

"Math is Cool" Masters - 2006-07

Algebra II, Pre-Calculus, Calculus - January 13, 2007
Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by all involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
 - *For problems dealing with money, a decimal answer should be given rounded to the nearest cent.*
 - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of π where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

"Math is Cool" Masters - 2006-07

Algebra II, Pre-Calculus, Calculus - January 13, 2007
Individual Contest

1	34 is what percent of 85?
2	What is the product of the number of days in a week, the number of vertices on a triangle, and the number of centimeters in a meter?
3	Express in simplest radical form: $\sqrt[3]{3024}$
4	Simplify by rationalizing the denominator: $\frac{12}{3 - \sqrt{5}}$
5	What value(s) of y satisfy $3(2y+1) + 5(y-3) = 65$?
6	What value(s) of n satisfy $2n^2 + 3n + 1 = 0$?
7	What value(s) of s satisfies $3s^2 + 4s + 5 = 0$?
8	If five engineers can design three circuits in six days, how many engineers would be needed to design twenty-eight circuits in eight days?
9	What is the slope of a line perpendicular to the line $y = 8x + 4$?
10	What is the distance between the points $(-3, -2)$ and $(5, 2)$?
11	What is the distance from the point $(-3, 8)$ to the line $3x - 4y = 19$?
12	Beau's rectangular lawn is fenced on all sides and measures 40 meters by 50 meters. When Beau mows his lawn, he mows everything that is not within two meters of the fence. What is the area, in square meters, of the region that Beau does not mow?
13	Express the base thirteen number 234_{13} as a base ten number.
14	Sam's bug collection consists of spiders (8 legs each) and beetles (6 legs each). If Sam has a total of 47 bugs with a total of 350 legs, how many beetles does he have?
15	What is the length, in centimeters, of the hypotenuse of a right triangle with legs measuring 12 and 18 cm?

16	An isosceles triangle has a base angle of 32 degrees. What is the measure, in degrees, of its vertex angle?
17	What is the perimeter, in centimeters, of a rectangle with an area of 200 cm^2 and a side measuring 5 cm?
18	What is the area, in square centimeters, of a rhombus with sides measuring 18 cm and an interior angle measuring 60 degrees?
19	A regular n-gon has interior angles measuring 160 degrees. How many diagonals can be drawn in this n-gon?
20	What is the height, in centimeters, of a right circular cylinder with a base radius of 8 cm and a surface area of $168\pi \text{ cm}^2$?
21	When three standard six-sided dice are rolled, what is the probability that the product of the numbers shown is six?
22	How many edges does a regular octahedron have?
23	In how many ways can a committee of four women and one man be chosen from a group of ten women and five men?
24	What is the fifth term of an arithmetic sequence with a first term of 18 and a common difference of 31?
25	If $C = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & -3 \\ 2 & 3 & -4 \end{bmatrix}$, determine $ C^{-1} $.
26	If $w(v) = 3v^2 - 12v + 5$, evaluate $w^{-1}(-7)$.
27	Chuck invests \$2500 in an account that receives 24% annual interest compounded monthly. How much money will be in the account after three months?
28	What is the value of the constant term in the expansion of $\left(p - \frac{1}{p^2}\right)^6$ after like terms are combined?
29	How many positive integer factors does 208 have?

Challenge Questions

30	If two "zoinks" are worth three "yikes", four "xylos" are worth five "womps", and six "yikes" are worth seven "womps", how many "zoinks" are twenty-one "xylos" worth?
31	If A is the set of all positive multiples of five less than 1000, B is the set of all composite numbers between 120 and 140 inclusive, and C is the set of all multiples of two between 150 and 350, how many elements does the set $(B \cup C) \cap A$ contain?
32	Two right rectangular pyramids are similar to one another, having volumes of 8 and 64 cm^3 . If the larger pyramid has a surface area of 100 cm^2 , what is the surface area, in square centimeters, of the smaller pyramid?
33	In the cryptarithm below, in which each letter represents a distinct digit and all instances of the same letter represent the same digit, what is the smallest possible value of the four-digit number $ABCD$? $\begin{array}{r} ABC \\ -BCA \\ \hline BDA \end{array}$
34	A coin is flipped five times, and you are told (truthfully) that there were at least two tails. What is the probability that there were exactly four tails?
35	A triangle has sides measuring 5, 6, and 7 cm. What is the length, in centimeters, of the median to the 6 cm side?
36	A circle is inscribed in a square with sides measuring 24 cm. What is the radius, in centimeters, of the largest circle that can be inscribed in the corner of the square between the circle and the square?
37	When two real numbers are selected between 5 and 8, what is the probability that they differ by less than one?
38	A teacher administers a make-up test to five students, each of whom earns an integer score between 0 and 100 inclusive. If the mean of their scores is 61, their unique mode is 42, and their median is 70, what is the maximum value of the range of the scores?
39	What real value(s) of h satisfy $e^{2h} - 2e^h = 3$?
40	How many positive three-digit even multiples of three are comprised of three distinct digits?

"Math is Cool" Masters - 2006-07

9th & 10th Grade - January 13, 2007

Individual Multiple Choice Contest

1	What is the least common multiple of 2007 and 12? A) 24084 B) 3 C) 892 D) 8028 E) Answer not given
2	What is the sum of the positive factors of 20? A) 36 B) 21 C) 42 D) 48 E) Answer not given
3	Let line l be defined by the equation $y = -3x + 8$. If line m intersects l perpendicularly at $x = 2$, at what value of x does m intersect the x -axis? A) $\frac{3}{2}$ B) -4 C) 4 D) 2 E) Answer not given
4	Let $f(x)$ be an even function and $g(x)$ be an odd function, both defined on all integers. If $f(2) = 5$ and $g(-5) = 2$. What is the sum $g(f(-2)) + f(g(-f(2)))$? A) 3 B) 12 C) -5 D) 7 E) Answer not given
5	Anna is walking laps around a park. She runs the first lap at 5 mph, the second at $\frac{5}{2}$ mph, the third at $\frac{5}{3}$ mph, the fourth at $\frac{5}{4}$ mph and the last at 1 mph. What was her average speed? A) $\frac{5}{4}$ B) $\frac{5}{3}$ C) $\frac{3}{5}$ D) $\frac{137}{60}$ E) Answer not given
6	What is the volume of a sphere that has the same surface area as a unit cube? A) $\pi\sqrt{2}$ B) $\sqrt{\frac{6}{\pi}}$ C) $\frac{\pi\sqrt{3}}{2}$ D) $\sqrt{\frac{3}{2\pi}}$ E) Answer not given
7	When 37_{10} is expressed in base n , $1 < n \leq 10$, let a represent the sum of the digits expressed in base 10. What is the smallest possible value of a ? A) 7 B) 4 C) 6 D) 5 E) Answer not given
8	A solid sphere has radius r and surface area SA . A smaller concentric sphere is hollowed out, with radius $r/2$. The larger sphere is then bisected. What is the ratio between the surface area of one of the new hemispheres and the original surface area SA ? A) $\frac{7}{8}$ B) $\frac{3}{2}$ C) $\frac{5}{7}$ D) $\frac{13}{16}$ E) Answer not given
9	What is the area of the largest circle that can be inscribed in a regular hexagon with side length s ? A) πs^2 B) $\pi s^2(\sqrt{3} - 1)$ C) $\pi s^2\left(\frac{2}{3}\right)$ D) $\pi s^2\left(\frac{3}{4}\right)$ E) Answer not given

"Math is Cool" Masters - 2006-07

9th & 10th Grade - January 13, 2007

Team Contest

1	Evaluate: $108^3 + 92^3$
2	Last night when Cherie turned on the faucet to fill the bathtub, she forgot to plug the drain and it took 30 minutes for the tub to fill. As soon as she turned off the faucet, the tub began to drain, and was empty in 20 minutes. Cherie then plugged the drain and turned on the faucet; how many minutes did it then take the tub to fill?
3	What point, in the form (x, y) , is the reflection of the point $(-4, 13)$ across the line $x + y = 4$?
4	An isosceles right triangle has legs measuring eight centimeters each. What is the area, in square centimeters, of the region within the triangle that is at least one centimeter from each side?
5	A group of thirsty hikers split the cost of a Huge Glug soft drink, splitting the cost evenly. If there had been four fewer people in the group, each person would have paid three more cents, while if there had been two more people in the group, each person would have paid one cent less. What was the total cost of the Huge Glug, in dollars rounded to the nearest hundredth (cent)?
6	A circle is inscribed in a square with sides measuring 12 cm. What is the area, in square centimeters, of the largest square that can be inscribed in the corner between the original circle and square?
7	What is the missing term of the sequence 1, 4, 15, 40, ____, 156, 259?
8	The probability that Sam drinks a soda on Wednesday is $\frac{1}{4}$, the probability that he passes his calculus test on Wednesday is $\frac{1}{3}$, and the probability that both occur is $\frac{1}{5}$. If Sam does not drink a soda on Wednesday, what is the probability he passes his calculus test that day?
9	What value(s) of y satisfy $\log_4 y^{\log_2 y} = 2 - 3\log_4 y$?
10	How many positive integers less than 100 have exactly four positive integer factors?

"Math is Cool" Masters - 2006-07

9th & 10th Grade - January 13, 2007

Pressure Round Contest

1	When 2 lines intersect at a point, two pairs of vertical angles are formed. When 3 lines intersect at a single point, 6 pairs of vertical angles are formed. When n lines intersect at a single point, 272 pairs of vertical angles are created. What is n ?
2	The sum of the digits of the base-10 number 2006 is 8. When 2006_{10} is represented in base b (where b is a positive integer less than 10), the sum of the digits is also 8_{10} . List all possible values for b . (Give your answer in base 10.)
3	A cubical box 1 foot on each edge is subdivided by thin partitions into cubical cells each 2 inches on an edge. (Assume these partitions to have 0 thickness.) Each cell holds a glass sphere of diameter 2 inches. A second box the same size is subdivided into cubes 3 inches on an edge, and it is full of spheres of diameter 3 inches. Find the total volume, in cubic inches, of all the spheres in both boxes combined.
4	Flip a coin 6 times. Of the 64 possible outcome sequences, how many have at least 4 heads in a row?
5	Find the largest integer that divides the following three numbers with the same remainder in each case: 54437, 20093, and 52997.

"Math is Cool" Masters - 2006-07

9th & 10th Grade - January 13, 2007

Mental Math Contest

PERSON 1		
1.1	What is the remainder when $x^2 - 5x + 7$ is divided by $x - 1$?	3
1.2	How many positive even factors does the number 60 have?	8 [fac]
1.3	Alice, Bob, Carol and David ran a race and there were no ties. If David did not get second, in how many ways could they have finished?	18 [ways]
1.4	The vertices of a regular tetrahedron of side length 3 are cut by planes that are one unit from the vertex. How many vertices are in the resulting figure?	12 [vert]
PERSON 2		
2.1	A triangle has sides of lengths 6, 8 and 10, what is its area?	24 [un ²]
2.2	What is the largest four-digit base 4 number when expressed in base 10?	255 [10]
2.3	Trevor rolls two fair six-sided dice. What is the probability that the product of the two numbers showing will be prime?	1/6
2.4	The middle third of a line segment of length 1 is discarded. In the second series of cuts, the middle third of the two remaining segments is removed. After the fourth series of cuts, what is the total length of the remaining pieces?	16/81 [un]
PERSON 3		
3.1	What is the highest number of non-overlapping equilateral triangles of side length 1 that can be placed inside an equilateral triangle of side length 3?	9 [tri]
3.2	Suppose a coin is weighted so that the probability of a head is two-thirds. What is the probability of getting exactly 2 heads out of 3 coin flips?	4/9
3.3	What is the average of the two missing values in the arithmetic sequence: 5, blank, blank, 10?	15/2
3.4	What is the smallest positive integer that has exactly six positive integer factors?	12 [int]
PERSON 4		
4.1	What is the log of 2 divided by the log of 8?	1/3
4.2	A fair coin is tossed five times. I know it came up heads at least four times, what is the probability it was heads all 5 times?	1/6
4.3	In a 3 by 4 by 5 prism, what is the distance between the two farthest corners?	$5\sqrt{2}$
4.4	How many two digit numbers contain at least one 2?	18

"Math is Cool" Masters - 2006-07

9th & 10th Grade - January 13, 2007

COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	A right triangle has sides 5 and 12. What is the sum of all possible lengths for the third side?	$13 + \sqrt{119}$ [un]
2	What is the sum of the factors of 36?	91
3	The cube root of the square root of x is 3. What is the sum of the digits of x ?	18
4	Three fair four-sided dice with faces numbered one through four are rolled. What is the probability that the product of the numbers shown is two?	$3/64$
5	f of x equals x squared plus $6x$ plus 8 . g of x equals two x plus 4 . For what value(s) of x does f of x equal g of x ?	-2
6	The angles of a convex pentagon form an arithmetic sequence with a difference of one. What is the smallest of these angles in degrees?	106°
7	a squared plus b squared is equal to 24 . The quantity a plus b squared is equal to 8 . What is the product of a and b ?	-8
	Extra Problem - Only if Needed	
8	2007 in base 8 is what number expressed in base 10?	$1031_{[10]}$

"Math is Cool" Masters - 2006-07

9th & 10th Grade - January 13, 2007

COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	When $2x$ plus the quantity 1 over x squared is raised to the sixth power, what is the constant term?	240
2	What fraction of the numbers, from 20 to 40 inclusive, have digits whose product is even?	$16/21$
3	What is the largest area, in square meters, that can be enclosed using 20 meters of fencing?	$100/\pi$ [m^2]
4	What is the sum of 1 over 11 plus 1 over 121 plus 1 over 1331 plus and so on?	$1/10$
5	What is the product of the two largest prime numbers less than 40?	1147
6	This year is 2007. There is no polygon whose interior angles sum to 2007 degrees. How many years ago was the year equal to the sum, in degrees, of the interior angles of a convex polygon?	27 [yrs]
7	Rebecca has an infinitely large supply of 2 kinds of stamps, 17-cent stamps and stamps worth x cents. She cannot make postage worth exactly \$2.71, but can make any amount greater than \$2.71. What is x ?	18
	Extra Problem - Only if Needed	
8	What is the surface area of the shape determined by the equation: x squared plus y squared plus z squared equals 4?	64π [un^2]

"Math is Cool" Masters - 2006-07

9th & 10th Grade - January 13, 2007

COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	A pizza with an 18-inch diameter is cut into six equal pieces. What is the area remaining after a single piece is removed?	$\frac{135\pi}{2}$ [in ²]
2	7 factorial times 8 factorial is equal to c times n factorial, where c is a positive integer and n is as large as possible. What is the sum of c and n?	66
3	A circle is inscribed in an equilateral triangle and a square is inscribed in that circle. What is the ratio of the area of the square to the triangle expressed as a fraction?	$\frac{2\sqrt{3}}{9}$ [or 2 root 3 over 9]
4	What is the sum of the positive real solutions of $2x$ to the fourth minus $5x$ squared plus 2 equals 0?	$\frac{3\sqrt{2}}{2}$ [or 3 (root 2)/2]
5	What is the quantity i plus 1 to the fourth power, where i is the square root of -1?	-4
6	The rectangular coordinates 3 comma root 3 can be rewritten as what polar coordinates using radians?	$\left(2\sqrt{3}, \frac{\pi}{6}\right)$
7	The radius of a circle is 5. 108 degrees is subtended by arc AB. What is the arc length of AB?	3π
Extra Problem - Only if Needed		
8	What is the sum of 5 plus 10 plus 15 and so on until 100?	1050

"Math is Cool" Masters - 2006-07

Algebra II, Pre-Calculus, Calculus - January 13, 2007

School Name _____ Team # _____

Proctor Name _____ Room # _____

Final Score:

KEY

First Score

STUDENT NAME _____

Individual Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	40 [%]		
2	2100		
3	$6\sqrt[3]{14}$		
4	$9 + 3\sqrt{5}$		
5	7		
6	-1/2, -1 [in any order]		
7	$(-2 \pm i\sqrt{11})/3$		
8	35 [engin]		
9	-1/8		
10	$4\sqrt{5}$ [un]		
11	12 [un]		
12	344 [m ²]		
13	381 _[10]		
14	13 [beetles]		
15	$6\sqrt{13}$ [cm]		
16	116 [°]		
17	90 [cm]		
18	$162\sqrt{3}$ [cm ²]		
19	135 [diag]		
20	5/2 [cm]		

	Answer	1 or 0	1 or 0
21	1/24		
22	12 [edges]		
23	1050 [ways]		
24	142		
25	-1/40		
26	2		
27	[\$] 2653.02		
28	15		
29	10 [factors]		
30	15 [zoinks]		
31	26 [elements]		
32	25 [cm ²]		
33	3165		
34	5/26		
35	$2\sqrt{7}$ [cm]		
36	$36 - 24\sqrt{2}$ [cm]		
37	5/9		
38	38		
39	ln 3		
40	115 [numbers]		

"Math is Cool" Masters - 2006-07

9th & 10th Grade - January 13, 2007

Final Score:

KEY

School Name _____ Team # _____

Proctor Name _____ Room # _____

First Score

(out of 18)

STUDENT NAME _____

INDIVIDUAL MULTIPLE CHOICE - 15 minutes

*This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. **Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet. No talking during the test.***

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	D		
2	C		
3	B		
4	A		
5	B		
6	B		
7	E [2]		
8	D		
9	D		

"Math is Cool" Masters - 2006-07

9th & 10th Grade - January 13, 2007

Final Score:

KEY

School Name _____ Team # _____

Proctor Name _____ Room # _____

First Score

(out of 20)

STUDENT NAME _____

Team Contest - Score Sheet

TEAM TEST - 15 minutes

When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as 2 or 0. Record all answers on the colored answer sheet.

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	2038400		
2	12 [min]		
3	(-9, 8)		
4	$19 - 6\sqrt{2}$ [cm ²]		
5	[\$]1.44		
6	$54 - 36\sqrt{2}$ [cm ²]		
7	85		
8	$\frac{8}{45}$		
9	$2, \frac{1}{16}$		
10	32 [int]		

"Math is Cool" Masters - 2006-07
9th & 10th Grade - January 13, 2007

KEY

School Name _____ Team # _____

Proctor Name _____ Room # _____

STUDENT NAME _____

PRESSURE ROUND - 10 minutes

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

Pressure Round Answers

Answer	
1	17 [lines]
2	2
3	576π [in ³]
4	8
5	72