"Math is Cool" Masters – 2007-08

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9th & 10th Grade – January 26, 2008

Individual Multiple Choice Contest

1	Evaluate: $(\log_3 128)(\log_2 243)$ A) 35 B) 42 C) 12 D) 36 E) NOTA
2	What is the sum of the roots of the following function? $x^2 - 56x + 71 = 0$ A) -23 B) 14 C) 56 D) 71 E) NOTA
3	Evaluate in base 4: 101110111_2 101001110_2 101110001_2 $+1011101111_2$ A) 211231_4 B) 120231_4 C) 121211_4 D) 130211_4 E) NOTA
4	Rationalize the denominator of the following and simplify. What is the resulting denominator? $\frac{\sqrt{2}}{5-\sqrt{2}-\sqrt{3}}$ A)41 B) 94 C) 122 D) 180 E) NOTA
5	Solve for x: $3^{27^x} = 27^{3^x}$ A) 0 B) 1/3 C) 1/2 D) 1 E) NOTA
6	Solve for n: n! = (n-1)! + (n-2)! A) 1 B) 2 C) 6 D) 12 E) NOTA
7	Solve the following equation for x: 3(2x-4)-7(3x-1) = 4(5x+3) A) $\frac{17}{35}$ B) $-\frac{17}{35}$ C) $-\frac{27}{35}$ D) -2 E) NOTA
8	Find the area of a sector of a circle with a central angle of 70° and a radius of 6. A) 7π B) $\frac{18\pi}{35}$ C) $\frac{7\pi}{3}$ D) $\frac{7\pi}{6}$ E) NOTA
9	Paul has a cube of side length 4 inches. He doubles one side length and triples another. What is the ratio of the surface area of the cube to the surface area of the new rectangular prism? A) 1:6 B) 1:5 C) 3:11 D) 1:36 E) NOTA

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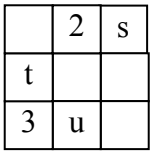
9^{th}, 10^{th}, 11^{th}, & 12^{th} Grade – January 26, 2008

Team Contest

1	A circle with a radius of 3 cm rolls around the inside of an equilateral triangle with sides measuring 12 cm. What is the distance, in centimeters, travelled by the center of the circle during one complete circuit?
2	In ΔPQR, $m\angle P = 45°$, $m\angle Q = 105°$, and $PQ = 6$ cm. What is the length of $\overline{QR}$, in centimeters?
3	If set W is the set of all positive multiples of 6 less than 10,000 and set V is the set of all positive perfect squares, how many elements are in the set $V' \cap W$?
4	My coin collection consists of only nickels, dimes, and quarters, and has a value of $2.55. If twice the number of nickels is equal to the absolute value of the difference between the number of dimes and the number of quarters, what is the largest number of coins that can be in my collection?
5	Tony and Albert are setting up a meeting. They decide to meet sometime between 7:00 PM and 8:00 PM. When Tony arrives, he will wait 10 minutes for Albert to show up, then leave if Albert hasn't arrived by then. When Albert arrives, he will wait 5 minutes for Tony to show up, then leave if Tony hasn't arrived by then. If they both arrive at random times between 7:00 and 8:00 PM, what is the probability that they will meet?
6	The integers from 2 to 10 are each put in a cell of a 3x3 magic square so that each cell receives a number. If each row, column, and diagonal has the same sum, what is the sum of s, t, and u? 
7	The Clymer test detects MIC syndrome, which affects one person in a million. A person with MIC will test positive 99% of the time and a person without it will test negative 97% of the time. What is the probability that you have MIC if you test positive for it using the Clymer test?
8	In an old-fashioned mill, eight horses walking for nine hours can grind 300 kg of flour. How many minutes, would it take six horses to grind 120 kg of flour?
9	Let n be a natural number whose square is greater than 100 and less than 300. Find the sum of all possible values of n, given that ${}_nC_{n-10}$ is not a palindrome. (A palindrome is a number that remains the same when its digits are reversed. The notation ${}_nC_k$ indicates the number of ways to choose k objects from a set of n distinct objects, without regard to order.)
10	What is the area of the triangle with vertices at the points $(4,-1)$, $(-2,5)$, and $(-3,-3)$?

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9^{th} & 10^{th} Grade – January 26, 2008

Pressure Round Contest

1	Put the following 6 values in order from least to greatest. Your answer should consist of 6 letters in the correct order. $A = 999 \quad B = 9^{99} \quad C = 99^{9} \quad D = 9^{9^{9}} \quad E = 9 \cdot 9 \cdot 9 \quad F = \left(9^{9}\right)^{9}$
2	Find the sum of all 3-digit positive integers divisible without remainder by exactly 8 out of 9 single-digit positive integers.
3	A triangle with sides measuring 6, 9, and 12 cm is similar to a triangle with sides measuring 66, 88, and y cm. What is the value of y?
4	175 people stand equally spaced in a circle. They count off consecutively, starting with 1 (1, 2, 3, and so on). The person who says "63" is standing opposite a gap. If the two people next to this gap then leave the circle, what is the sum of the numbers that were called out by all the people still in the circle?
5	How many positive integer values of x less than 1000 satisfy $x \equiv 4 \bmod 9$ and $x \equiv 5 \bmod 12$?

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9th & 10th Grade – January 26, 2008

Mental Math Contest

PERSON 1		
1.1	Evaluate 63 squared minus 37 squared.	2600
1.2	What is the probability of flipping exactly 4 heads when flipping 7 fair coins?	$\frac{35}{128}$
1.3	What is the sum of the first 6 terms of the geometric sequence beginning 4, 2, 1 and so on? Express your answer as a mixed number.	$7\frac{7}{8}$
1.4	What is the total surface area of a right circular cylinder with radius two-thirds and height five-sixths?	2π [un^2]
PERSON 2		
2.1	What is the area of the triangle bounded by the zeros and vertex of the function y equals "x" squared minus eight "x"?	64 [un^2]
2.2	Find the sum of the first hundred even natural numbers.	10100
2.3	Evaluate 216 to the negative two-thirds power.	1/36
2.4	Spiders and dogs are in a room. If Jim sees 23 heads and 160 legs, how many dogs are in the room?	6 [dogs]
PERSON 3		
3.1	Find the least common multiple of 187 and 68.	748
3.2	What is the perimeter of a rhombus with sides measuring 23 centimeters?	92 [cm]
3.3	What is the area of a triangle with sides of length 4, 5, and 7?	$4\sqrt{6}$ [un^2]
3.4	A circle of radius 3 is inscribed in a square of side length 6. What is the least distance from a vertex of the square to a point on the circle?	$3\sqrt{2}-3$ [un]
PERSON 4		
4.1	What value of "X" satisfies the equation three "X" minus four equals thirty-eight?	14
4.2	How many seating arrangements are possible in a small classroom with 6 chairs in a line, but only 5 students?	720 [arrangements]
4.3	What is the sum of the first 10 perfect squares?	385
4.4	A motorist decides to change his wheels. He replaces his 30 inch wheels with new 3 foot wheels. When his speedometer reads 45 miles per hour, how many miles per hour is he actually going?	54 [mph]

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COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	Find the distance between the point 15 comma 8 and the line Y equals three X plus eight.	$\frac{9\sqrt{10}}{2}$
2	Evaluate: the log base five of eight times the log base two of twenty-five.	6
3	Cam, Joel and Trevor are mowing the lawn. It takes Cam 2 hours to mow the lawn by himself and it takes Joel 90 minutes. If it takes the three of them working together 45 minutes, how long does it take Trevor to mow the lawn by himself, in hours?	6 [hours]
4	Two of the sides of a triangle have lengths 7 and 5. The angle between the two measures 60 degrees. Find the length of the side opposite this angle.	$\sqrt{39}$ [un]
5	A cube is inscribed in a sphere with volume 36 pi. Find the surface area of the cube.	72 [un]
6	The parabola given by the equation Y equals the quantity X minus three times the quantity two X plus eight has vertex with coordinates H comma K. Find H plus K.	-25
7	Andrew is lost in a forest. He wanders 3 miles north, 7 miles east, 6 miles south and then 2 miles east. How many miles away from his starting point is he?	$3\sqrt{10}$ [miles]
	Extra Problem – Only if Needed	
8	Find the sum of the four non-real fifth roots of 1.	-1

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COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	Find the smallest possible perimeter of a scalene triangle with integer side lengths.	9 [un]
2	What is the probability of drawing an ace, king or diamond from a standard deck of 52 cards?	$\frac{19}{52}$
3	What is the length of the altitude to the hypotenuse of a right triangle with legs of length 7 and 24?	$\frac{168}{25}$ [un]
4	Find the remainder when 3 to the 18th power is divided by 5.	4
5	Evaluate the sum of the first 47 multiples of 6.	6768
6	How many two letter permutations can be made from the letters B O O T H?	13
7	Find the area of the shape described by the equation four X squared minus twenty-four X plus Y squared minus four Y plus twenty-four equals zero.	8π [un^2]
	Extra Problem – Only if Needed	
8	Evaluate the quantity tangent squared of 15 degrees [PAUSE] plus one times the quantity sine squared of 105 degrees.	1

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COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	Find the coefficient of the X squared term in the expansion of X squared plus the quantity 3 over X, quantity to the seventh power.	2835
2	Jack is serving google-ade. It is composed of 80 percent google and 20 percent water. He currently has 30 cups of google-ade. How many ounces of water must Jack add to the google-ade in order to make it 60 percent google?	80 [ounces]
3	What is the radius of the circle inscribed in a triangle with side lengths of 3, 4 and 5?	1 [un]
4	How many different ways can you hand out 9 pinecones to 4 people if each person must receive at least 1 pinecone?	56 [ways]
5	Find the sum of the roots minus the product of the roots of the equation: the quantity X squared plus nine times the quantity X squared minus X plus ten equals zero.	-89
6	The probability that a windstorm occurs is one fifth. It is a widely known fact that the probability that pigs will fly given that a windstorm occurs is one twentieth. What is the probability that they both occur?	$\frac{1}{100}$
7	What is the distance between the line described by 3 "Y" equals 4 "X" plus 1 and the line 3 "Y" equals 4 "X" plus 4?	3/5
	Extra Problem – Only if Needed	
8	An airplane is flying north at 100 miles per hour, its maximum speed. At this rate, it uses 1 gallon of fuel per hour. It encounters a 10 root 2 mile per hour head wind coming from directly northeast. If it has 2 gallons of fuel left, how far will it travel, assuming the pilot maintains maximum speed?	$20\sqrt{82}$ [miles]

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9^{th} & 10^{th} Grade – January 26, 2008

KEY

School Name_____Team #_____

Proctor Name_____Room #_____

First Score

(out of 18)

STUDENT NAME_____

INDIVIDUAL MULTIPLE CHOICE - 15 minutes

This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. ***Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet.*** *No talking during the test.*

DO NOT WRITE IN SHADED REGIONS

Answer		-1, 0 or 2	-1, 0 or 2
1	A		
2	C		
3	D		
4	B		
5	C		
6	B		
7	B		
8	A		
9	C		

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9^{th} & 10^{th} Grade – January 26, 2008

KEY

School Name_____Team #_____

Proctor Name_____Room #_____

First Score

(out of 20)

STUDENT NAME_____

Team Contest – Score Sheet

TEAM TEST - 15 minutes

When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as ***2 or 0****. Record all answers on the colored answer sheet.*

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	$36-18\sqrt{3}$ [cm]		
2	$6\sqrt{2}$ [cm]		
3	1650 [elements]		
4	24 [coins]		
5	$\frac{67}{288}$		
6	27		
7	1/30304		
8	288 [min]		
9	30		
10	27 [un^2]		

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9th & 10th Grade – January 26, 2008

KEY

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

PRESSURE ROUND - 10 minutes

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

Pressure Round Answers

	Answer
1	*E A C F B D* [in order]
2	2424
3	44
4	15,099
5	0