"Math is Cool" Championships – 2006-07

Sponsored by: Basic American Foods

Geometry & Algebra II – November 1, 2006

Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by <u>all</u> involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
  - *For problems dealing with money, a decimal answer should be given.*
  - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of π where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

"Math is Cool" Championships – 2006-07

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Geometry & Algebra II – November 1, 2006

Individual Contest

1	Evaluate as a mixed number: $5\frac{2}{3} - 2\frac{3}{4}$
2	How many minutes are there in one day?
3	Evaluate: $(9 + 8 \cdot 7 - 6 \cdot 5 \div 4 - 2) \div 3$
4	In how many points do the graphs of $y = e^x + 1$ and $y = 2\sin(2\pi x) + 5$ intersect?
5	What value(s) of a satisfy $3a - 8 = 34$?
6	What value(s) of *m* satisfy $m^2 + 12m - 28 = 0$?
7	At what point, in the form (x, y), does the line $y = 4x + 15$ intersect the line $-2x + 3y = 15$?
8	If u, v, and w are all positive integers, the average of u and v is 47 and the average of v and w is 38, what is the smallest possible value of u?
9	Jan drove 700 km at 140 kmph, then 200 km at 80 kmph. What was her average speed, in kmph, for the entire trip?
10	*C* is 67 less than D. If *C* is 44, what is the sum of *C* and D?
11	What quadrant is the point (15, -8) in?
12	What are the coordinates, in (*x*, *y*) form, of the vertex of the parabola $y = x^2 + 4x - 9$?
13	If $s(r) = 3e^{2r} + 1$, determine $s^{-1}(r)$.
14	What is the area, in square centimeters, of a triangle with sides measuring 5, 8, and 9 cm?
15	What is the smaller angle, in degrees, between the hands of a standard twelve-hour clock at 5:50?
16	What is the area, in square centimeters, of an equilateral triangle with sides measuring 12 cm?
17	A non-degenerate triangle has two sides measuring 8 and 15 cm. How many integers could be the length, in centimeters, of the third side?
18	A pentagon has interior angles measuring 80, 90, 100, and 110 degrees. What is the measure, in degrees, of the fifth interior angle?
19	What is the area, in square centimeters, of a 40 degree sector of a circle with a radius of 6 cm?
20	What is the surface area, in square centimeters, of a right rectangular prism with edges measuring 2, 3, and 4 cm?

21	A triangle has sides measuring 6, 8, and 9 cm, while a similar triangle has two sides measuring 12 and 9 cm. What is the length, in centimeters, of the third side of the larger triangle?
22	A triangle has sides measuring 6, 7, and 8 cm. An angle bisector is drawn to the 6 cm side. What is the length, in centimeters, of the smaller of the two segments into which the 6 cm side is divided?
23	How many of the numbers between 40 and 70 are prime?
24	Evaluate: $i^{147}\left(i^{222}\right)-\frac{i^{213}}{i^{344}}$, where $i=\sqrt{-1}$
25	Evaluate: $\log_4 128$
26	If $d(c)=(2c+1)(5c-6)+\frac{8}{c}$, evaluate $d(2)$.
27	If a is directly proportional to b and a = 4 when b = 3, what is the value of a when b = 5?
28	What value(s) of g satisfy $2^{2g+1}-5\cdot 2^{g}=3$?
29	What is the greatest common factor of 108 and 240?

Challenge Questions

30	What is the area of the ellipse represented by $2x^2-28x+3y^2+24y=-140$?
31	What is the length of a latus rectum of the hyperbola represented by $9y^2-54y-16x^2+64x=127$?
32	Two circles with radii of 9 and 12 cm have their centers separated by 29 cm. What is the length, in centimeters, of one of their common internal tangents?
33	What is the sum of the squares of the roots of $5k^2+3k-1=0$
34	How many positive five digit palindromes are multiples of four?
35	Evaluate: $\cfrac{3}{2+\cfrac{3}{2+\cdots}}$
36	Evaluate: $1324\cdot 1276$
37	My favorite number is a positive four-digit integer. My second favorite number is also a positive four-digit integer, and uses the same four digits. When I subtract the two, I get a third four-digit number, three of whose digits are 2, 3, and 9. What is the fourth digit of the difference?
38	Two circles with radii of 24 and 54 are tangent to one another, and one of their common external tangents is drawn. What is the radius of a third circle inscribed tangent to both original circles and their common external tangent?
39	How many integers are in the range of the function $y(x)=\frac{4x^2+75}{2x^2+3}$?
40	In the cryptarithm shown below, each instance of a particular letter represents the same digit (0-9), and no two different letters represent the same digit. What is the maximum possible value of the five-digit number ABCDE? $\begin{array}{r} ABCD \\ -\,BDEA \\ \hline CBA \end{array}$

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9th & 10th Grade – November 1, 2006

Individual Multiple Choice Contest

NOTA = None of the Above

1 What is the sum of the roots of $x^3 + 6x^2 + 11x + 6$?

A) 6 B) 5 C) $\frac{11}{6}$ D) $\frac{1}{6}$ E) NOTA

2 If a circle of radius 2 rolls (like a gear) around a circle of radius 80, how many revolutions will it make before returning to the place it started?

A) $4\sqrt{5}$ B) 160 C) 41 D) 40 E) NOTA

3 Evaluate: $\sqrt{9+\sqrt{9+\sqrt{9+\cdots}}}$

A) 3 B) $3+\frac{\sqrt{37}}{9}$ C) $\frac{1+\sqrt{37}}{2}$ D) $1+\sqrt{37}$ E) NOTA

4 When 200^6 is converted to base 4, how many zeroes does it have at the end?

A) 3 B) 12 C) 6 D) 9 E) NOTA

5 A certain trapezoid can be decomposed into a unit square and two similar, but incongruent, triangles. One side of one of the triangles has length $\frac{1}{2}$. What is the trapezoid's perimeter?

A) $3+\sqrt{5}$ B) $\frac{9+3\sqrt{5}}{2}$ C) $\frac{13+3\sqrt{5}}{2}$ D) $3\frac{1}{2}$ E) NOTA

6 John flips two coins and Sarah flips one coin. What is the probability that John gets more heads than Sarah?

A) $\frac{1}{4}$ B) $\frac{3}{16}$ C) $\frac{1}{2}$ D) $\frac{1}{16}$ E) NOTA

7 What is the area of a circle inscribed in a regular hexagon with perimeter 12?

A) $\pi\sqrt{3}$ B) $3\pi\sqrt{3}$ C) $3\pi\sqrt{2}$ D) 3π E) NOTA

8 Below is a board from the game of Minemop. Numbers signify the number of adjacent grid squares, including diagonals that have mines in them. Squares with numbers in them never have mines in them. How many mines are adjacent to **x**?

1	2	2	2	1
3		X		
				1
3			1	1

A) 0 B) 1 or 2 C) 3 or 4 D) 5 or 6 E) NOTA

9 Eustace and Elizabeth will each show up for a date at a random instant during an hour and wait fifteen minutes for the other, what is the probability that they will meet?

A) $\frac{7}{16}$ B) $\frac{1}{2}$ C) $\frac{7}{8}$ D) $\frac{5}{16}$ E) NOTA

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9^{th} & 10^{th} Grade – November 1, 2006

Team Contest

1	Express 45,629 in scientific notation rounded to two significant figures.
2	Tom's age is nine years less than the sum of Cherie and Katie's ages, while Cherie's age, in years, is eight more than the average of Tom and Katie's ages. If Tom's age minus Cherie's age is half of Katie's age, what is Tom's age, in years?
3	Lori likes extra-strength fruit punch that is 120% the strength of normal fruit punch. Tom likes weak fruit punch that is 75% the strength of normal fruit punch. How many **milliliters** of water should Tom add to three liters of Lori's punch to produce his desired punch strength?
4	Point P is 6 cm from the center of circle O, which has a diameter of 8 cm. What is the length, in centimeters, of the tangent from P to circle O?
5	When five cards are drawn from a standard 52-card deck, what is the probability that none of the five cards have the same rank?
6	Circles A and B are concentric, and a 10 m chord of circle B has a segment which is an 8 m chord of circle A. What is the area, in square meters, of the region between the two circles?
7	The sum of the first five terms of an arithmetic sequence is 2280, while the sum of the first fifteen terms of the sequence is 6240. What is the sum of the first twenty terms of the sequence?
8	How many terms are there in the expansion of $(2n+3o-4)^5$ after like terms are combined?
9	Seven students take a test, each earning an integer score between 0 and 100 inclusive. If the mean of their scores is 68, the mode of their scores is 64, and the range of their scores is 28, what is the lowest possible value of the median of their scores?
10	What is the product, in base seven, of the base seven numbers 23_7 and 34_7?

"Math is Cool" Championships – 2006-07

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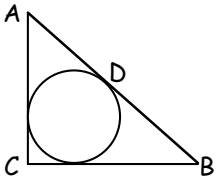
9^{th} & 10^{th} Grade – November 1, 2006

Pressure Round Contest

1	Give the letters of all of the following expressions that are equal to 1. (A) $\left(\left(2^0\right)^0\right)^6$ (B) $2^{0^{0^6}}$ (C) $\left(2^0\right)^{0^6}$ (D) $\left(20^0\right)^6$ (E) $\left(2^0\right)\left(0^6\right)$
2	Julian has at least 200 coins in his collection. Together, dimes and half-dollars make up exactly $\frac{1}{8}$ of the total number of coins. If dimes and half-dollars separately make up $\frac{1}{x}$ and $\frac{1}{y}$ of the total, respectively, where x and y are integers, what is the least number of half-dollars Julian could have in his coin collection?
3	Miya is making spinners for Games Day in a 5th grade math class. Each circular spinner card is divided into 4 equal quadrants. She uses purple, orange, green, and black to color the spinners. Each spinner uses at least 2 different colors and each quadrant is a single color. How many different spinners can Miya make? (Two spinners are different if they cannot be made to appear the same by rotation in the plane.)
4	A circle inscribed in right triangle ABC is tangent to the hypotenuse at point D. Find the area of the triangle, in square cm, if AD = 4 cm and BD = 21 cm. 
5	When $(ax+by)^2$ is expanded, the expansion is $a^2x^2+2abxy+b^2y^2$, and the sum of the coefficients of this expansion is found by adding $a^2+2ab+b^2$. What is the sum of the coefficients of the expansion of $(3x-4y)^5$?

"Math is Cool" Championships – 2006-07

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9th & 10th Grade – November 1, 2006

Mental Math Contest

PERSON 1		
1.1	When the quantity "a" plus two "b" is raised to the fifth power and expanded, what is the sum of the coefficients of the resulting polynomial?	243
1.2	How many possible integer lengths are there for the third side of a triangle whose other two sides are 8 and 11?	15
1.3	How many terms are there in the arithmetic sequence, 3, 4.5, …, 63?	41
1.4	What is the probability of getting at least 3 heads on 5 flips of a fair coin?	1/2
PERSON 2		
2.1	What is the sum of the solutions to the equation: 2 x squared plus 9 x equals 13.	-9/2
2.2	For what value of x is the log base 4 of x equal to negative one-half?	1/2
2.3	What is the sum: 2 plus two-thirds plus two-ninths plus two-twenty-sevenths and so on?	3
2.4	What is the lateral surface area of a right cone with base radius equals 6 and height equal 8?	60π
PERSON 3		
3.1	How many terms will be in the simplified expansion of the quantity a plus b plus c, squared?	6
3.2	What is the sum of the numbers 2 plus 4 plus 8 plus 16 plus … plus 512?	1022
3.3	The number of diagonals that can be drawn in a certain regular polygon is 35, how many sides does the polygon have?	10
3.4	John travels 40 miles per hour to visit his aunt and 60 miles per hour home, what was his average speed?	48 [mph]
PERSON 4		
4.1	A square is inscribed in a circle of radius 2, what is the area inside the circle but outside the square?	$4\pi - 8$
4.2	What is 35 times 45?	1575
4.3	What is the sum of the positive integers from 1 to 40 minus the sum of the positive integers from 1 to 20?	610
4.4	A bag has 6 blue marbles and 4 red marbles, what is the probability of drawing two marbles of the same color without replacement?	7/15

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9th & 10th Grade – November 1, 2006

COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	What is the surface area of a hemisphere with radius 4?	48 π [un^2]
2	How many positive factors does the number 36 have?	9 [factors]
3	What is the sum 5 plus 10 plus 15 plus … plus 45?	225
4	If the square root of the quantity, a squared plus b squared, equals a plus b then what is a times b?	0
5	If the probability of event A is two-fifths and the probability of event B is also two-fifths, what is the smallest value that the probability of A or B could be?	2/5
6	What are the coordinates of the vertex of the parabola: y equals x squared minus 6 x plus 4?	(3,-5)
7	In an arithmetic sequence, the third element is 10 and the eleventh element is 14; what is the seventeenth element?	17
	Extra Problem – Only if Needed	
8	What is the area of an isosceles trapezoid with one base of 8, the other base is 4 and the height is 3?	18 [un^2]

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9th & 10th Grade – November 1, 2006

COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	What is the largest triangular area that can be made with 20 meters of fencing?	$\frac{100\sqrt{3}}{9}$ [cm^2]
2	If the probability of A is one-fourth and the probability of B is three-tenths, what is the largest value that the probability of A and B could be?	$\frac{1}{4}$
3	What is the sum of 7 choose 0, 7 choose 2, 7 choose 4, and 7 choose 6?	64
4	What are the coordinates of the point that is two-thirds of the way along the line segment going from the point 2 comma 5 to the point 5 comma 2?	(4,3)
5	What is the sum of the distinct prime factors of 2007?	226
6	All face cards, aces and jokers are removed from a standard deck of cards. What is the probability of drawing two prime numbers without replacement?	4/21
7	What is the highest number of non-overlapping circles of radius 1 that will fit inside a circle of radius 3?	7
	Extra Problem – Only if Needed	
8	What is the least common multiple of 27 and 2007?	6021

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9th & 10th Grade – November 1, 2006

COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	What is the sum of the measures of the interior and exterior angles, in degrees, of an equilateral triangle?	540 [°]
2	What is the remainder when 7 to the power 7 is divided by 3?	1
3	When the base 10 number 12 factorial is written in base 8, how many zeroes are on the right-hand end?	3
4	What is the sum of the positive odd integers less than 100 minus the sum of the positive even integers less than 100?	50
5	What is 2 factorial plus 0 factorial plus 0 factorial plus 7 factorial?	5044
6	What is x, if 10 to the quantity x minus 1 equals 100 to the quantity x plus 1?	-3
7	In a special deck of cards, the probability of drawing a red card is 3/5 and the probability of drawing an ace given you draw a red card is 1/3. What is the probability of drawing a red ace?	1/5
	Extra Problem – Only if Needed	
8	What is the sum of zero point four plus zero point zero four plus zero point zero zero four and so on?	4/9

"Math is Cool" Championships – 2006-07

Geometry & Algebra II – November 1, 2006

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

KEY

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	$2\frac{11}{12}$		
2	1440 [min]		
3	$\frac{37}{2}$		
4	3 [points]		
5	14		
6	-14, 2 [need both]		
7	(-3, 3)		
8	19		
9	120 [kmph]		
10	155		
11	IV or 4[th]		
12	(-2, -13)		
13	$\frac{1}{2}\ln\left(\frac{r-1}{3}\right)$		
14	$6\sqrt{11}$ [cm^2]		
15	125 [°]		
16	$36\sqrt{3}$ [cm^2]		
17	15 [integers]		
18	160 [°]		
19	4π [cm^2]		
20	52 [cm^2]		

	Answer	1 or 0	1 or 0
21	$\frac{27}{2}$ [cm]		
22	$\frac{14}{5}$ [cm]		
23	7 [numbers]		
24	0		
25	$\frac{7}{2}$		
26	24		
27	$\frac{20}{3}$		
28	$\log_2 3$		
29	12		
30	$\pi\sqrt{6}$ [un^2]		
31	9/2 [un]		
32	20 [cm]		
33	$\frac{19}{25}$		
34	200		
35	1		
36	1689424		
37	4		
38	$\frac{216}{25}$		
39	23		
40	43582		

"Math is Cool" Championships – 2006-07

9^{th} & 10^{th} Grade – November 1, 2006

First Score KEY (out of 18)

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

INDIVIDUAL MULTIPLE CHOICE - 15 minutes

This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. ***Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet.*** *No talking during the test.*

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	E (-6)		
2	D		
3	C		
4	D		
5	B		
6	C		
7	D		
8	C		
9	A		

"Math is Cool" Championships – 2006-07

9^{th} & 10^{th} Grade – November 1, 2006

First Score KEY (out of 20)

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Team Contest – Score Sheet

TEAM TEST - 15 minutes

*When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as **2 or 0**. Record all answers on the colored answer sheet.*

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	4.6×10^4		
2	52 [yrs]		
3	1800 [mm]		
4	$2\sqrt{5}$ [cm]		
5	$\frac{2112}{4165}$		
6	$9\ \pi$ [m^2]		
7	7920		
8	21		
9	64		
10	$1145_{[7]}$		

"Math is Cool" Championships – 2006-07

9th & 10th Grade – November 1, 2006

First Score KEY

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

PRESSURE ROUND - 10 minutes

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

Pressure Round Answers

	Answer
1	A, C, D [any order]
2	3 [half-dollars]
3	66 [spinners]
4	84 [cm^2]
5	-1