"Math is Cool" Masters – 2007-08

Sponsored by:

Algebra II – January 26, 2008

Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by all involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
  - *For problems dealing with money, a decimal answer should be given.*
  - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of π where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

"Math is Cool" Masters – 2007-08

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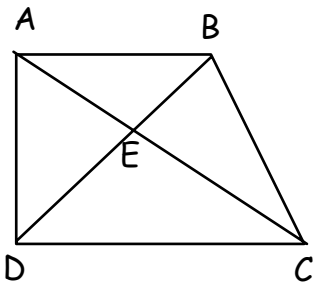
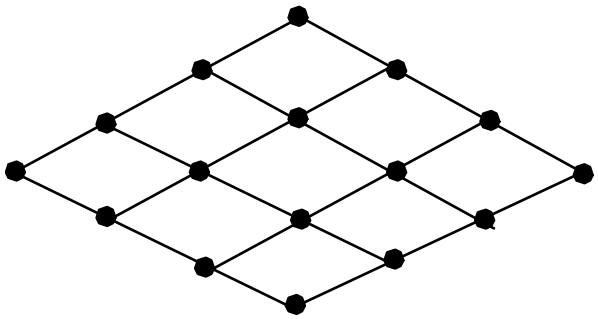
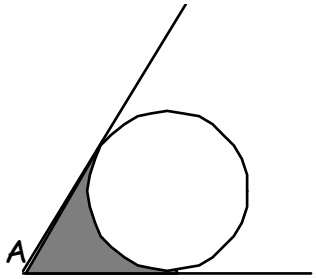
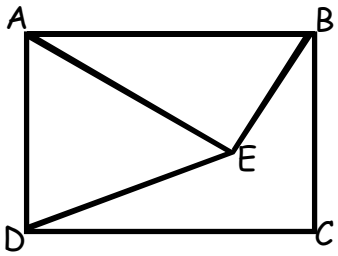
Algebra II – January 26, 2008

Individual Contest

1	Evaluate: $(4+6/3^2-7+2\bullet 5)$
2	What is the sum of the number of faces plus the number of edges plus the number of corners of a cube?
3	How many permutations are there of the letters in the word "FOOD"?
4	What is the sum 1 + 3 + 5 + 7 + 9 + 11 + 13?
5	What is the number of inches in a foot times the number of seconds in a minute?
6	What is the probability of getting 3 consecutive heads while flipping a fair coin?
7	What is the area of a circle with circumference 8π?
8	I roll two fair six-sided dice and multiply the top numbers showing. What is the probability that the product is odd?
9	What is the area of the largest square that can be put in a circle of radius 1?
10	What are all the solutions to the equation $2x^2-3x=-1$?
11	There are four colors available to paint my house. If I want to use one color for the sides and another for the trim, how many distinct ways can I paint my house?
12	What is the shortest side of a right triangle whose other two sides are 15 and 17?
13	I brought my apples to the market to sell. I sold half of them to Mr. Smith and then two-thirds of what I had left over to Mrs. Jones. I ate the last two apples. How many apples did I sell?
14	If AB is parallel to CD and AB=2, CD=3 and AE=1, what is CE? 
15	What is the smallest number of 6-inch by 8-inch tiles needed to form a square with no overlaps or empty areas?

16	Four couples sit in a row for the theater. In how many ways can they be seated so that the husbands sit next to their wives?
17	How many paths of length 6 are there from S to F in the figure below. S  F
18	What is the average of the arithmetic sequence: 28, a, b, 35; where a and b are unspecified numbers?
19	During my last road trip, I went 50 mph for 75 miles, then 60 mph for the next 75 miles while I maintained 56 mph the whole 150 miles coming home. What was my average speed for the trip, in mph?
20	The cost of producing x books is given by the expression: 11x+2200. If I have already produced 68 books, how much more will it cost to produce 2 more books?
21	I randomly open a book and the product of the page numbers showing is 6162. What is the larger page number?
22	What is the units digit of $13^{19} - 19^{13}$?
23	How many different isosceles triangles can be constructed with integral sides and a perimeter equal to 79?
24	When 5a7 base 8 is written in base 9, the units digit is 0. What are the possible choices for the digit 'a'?
25	What is the radius of the circle described by: $x^2 + y^2 - 6x + 8y - 8 = 0$
26	What is the minimum distance between the origin and a point on the line 3x+4y-5=0?
27	If 2 hens can lay 6 eggs in 5 days, how many eggs can 5 hens lay in 8 days?
28	Solve for x: $\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\ldots}}}} = 8$
29	If x and y are rational numbers, and $12^{3x+y+2} = 18^{4x-y-1}$, what is x/y?

Challenge Questions

30	Let $i=\sqrt{-1}$, express $\frac{3+2i}{5-i}$, in the form $a+bi$.
31	I draw a line in the coordinate plane starting at (0,0) and going to (0,1). I then turn 90 degrees to the right and proceed a distance of .5. I then turn 90 degrees to the right and proceed a distance of .25. If I continue this process of turning 90 degrees to the right and then drawing 1/2 the distance that I have just traveled indefinitely, at what point, (x,y) form, will I end up?
32	If p and p+2 are both prime numbers greater than 10, what is the largest integer guaranteed to divide p^2+2p+1 evenly?
33	In the figure below, angle A has a measure of 60 degrees and the radius of the circle is 3. What is the area of the shaded region? 
34	The probability of event A given event B is 3/4, the probability of A is 1/2 and the probability of A and B is 1/3. What is the probability of A given B^c, where B^c is the complement of B?
35	Sedgwick will descend a flight of nine stairs, taking at least two stairs at each step. In how many ways can Sedgwick descend?
36	What is the sum of the squares of the solutions to $x^2+x-1=0$?
37	There are 12 members of LMS 12^{th} grade Math Team. In how many ways can they be sorted into three teams, each with a captain and three other team members?
38	For what integers, n, is the expression $\frac{5n+26}{2n+3}$ an integer?
39	In the figure, AE=8, BE=4, DE=7, what is CE? [ABCD is a rectangle.] 
40	In how many ways can I choose seven distinct numbers from the set {1,2,3,4,5,6,7,8,9} so that their sum is a multiple of 3?

"Math is Cool" Masters – 2007-08

Algebra II – January 26, 2008

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

KEY

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	23/3		
2	26		
3	12		
4	49		
5	720		
6	1/8		
7	16π [un^2]		
8	1/4		
9	2 [un^2]		
10	[x=] 1, 1/2		
11	12 [ways]		
12	8		
13	10 [apples]		
14	3/2		
15	12 [tiles]		
16	384 [ways]		
17	20 [paths]		
18	63/2		
19	1050/19 [mph]		
20	[$] 22		

	Answer	1 or 0	1 or 0
21	79		
22	8		
23	20 [sides]		
24	[a=] 3		
25	$\sqrt{17}$		
26	1 [un]		
27	24 [eggs]		
28	$2\sqrt{14}$		
29	1/11		
30	$\frac{1}{2}+\frac{i}{2}$		
31	(2/5, 4/5)		
32	36		
33	$9\sqrt{3}-3\pi$		
34	3/10		
35	21 [ways]		
36	3		
37	369,600 [ways]		
38	-20, -2, -1 AND 17		
39	1		
40	12 [ways]		

"Math is Cool" Masters – 2007-08

Algebra II – January 26, 2008

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	Answer	1 or 0	1 or 0
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"Math is Cool" Masters – 2007-08

Sponsored by:

Pre-Calculus – January 26, 2008

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Sponsored by:

Pre-Calculus – January 26, 2008

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5	What is the value of $\log 4+\log 5-\log 2$?
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8	I roll two fair six-sided dice and multiply the top numbers showing. What is the probability that the product is odd?
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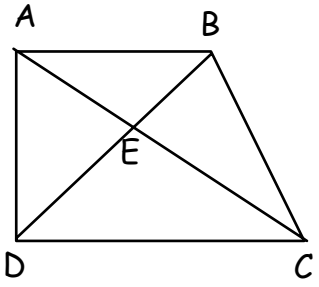
14	If AB is parallel to CD and AB=2, CD=3 and AE=1, what is CE? 
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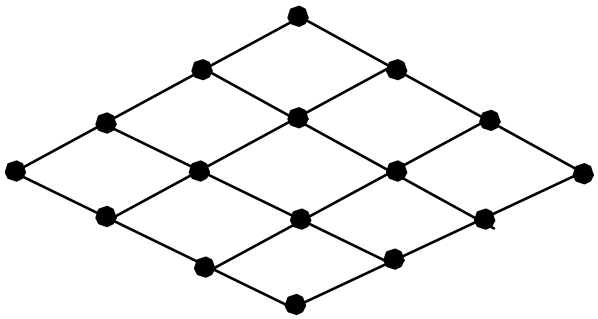
17	How many paths of length 6 are there from S to F in the figure below. S  F
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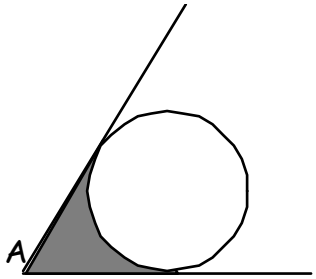
33	In the figure below, angle A has a measure of 60 degrees and the radius of the circle is 3. What is the area of the shaded region? 
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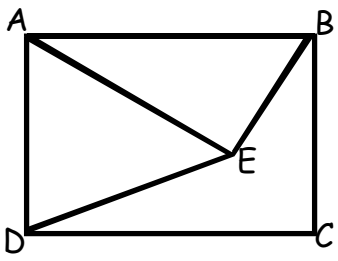
39	In the figure, AE=8, BE=4, DE=7, what is CE? [ABCD is a rectangle.] 
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"Math is Cool" Masters – 2007-08

Pre-Calculus – January 26, 2008

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4	49		
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17	20 [paths]		
18	$4\sqrt{5}$		
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	Answer	1 or 0	1 or 0
21	48 [zeroes]		
22	8		
23	20 [sides]		
24	[a=] 3		
25	20π [un^2]		
26	1 [un]		
27	24 [eggs]		
28	$2\sqrt{14}$		
29	1/11		
30	$\frac{1}{2} + \frac{i}{2}$		
31	(2/5, 4/5)		
32	91/2		
33	$9\sqrt{3} - 3\pi$		
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36	-4		
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Pre-Calculus – January 26, 2008

School Name_____Team #_____

Proctor Name_____Room #_____

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First Score

STUDENT NAME_____

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Calculus – January 26, 2008

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9	What is the area of the largest square that can be put in a circle of radius 1?
10	How many negative real solutions are there to the equation: $3x^4-x^3+2x^2-4x=2$
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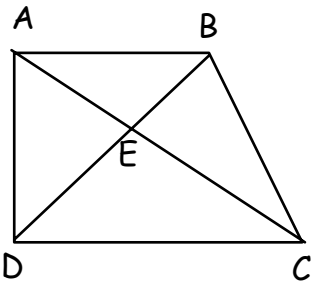
14	If AB is parallel to CD and AB=2, CD=3 and AE=1, what is CE? 
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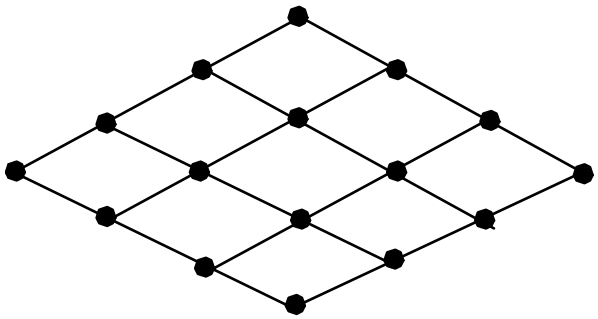
17	How many paths of length 6 are there from S to F in the figure below. S  F
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20	What is the height, in units, of the rectangle with the largest area that can be drawn in the region bounded by $y = 9 - x^2$ and y=0?
21	When 100! in base 10 is written in base 12, how many zeroes will be on the right?
22	What is the units digit of $13^{19} - 19^{13}$?
23	How many different isosceles triangles can be constructed with integral sides and a perimeter equal to 79?
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26	What is the minimum distance between the origin and a point on the line 3x+4y-5=0?
27	If the radius of a sphere is changing at the rate of 4 cm/min, how fast is the surface area changing, in cm^2/min, when the radius is 20 cm?
28	What is the $\lim_{x\to\infty}\left(x - \sqrt{x^2 - x}\right)$?
29	If x and y are rational numbers, and $12^{3x+y+2} = 18^{4x-y-1}$, what is x/y?

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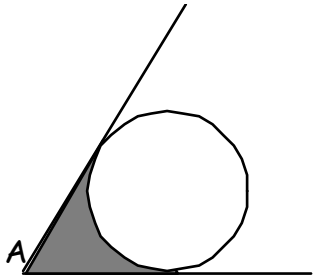
33	In the figure below, angle A has a measure of 60 degrees and the radius of the circle is 3. What is the area of the shaded region? 
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36	What is the sum of the cubes of the solutions to $x^2+x-1=0$?
37	There are 12 members of LMS 12^{th} grade Math Team. In how many ways can they be sorted into three teams, each with a captain and three other team members?
38	For what integers, n, is the expression $\frac{5n+26}{2n+3}$ an integer?
39	Including the complex values, list the cube roots of -1.
40	In how many ways can I choose seven distinct numbers from the set {1,2,3,4,5,6,7,8,9} so that their sum is a multiple of 3?

"Math is Cool" Masters – 2007-08

Calculus – January 26, 2008

Final Score:

KEY

First Score

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	23/3		
2	26		
3	12		
4	49		
5	1		
6	3/8		
7	16π [un^2]		
8	1/4		
9	2 [un^2]		
10	1		
11	12 [ways]		
12	$4x^2 - 15$		
13	10 [apples]		
14	3/2		
15	12 [tiles]		
16	384 [ways]		
17	20 [paths]		
18	$4\sqrt{5}$		
19	1050/19 [mph]		
20	6 [un]		

	Answer	1 or 0	1 or 0
21	48 [zeroes]		
22	8		
23	20 [sides]		
24	[a=] 3		
25	20π [un^2]		
26	1 [un]		
27	640π [cm^2/min]		
28	1/2		
29	1/11		
30	$\frac{1}{2}+\frac{i}{2}$		
31	(2/5, 4/5)		
32	91/2		
33	$9\sqrt{3}-3\pi$		
34	3/10		
35	21 [ways]		
36	-4		
37	369,600 [ways]		
38	-20, -2, -1 AND 17		
39	$-1, \frac{1}{2}+\frac{\sqrt{3}}{2}i, \frac{1}{2}-\frac{\sqrt{3}}{2}i$		
40	12 [ways]		

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First Score

School Name_____Team #_____

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Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1			
2			
3			
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	Answer	1 or 0	1 or 0
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