collection contains only pennies, nickels, and dimes, and the number of nickels is one more than the combined number of dimes and pennies, how many nickels are there?
4	How many times a day do the hands of a standard twelve-hour clock make an angle of eighteen degrees?
5	How many three-digit numbers have no 3s or 5s and at least one 2?
6	Suppose that as we come to each Δ in the expression that follows, we replace each Δ with a plus sign or a minus sign, chosen at random. What is the probability that the result is positive? 11 Δ 10 Δ 8 Δ 6 Δ 2
7	In a survey, 50 people liked artichokes, 56 liked broccoli, 56 liked carrots, and 18 liked none of the above. If 11 liked all three, 25 liked both artichokes and broccoli, 27 liked both broccoli and carrots, and 23 liked both artichokes and carrots, how many people were surveyed?
8	What is the largest number less than 1000 that leaves a remainder of 17 when divided by 23?
9	Each successive term of a geometric sequence is found by multiplying the preceding term by a common ratio. For example, in the sequence {1, 3, 9, 27, 81, . . .} the common ratio is 3. What is the sum of the first six terms of a geometric sequence with a first term of 54 and a common ratio of $\frac{2}{3}$?
10	The first digit of a string of 2007digits is a 2. Any two-digit number formed by consecutive digits within this string is divisible by 17 or 23. What is final digit?

"Math is Cool" Championships – 2006-07

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8th Grade – November 17, 2006

Pressure Round Contest

1	An airplane moving away from Dr. Quest travels at $\frac{1}{4}$ the speed of Dr. Quest's own supersonic jet. If it takes Dr. Quest 400 minutes to catch the plane from a starting distance of 4000 miles, what is the speed of Dr. Quest's jet, in miles per hour?
2	When the base-10 number 2006 is expressed in base 4, find the sum of its digits, and express this sum in base 4.
3	Let m and n be 2-digit positive integers. If m! divided by n! is a positive integer ending in 11 zeros, find the largest possible value for n.
4	Imagine that a perfectly spherical orange is cut into 8 congruent pieces with 3 cuts at right angles to each other, each passing through the center of the orange. What is the percent change in the surface area of the cut orange, compared to the original uncut orange?
5	How many odd integer multiples of 13 are greater than 26 but less than 2006?

"Math is Cool" Championships – 2006-07

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8th Grade – November 17, 2006

Mental Math Contest

PERSON 1		
1.1	What power of 2 is .125?	-3
1.2	What is the sum of the measures of three interior angles of a regular pentagon?	324[o]
1.3	What is the probability that a randomly chosen integer from 1 to 11, inclusive, is prime?	5/11
1.4	What is the positive difference between 5 to the third power and 2 to the seventh power.	3
PERSON 2		
2.1	Name the sixteenth positive odd number.	31
2.2	How many zeros does the product of the first twenty even numbers end in?	4
2.3	What is the product of the slope and the y-intercept for the graph of the equation y equals negative three-fifths x minus fifteen?	9
2.4	Calculate the length of the hypotenuse of a right triangle with legs of length 2 cm and 4 cm. Answer in simplest radical form.	$2\sqrt{5}$
PERSON 3		
3.1	When two standard dice are rolled, what is the probability that their sum is 10 or 12?	1/9
3.2	Solve for x: x over 72 equals 60 over 96.	45
3.3	In how many different ways can the letters in the word cha-cha, C-H-A-C-H-A, be arranged?	90
3.4	What is the measure of one exterior angle of a regular octagon?	45[o]
PERSON 4		
4.1	What is the area of a square with diagonal $8\sqrt{2}$ inches?	64 [in^2]
4.2	If five x equals two x minus thirty-three, what is x?	-11
4.3	How many multiples of three, that are not multiples of eighteen, are between one and one hundred?	28
4.4	What is the remainder when you divide 14,283 by 7?	3

"Math is Cool" Championships – 2006-07

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8th Grade – November 17, 2006

COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	In this problem, a different letter represents a different digit and the same letter represents the same digit. If MAMA (spelled M-A-M-A) represents a 4-digit integer that is divisible by 3 and 5 but not by 6, what is the largest that MAMA can be?	7575
2	What is the fourth root of the square root of 256?	2
3	Ashley's TV has six channels, two of which have bad reception. She randomly checks two channels. What is the probability that at least one has bad reception?	3/5
4	A piece of treated wood has a density of 3 ounces per cubic inch. How many pounds does a 2-inch by 4-inch by 3-foot board made of this wood weigh?	54 [pounds]
5	Cindy has a big box of videotapes to shelve. When she tries to put them into stacks of 4, 5, 6, or 7, she is always one short in the last stack. What is the minimum number of videotapes Cindy could possibly have?	419 [videotapes]
6	The side opposite the 60-degree angle in a right triangle is 8 times the square root of 3 inches long. What is the length of the hypotenuse, in inches?	16 [inches]
7	There are eight chairs around a circular table. Two married couples and four single people are to sit around the table so that each married person sits next to his or her spouse. How many ways can this be done?	480 [ways]
	Extra Problem – Only if Needed	
8	Evaluate the following expression for x equals 2: Two times the cube of x minus x plus eighteen.	32

"Math is Cool" Championships – 2006-07

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8th Grade – November 17, 2006

COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	The product of the weight of a rock and the distance a catapult can throw the rock is constant. Cody catapults a 4-pound rock at a target 80 feet away, but it falls 20 feet short. What is the weight, in pounds, of the heaviest rock with which Cody's catapult can reach the target?	3 [pounds]
2	What is the 2006th digit after the decimal point in the decimal representation of 1 over 7?	4
3	What is the least possible perimeter, in inches, for an isosceles triangle with sides of 9 and 4 inches?	22 [inches]
4	A ladder is leaning against a wall. The base of the ladder suddenly slips and slides fourteen feet further from the base of the wall and the top of the ladder also slides fourteen feet down. If the top of the ladder is now ten feet above the ground, how many feet long is the ladder?	26 [feet]
5	The length of a square is increased by twenty percent but the width is decreased by twenty percent. By what percent does the area of the square change?	4 [or accept negative 4] [percent]
6	Find the tenth term of an arithmetic series whose first term is 10 and for which the sum of the first 5 terms is 80.	37
7	Find the largest fraction that produces an integer quotient when divided into each of one-half, two-thirds, and five-eighths.	$\frac{1}{24}$
	Extra Problem – Only if Needed	
8	How many diagonals can be drawn in a heptagon?	14 [diagonals]

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8th Grade – November 17, 2006

COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	If *x* plus *y* equals 20, and *x* squared plus *y* squared equals 300, what is the value of *x* times *y*?	50
2	Anna has 3 stacks of checkers. She has 36 checkers total. Stack 1 is the largest and has 3 times as many checkers as Stack 2. Stack 3 is the smallest and only has half as many checkers as Stack 2. How many checkers are in Stack 2?	8 [checkers]
3	For what value or values of *x* is the rational expression x-squared minus 4 over the quantity 3x plus 9 undefined?	negative 3 [or minus 3]
4	Cam has 10 blue socks, 2 red socks, and 14 gray socks. One morning, Cam realizes that he is wearing a blue sock and a red sock. He throws these two socks away and randomly draws two new socks from his drawer. What is the probability that the two new socks match?	$\frac{127}{276}$
5	Divide 15 factorial by the product of 10 factorial and 5 factorial.	3003
6	Find the volume, in cubic centimeters, of a right circular cone with diameter 40 centimeters and height 30 centimeters.	4000 pi [cubic cm]
7	Heads up! If this is this more than that, and that is this more than those, what is this plus those? I said, If this is this more than that, and that is this more than those, what is this plus those?	0
	Extra Problem – Only if Needed	
8	What is the area in square units of a regular hexagon with side length 4 units?	$24\sqrt{3}$ [or 24 root 3] [sq units]

"Math is Cool" Championships – 2006-07

Geometry – November 17, 2006

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

KEY

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	3.32		
2	9/35		
3	-56		
4	3000		
5	Decagon		
6	3^7		
7	[y =] – 16/3		
8	4 [factors]		
9	-4		
10	4,444,444		
11	180		
12	$\frac{1}{8}$ or 1:8		
13	$3m^2 + 31m - 60$		
14	3/26		
15	100 [gallons]		
16	$223_{[10]}$		
17	12		
18	$5\sqrt{5}$ [units]		
19	216		
20	40 [mm]		

	Answer	1 or 0	1 or 0
21	-217		
22	[$] 2.50		
23	512 [integers]		
24	$\frac{58}{41}$		
25	$\frac{3}{14}$		
26	7 [cm]		
27	8/3		
28	7.2×10^{10}		
29	81		
30	38 [ways]		
31	-223		
32	$\frac{63}{2}\pi$ [sq in]		
33	2r + 3		
34	7/25		
35	$101^{[o]}$		
36	222		
37	102		
38	$y = 2x + \frac{9}{2}$		
39	5 [hours]		
40	576π [ft^3]		

First Score

(out of 18)

"Math is Cool" Championships – 2006-07

8th Grade – November 17, 2006

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

INDIVIDUAL MULTIPLE CHOICE - 15 minutes

This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. ***Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet.*** *No talking during the test.*

DO NOT WRITE IN SHADED REGIONS

Answer		-1, 0 or 2	-1, 0 or 2
1	A		
2	D		
3	B		
4	B		
5	C		
6	B		
7	C		
8	B		
9	E		

"Math is Cool" Championships – 2006-07

8th Grade – November 17, 2006

First Score
(out of 20)

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Team Contest – Score Sheet

TEAM TEST - 15 minutes

When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as ***2 or 0****. Record all answers on the colored answer sheet.*

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	$34^{[o]}$		
2	3		
3	11		
4	44		
5	154		
6	13/16		
7	116		
8	983		
9	1330/9		
10	3		

First Score

"Math is Cool" Championships – 2006-07

8th Grade – November 17, 2006

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

PRESSURE ROUND - 10 minutes

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

Pressure Round Answers

	Answer
1	800 [mph]
2	$23_{[4]}$
3	49
4	150 [%]
5	76 [multiples]