"Math is Cool" Championships – 2007-08

Sponsored by:

Geometry & Algebra II – November 14, 2007

Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by all involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
  - *For problems dealing with money, a decimal answer should be given.*
  - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of π where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

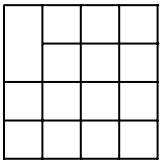
"Math is Cool" Championships – 2007-08

Sponsored by:

Geometry & Algebra II – November 14, 2007

Individual Contest

1	Evaluate: 4147 + 3284
2	What is 40% of the number 25% larger than 180?
3	Evaluate as a mixed number: $5\frac{1}{4} \div 3\frac{1}{2}$
4	For what value of a does 6a + 4 = 46?
5	What are the coordinates, in the form (x,y), of the reflection of the point (7,-8) across the line y = 22?
6	Sam walks 3 kilometers to school in forty minutes. What was Sam's average speed in kilometers per hour?
7	For what value(s) of d does $2d^2 - 3d - 1 = 0$?
8	What are the coordinates, in the form (x,y), of the point of intersection of the lines y = 3x+4 and 2x + 3y = -21?
9	If you can buy K kilograms of flour with N nickels, how many dimes would be necessary to buy G grams of flour?
10	A right triangle has a hypotenuse measuring 18 cm and a leg measuring 8 cm. What is the length of the other leg, in centimeters?
11	What is the volume, in cubic centimeters, of a right rectangular pyramid with a height measuring 8 cm and a base measuring 6 cm by 10 cm?
12	What is the measure of an interior angle, in degrees, in a regular 24-gon?
13	What is the area, in square centimeters, of a 30° sector of a circle with a radius of 6 cm?
14	What is the surface area, in square centimeters, of an octahedron with edges measuring 4 cm?
15	What is the smallest number of regions into which four distinct lines can divide a plane?
16	Simplify, where $i = \sqrt{-1}$: $(3-4i)-(-1+2i)(-5-6i)$
17	What are the coordinates, in the form (x,y), of the vertex of the given conic section that is farthest from the origin? $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{36} = 1$
18	What is the sum of all values of k satisfying $3^{k+1} + 9^k = 4$?

19	What is the least common multiple of 48 and 50?
20	What is the third-largest six-digit palindrome that can be formed using the digits 2, 3, and 5 exactly twice each?
21	In an election with six candidates, how many ways can a Secretary and an Assistant Secretary be chosen?
22	In how many ways can the letters in the phrase "MAMA LLAMA" be arranged? (Ignore the space.)
23	What is the tenth term of an arithmetic sequence with a first term of 84 and a common difference of –9?
24	What is the sixth term of a recursively defined sequence with its first term defined as $r_1 = 8$ and subsequent terms defined as $r_n = (8 - r_{n-1})^2 + 7$?
25	What is the mean of the data set 1, 2, 2, 3, 3, 3, 4, 6, 6?
26	How many subsets of {1,2,3,4,5,6,7} are supersets of {2,3,5}?
27	Evaluate: $\begin{bmatrix} 1 & 3 & -5 \\ 0 & -4 & -1 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$
28	Evaluate: $\cfrac{3}{1+\cfrac{3}{1+\cdots}}$
29	How many squares of any size can be drawn along the lines of the grid of unit squares shown with one missing grid line? 

Challenge Questions

30 Evaluate: $\left(1-2^3\right)-\left(4(5-6)^7-(-1)^8\right)\left(9-1^0\right)$

31 A fraternity pooled their money and bought a car. If they had had four more members, each member would have paid ten dollars less. If they had had six fewer members, each member would have paid twenty dollars more. How many dollars did the car cost?

32 What is the minimum distance, in centimeters, an ant can crawl along the surface of a regular tetrahedron with edges measuring 8 cm if he is on an edge 1 cm from its midpoint and is trying to reach a point on the opposite edge 2 cm from its midpoint?

33 Which of the following is an eigenvector of $\begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$?

A) $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ B) $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$ C) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ D) $\begin{bmatrix} -5 \\ 2 \end{bmatrix}$ E) $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

34 What is the sum of the perfect squares between 101 and 1000?

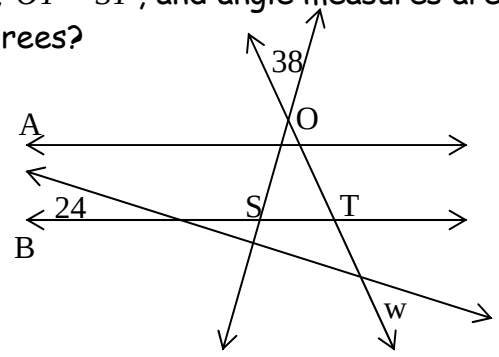
35 In a survey of 60 people, 50 liked sausage on their pizzas, 40 liked mushrooms, 30 liked olives, and 20 liked all three. What is the largest number of people that could have liked only sausage?

36 What is the quotient, expressed in base 9, when 44684_9 is divided by 234_9?

37 If $\log_4 6 = p$, express $\log_3 12$ in terms of p.

38 A right triangular prism with two faces that are equilateral triangles is sized so that its inscribed sphere touches all five of its faces. A monolith (a right rectangular prism with edges in the ratio of 1:4:9) is inscribed in the sphere. What is the ratio, expressed as a fraction, of the surface area of the monolith to that of the right triangular prism?

39 In the figure shown, lines $\overleftrightarrow{A}$ and $\overleftrightarrow{B}$ are parallel, $OT = ST$, and angle measures are given in degrees. What is the value of w, in degrees?



40 Evaluate: $43^3 + 27^3$

"Math is Cool" Championships – 2007-08

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9th & 10th Grade – November 14, 2007

Individual Multiple Choice Contest

1	Evaluate: $111^2 - 89^2$ A) 3850 B) 4000 C) 4180 D) 4400 E) 4650
2	What ordered pair (e, f) satisfies the equations $3e + 2f = -2$ and $2e - f = 15$? A) $(6, -10)$ B) $(4, -7)$ C) $(-2, 2)$ D) $(0, -15)$ E) $(-6, 27)$
3	Using a 50 m rope, a horse is tied to the inner corner of an L-shaped fence with two sections measuring 20 m and 40 m meeting at a right angle. What is the total area, in square meters, the horse can graze? A) 1150π B) 1200π C) 1250π D) 1500π E) 1675π
4	Express the range of the function $h(j) = \sqrt{-2j^2 + 8j - 3}$ in interval notation. A) $\left[0, \sqrt{5}\right]$ B) $\left[\frac{4-\sqrt{10}}{2}, \frac{4+\sqrt{10}}{2}\right]$ C) $\left[0, \frac{4+\sqrt{10}}{2}\right)$ D) $(0, 2)$ E) $\left(\frac{4-\sqrt{10}}{2}, \frac{4+\sqrt{10}}{2}\right]$
5	Which of the following numbers is prime? A) 148 B) 237 C) 355 D) 707 E) 1117
6	When one card is drawn from a standard 52-card deck, what is the probability that it is black or a face card (J, Q, or K)? A) $\frac{15}{26}$ B) $\frac{8}{13}$ C) $\frac{17}{26}$ D) $\frac{9}{13}$ E) $\frac{19}{26}$
7	In a set of 7 integers from 0 to 100 inclusive, the range is 33, the mean is 72, and the mode is 85. What is the largest possible value of the median? A) 72 B) 78 C) 85 D) 87 E) 88
8	What is the coefficient of the f^3 term in the expansion of $(3f - 2)^4$? A) -156 B) -168 C) -180 D) 192 E) -216
9	What is the length, in centimeters, of the altitude to the longest side of a triangle with sides measuring 4, 7, and 9 cm? A) $\frac{3\sqrt{5}}{4}$ B) π C) $\frac{4\sqrt{5}}{3}$ D) $\frac{3\sqrt{15}}{4}$ E) $\frac{5\sqrt{3}}{4}$

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9th & 10th Grade – November 14, 2007

Team Contest

1	Express in simplest radical form: $\sqrt[3]{6048}$
2	Cherie bikes downhill to work on the shortest route, going five kilometers at a speed of 30 kilometers per hour. She bikes home on a less steep route, going seven kilometers at 20 kilometers per hour. What is her average speed for the round trip, in kilometers per hour?
3	A log with a radius of 30 cm is rolled against a wall and chocked, after which a log with a radius of 20 cm is placed resting on the first log and touching the wall. How many cm above the ground is the top of the second log?
4	Simplify: $\dfrac{6g^5+9g^4-14g^3+11g^2+25g-28}{2g^2+3g-4}$
5	How many positive four-digit integers satisfy the conditions that their first digit is a multiple of 2, their first two digits can be read as a two-digit number divisible by 3, their first three digits can be read as a three-digit number divisible by 5, and the entire number is divisible by 6?
6	What is the next term of the sequence 56, 69, 84, 101, 120, ___?
7	Evaluate: $2.4 \div 2\frac{1}{4} - \frac{4}{21} \cdot \frac{14}{20}$
8	What is the largest possible difference between the positive four-digit integer GHIJ and its reversal, the positive four-digit integer JIHG, where G, H, I, and J each represent a unique digit 0-9?
9	In the following cryptarithm, each instance of a letter stands for the same digit (0-9), and different letters stand for different digits. E.g. if one D is a 1, then all D's are 1's and E cannot be 1. What is the value of the seven-digit number ABCDEFG? $\begin{array}{r} AB \\ \times CA \\ \hline DEF \\ GF \\ \hline DFEF \end{array}$
10	When two concentric circles are drawn, the area of the annulus (the region outside one circle but inside the other) is 64π cm^2. What is the length, in centimeters, of a chord drawn in the larger circle tangent to the smaller circle?

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Pressure Round Contest

1	In a regular hexagon, three diagonals are drawn from the same vertex, dividing the hexagon into four triangular areas. What is the area of the largest triangle divided by the area of the smallest triangle?
2	What is the most specific name for the shape of the locus of all points that are four times as far from the point $(3,3)$ as they are from the origin?
3	When three liters of a 40% sodium solution are mixed with two liters of a 25% sodium solution, what percent of the resulting solution is sodium?
4	Write an expression which evaluates to 14 using the numbers 1, 2, 3, and 4 exactly once each, the operators +, -, $\times$, and $\div$ as much or as little as you like, and parentheses as much or as little as you like. E.g. if the numbers were 2, 3, 5, and 8, you could write $3\times 8-2\times 5$.
5	Five suspects are interrogated about a crime, and make the following statements: A) Person C did not do it. B) Person A did it. C) Person D did not do it. D) Person B did it. E) I did not do it. If exactly one of the five is guilty and only innocent people told the truth, list all the people you can be sure are innocent.

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Mental Math Contest

PERSON 1 Name:		
1.1	Express the number seventy-three point one eight five in scientific notation rounded to three significant figures.	7.32×10^1 7.32×10
1.2	What are the coordinates, in the form X comma Y, of the x-intercept of the line two X minus three Y equals negative 18?	(-9,0)
1.3	What is the area, in square centimeters, of a circle with a perimeter of twenty-two pi root two centimeters?	242π
1.4	What is the slope of the line three X plus five Y equals 28?	-3/5
PERSON 2 Name:		
2.1	Evaluate three [PAUSE] minus four times five [PAUSE] plus six squared.	19
2.2	What is the equation, in slope-intercept form (Y equals M X plus B), of the line through the points one comma one and two comma three?	Y=2x-1
2.3	What is the perimeter, in centimeters, of an equilateral triangle with an area of thirty-six root three square centimeters?	36
2.4	What is the distance between the points negative four comma nineteen and five comma ten?	$9\sqrt{2}$
PERSON 3 Name:		
3.1	What is the sum of the number of seconds in a minute, the number of days in a week, and the number of months in a year?	79
3.2	In which quadrant does the point five comma negative three lie?	IV or 4th
3.3	What is the perimeter, in centimeters, of a rectangle with sides measuring fourteen and thirty-seven centimeters?	102 [cm]
3.4	What is the sum of the terms of an infinite geometric sequence with a first term of twenty-seven and a common ratio of one-fourth?	36
PERSON 4 Name:		
4.1	The sum of two numbers is one-hundred eighty-four and their positive difference is eight. What is the larger of the two numbers?	96
4.2	If the vertex angle of an isosceles triangle measures sixty-six degrees, what is the measure of a base angle, in degrees?	57
4.3	Each side of a triangle measures an integer number of centimeters. If one of the sides measures eighteen centimeters, what is the minimum perimeter, in centimeters, of the triangle?	37
4.4	When two fair six-sided dice are rolled, what is the probability that the sum of the numbers shown is ten?	1/12

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COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	Evaluate eleven-thousand, eight-hundred fifty-three divided by twenty-seven.	439
2	What is the equation of the axis of symmetry of the parabola with equation X equals three Y squared minus five Y plus eight?	$y = \frac{5}{6}$
3	What is the volume, in cubic centimeters, of a sphere with a diameter of eighteen centimeters?	972π [cm^3]
4	Express the number five, six, seven base eight as a base ten number.	375
5	How many two-element subsets does a six-element data set have?	15
6	For what value(s) of B does B squared minus eight B minus twenty equal zero?	10, -2
7	Convert the polar coordinates ten comma five pi over six to rectangular coordinates in the form X comma Y.	$\left(-5\sqrt{3}, 5\right)$
	Extra Problem – Only if Needed	
8	Simplify fifty-five divided by the quantity five minus two root five by rationalizing the denominator.	$55 + 22\sqrt{5}$

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COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	Express the square root of six-hundred seventy-five in simplest radical form.	$15\sqrt{3}$
2	It takes Loki one hour to paint a masterpiece, but it only takes Vivian a half-hour to paint a masterpiece. How many **minutes** would it take the two of them, working together, to paint a masterpiece?	20 [min]
3	How many squares of any size can be drawn along the gridlines of a standard chessboard?	204 [squares]
4	What is the sum of the roots of the equation two D squared plus D minus five equals zero?	-1/2
5	How many positive integer factors of two-thousand eight-hundred eighty are multiples of forty-eight?	12
6	What is the perimeter of a 45-45-90 triangle with a hypotenuse measuring root eight centimeters?	$4+2\sqrt{2}$
7	If the cosine of an angle in the first quadrant is equal to two-fifths, what is the tangent of the angle?	$\frac{\sqrt{21}}{2}$
	Extra Problem – Only if Needed	
8	What is the total surface area, in square centimeters, of a right circular cylinder with a height of four centimeters and a base radius of four centimeters?	64π

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COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	Evaluate eight-hundred seventy-six times two-hundred thirty-four.	204984
2	For what value(s) of C does two times the quantity three C plus 5 [PAUSE] minus four times the quantity six plus C [PAUSE] equal seventy-eight?	46
3	Two circles with radii of fourteen and twenty-one centimeters are drawn tangent to one another. What is the length, in centimeters, of one of their common external tangents?	$14\sqrt{6}$ [cm]
4	If X of Y equals three Y minus two and Z of C equals two minus C squared, evaluate X of Z of four.	-44
5	If S is the set of all positive multiples of three less than twenty and T is the set of all even numbers between ten and thirty inclusive, what is the sum of the elements in the intersection of S and T?	30
6	If vector A is one comma four and vector B is three comma negative two, express five A minus six B in component form.	$\langle -13, 32 \rangle$
7	What are the coordinates, in the form X comma Y, of the vertex of the parabola with equation Y equals three X squared minus nine X plus two?	$\left(\frac{3}{2}, -\frac{19}{4}\right)$
	Extra Problem – Only if Needed	
8	Four times a number is one-hundred five more than the additive inverse of the number. What is the number?	21

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Geometry & Algebra II – November 14, 2007

Final Score:
KEY

Individual Contest – Score Sheet

	Answer	1 or 0	1 or 0
1	7431		
2	90		
3	$1\frac{1}{2}$		
4	7		
5	(7,52)		
6	9/2 [kph]		
7	$\frac{3\pm\sqrt{17}}{4}$		
8	(-3,-5)		
9	$\frac{GN}{2000K}$		
10	$2\sqrt{65}$ [cm]		
11	160 [cm^3]		
12	165 [°]		
13	3π [cm^2]		
14	$32\sqrt{3}$ [cm^2]		
15	5 [regions]		
16	-14		
17	(-2,11)		
18	0		
19	1200		
20	352253		

	Answer	1 or 0	1 or 0
21	30 [ways]		
22	1260 [ways]		
23	3		
24	7		
25	10/3		
26	16		
27	$\begin{bmatrix}-7\\-1\end{bmatrix}$		
28	$\frac{-1+\sqrt{13}}{2}$		
29	26 [squares]		
30	33		
31	[$] 3600		
32	7 [cm]		
33	B		
34	10031		
35	20 [people]		
36	$181_{[9]}$		
37	$\frac{2p+1}{2p-1}$		
38	$\frac{2\sqrt{3}}{27}$		
39	80 [°]		
40	99190		

"Math is Cool" Championships – 2007-08

KEY

9th & 10th Grade – November 14, 2007

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

INDIVIDUAL MULTIPLE CHOICE - 15 minutes

This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. ***Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet.*** *No talking during the test.*

DO NOT WRITE IN SHADED REGIONS

Answer		-1, 0 or 2	-1, 0 or 2
1	D		
2	B		
3	A		
4	A		
5	E		
6	B		
7	C		
8	E		
9	C		

"Math is Cool" Championships – 2007-08

9th & 10th Grade – November 14, 2007

KEY
(out of 20)

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Team Contest – Score Sheet

TEAM TEST - 15 minutes

When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as ***2 or 0****. Record all answers on the colored answer sheet.*

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	$6\sqrt[3]{28}$		
2	$\frac{720}{31}$		
3	$50 + 20\sqrt{6}$		
4	$3g^3 - g + 7$		
5	39		
6	141		
7	$\frac{14}{15}$		
8	8712		
9	4521809		
10	16		

"Math is Cool" Championships – 2007-08

9th & 10th Grade – November 14, 2007

KEY

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

PRESSURE ROUND - 10 minutes

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

Pressure Round Answers

	Answer
1	2
2	Circle
3	34 [%]
4	2x(3+4)x1 or 3x4+2x1
5	A, C