"Math is Cool" Championships – 2008-09

Sponsored by:

PreCalculus & Calculus – October 24, 2008

Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by all involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
  - *For problems dealing with money, a decimal answer should be given.*
  - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of π where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

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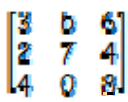
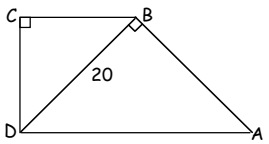
Individual Contest

1	How many cubic inches are there in a cubic foot?
2	How many two-digit prime numbers have a 4 as one of their digits?
3	What is the total measure, in degrees, of the interior angles of a regular pentagon?
4	What is the sum of the first 15 positive integers?
5	Simplify: $\frac{x^3x^{-2}}{x^{-1}}$
6	What is the sum of the distinct prime factors of 2008?
7	How many positive 4-digit palindromes are there? A palindrome is a number that is the same when it's digits are reversed.
8	What is the probability of rolling two dice and getting doubles (both numbers are the same)?
9	Find x if $294+x$ is 10% greater than $294-x$.
10	What is the sum $i^2+i^6+i^{10}+i^{14}$, if $i=\sqrt{-1}$?
11	If 2 is a solution of the equation $x^3-2x^2+kx+16=0$, what is k?
12	I put the numbers from 1 to 6 in a hat and randomly draw out two numbers. What is the probability that the numbers add to 7?
13	If the point (3,3) is the midpoint of the line segment between (1,2) and (x,y); what is the sum x+y?
14	What is the area, in square units, of an equilateral triangle inscribed in a circle of area 36π units squared?
15	What is the remainder when 123456789 is divided by 11?
16	Evaluate the determinant of the following matrix: 

17	My number is between 40 and 60 and has a remainder of 3 when I divide by 4 and, 4 when I divide by 5. What is my number?
18	In a rectangle, the length to width ratio is 4:1 and the area of the rectangle is the same as a square with perimeter 32. What is the perimeter of the rectangle?
19	In how many ways can two boys and two girls be seated in a row so that the boys are not seated next to each other and neither are the girls next to each other?
20	I randomly select two distinct positive integers less than 10, what is the probability that the sum of the pair of numbers is greater than 4?
21	Express the base 10 number: 2008_{10} in base 7.
22	I choose n, an integer between 1 and 2008 inclusive and $f(n)=ni^n$, where $i^2=-1$, what is the probability that f(n) is both real and positive?
23	Randy is reading a book, seeing a page on the left and another on the right of his view. If the difference between the sum and product of the page numbers is 131, what is the smaller of the page numbers he is reading?
24	All sides of trapezoid ABCD are integer lengths, in cm, with segment BD equal to 20 cm, as shown. Find the area of the trapezoid, in square cm. Figure not to scale! 
25	What is the sum of the coefficients of the expanded form of the expression $(3x-7)^5$?
26	What is $8^{\log_4 6}$?
27	What is the distance from the vertex to the focus of the parabola defined by the following equation: $x^2-8x-12y+16=0$
28	The weather on Monday has a 50% chance of rain. For the following days of the week, the chance of rain on any given day decreases to 40% following a rainy day and increases to 60% if it does not rain the day before. What is the probability that it is raining on Wednesday?
29	Evaluate $\cos(7\pi/12)$.
30	Timothy arrives on an island populated by 3 knights who always tell the truth, 3 knaves who always lie, and 3 pages who tell the truth only half the time. Meeting 3 people at random, what is the probability that they all tell the truth when asked a question?

Challenge Questions

31	What is the remainder when 10! is divided by 13?
32	How many ways can the letters in the word "MIXED" be rearranged such that no letter maintains its original position?
33	The complex number z satisfies $z+\lvert z\rvert = 1+7i$. Find $\lvert z\rvert^2$.
34	What is the shortest distance to the line y=3x+4 from the point (6,2)
35	In trapezoid ABCD, where $AB=20$, $BC=33$, $CD=25$, $AD=48$, and $\overline{AD}\parallel\overline{BC}$, a segment drawn from C intersects segment $\overline{AD}$ at point E such that $\angle CED$ is right. Segment $\overline{CE}$ intersects segment $\overline{BD}$ at point F. Find the length of segment $\overline{BF}$.
36	How many ordered integer triplets (a,b,c) are there such that abc=12?
37	If the smaller angle between the hour and minute hands of a 12-hour analog clock is 84° and the sum of the hours and minutes is 17, what time between noon and midnight is it?
38	What is the remainder when 2008^{2008} is divided by 2009?
39	A coin is flipped 10 times in a row. Given that exactly 7 heads appeared, find the probability that no two consecutive flips were both tails.
40	Both beginning on the starting-line, Sally and Sam run in opposite directions on a 450 meter-long circular track. Sally runs at 4 m/sec, Sam also runs at 4 m/sec, that is, until he passes Sally. Wanting to impress her, he speeds up by 1 m/sec each time he passes her and maintains that pace until their next meeting. How far has Sally run when Sam begins running at 7m/sec?

"Math is Cool" Championships – 2008-09

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11th & 12th Grade – October 24, 2008

Individual Multiple Choice Contest

1	Find the remainder when 7^{2008} is divided by 19. A) 1 B) 2 C) 7 D) 11 E) Answer not given
2	What is the sum of all the positive 2-digit multiples of 6? A) 720 B) 810 C) 840 D) 900 E) Answer not given
3	What is the sum of the x and y coordinates of the center plus the radius of the circle defined by: $x^2 + y^2 + 8x - 2y + 8 = 0$ A) -2 B) $-3+\sqrt{2}$ C) 0 D) 2 E) Answer not given
4	The measures of the angles of a triangle are in the ratio of 2:3:5. What is the measure of the smallest angle in degrees? A) 20 B) 36 C) 30 D) 72 E) Answer not given
5	Which of the following statements are true? I. The set {-1, 0, 1} is closed under addition. II. The set {-1.0,1} is closed under multiplication. III. The set of rational numbers is closed under addition. IV. The set of rational numbers is closed under multiplication. A) II, III B) I, II C) All of them D) II, III, IV E) Answer not given
6	Joe can mow the lawn in 2 hours but Mary takes 3 hours. If Joe starts mowing the lawn at 1:00pm and Mary joins him at 2:00pm, when will they finish? A) 2:15pm B) 2:20pm C) 2:30pm D) 2:45pm E) Answer not given
7	For how many values on the interval $[0,2\pi)$ is $\sin x + \cos x = 1$? A) 1 B) 2 C) 4 D) 8 E)
8	Ignoring order, in how many ways can the number 10 be represented as the sum of three positive integers? A) 6 B) 7 C) 8 D) 9 E) Answer not given
9	What is the radius of the circle inscribed in a triangle with sides of lengths 6, 7 and 7? A) 2 B) 5/2 C) 3 D) 7/2 E) Answer not given

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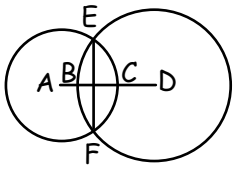
Team Contest

1	What is the area of the shape with vertices at the points (2,3), (2,8), (5,7), (5,3)?
2	For how many integers is $x^4 - 5x^2 + 4 \le 0$?
3	The DVD re-surfacer at Video Quest has a 10% chance of breaking down each time it is used. What is the expected number of times it will work before breaking down?
4	What is the largest integer that is a divisor of (n+1)(n+3)(n+5)(n+7)(n+9) for all positive even integers n?
5	How many distinct prime factors are there for the number $280371 = 23^4 + 23^2 + 1$.
6	Solve for x: $\sin(\arctan x) = \frac{1}{2}$
7	John has a 1/3 chance of getting an A and independently, Anne has a 3/5 chance of getting an A. I look at one of their papers on the teacher's desk and see an A. I can't see whose paper it is, what is the probability that it is Anne's?
8	If $\sin x + \cos x = \frac{1}{3}$, find all possible values of $\sin 2x + \cos 2x$.
9	In the diagram at the right, the circle with center A has radius 3 units, and the circle with center D has radius 4 units. Given that $AB = 1$ and $CD = 2$, find the length of EF. If your answer is not an integer, give it as a decimal. 
10	Each term in the series below has the form $\frac{1}{(n)(n+3)}$. Find the sum of this series: $\frac{1}{(1)(4)} + \frac{1}{(4)(7)} + \frac{1}{(7)(10)} + \ldots \frac{1}{(298)(301)}$

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9^{th}, 10^{th}, 11^{th} & 12^{th} Grade – October 24, 2008

Pressure Round Contest

1	A cube with a side length of 5 inches is painted and divided into unit cubes. What is the ratio, expressed as a fraction in lowest terms, between the number of cubes with 2 faces painted to the number of cubes with no faces painted?
2	What is the cubic polynomial P(x) that has zeros of -1, 3, and 5 and P(1) = 4?
3	John uses the average of the perimeters of the circumscribed and inscribed squares to a circle of radius 1 to estimate its circumference. Using this approximation, what is his estimate of π using the formula for the circumference?
4	A pair of fair six-sided dice is rolled. What is the probability the positive difference between the two numbers is a factor of one of the numbers rolled?
5	Find all integers m, so that there is an integer n such that $6m^2 + 2n^2 + 7mn = 5$.

"Math is Cool" Championships – 2008-09

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11th & 12th Grade – October 24, 2008

Mental Math Contest

PERSON 1		
1.1	Two fair six sided die are rolled. What is the probability of rolling a sum of 9 or greater?	5/18
1.2	Mary has a standard 8 by 8 checkerboard of black and white squares. Her little brother blackened all the squares down the left side. Now, what is the ratio of black to white squares in, a colon b, notation?	9:7
1.3	If B is in the second quadrant and sine B is four fifths, what is the tangent of B?	-4/3
1.4	Find the coordinates x comma y of the vertex of the parabola: negative x squared plus six times x plus nine.	(3,18)
PERSON 2		
2.1	Two angles of a triangle are 34 and 110 degrees. What is the measure of the third angle in degrees?	36[°]
2.2	What is the greatest common factor of 30, 12 and 72?	6
2.3	What is the sum of the infinite series: two thirds plus two ninths plus two twenty-sevenths and so on?	1
2.4	Randy has a 70% chance of telling a lie and Trevor has an 80% chance of lying. If their mother realizes that cookies are missing from the cookie jar but both of them say they didn't take any, then what is the probability that Trevor did take the cookies and Randy did not?	6/25
PERSON 3		
3.1	Evaluate seven factorial.	5040
3.2	If a ball is thrown at 36 meters per second and the ball travels 288 meters, then how many seconds does it take for the ball to reach the distance?	8 [seconds]
3.3	How many positive integer factors does 96 have?	12
3.4	How many positive real roots are there to the equation: x to the fourth plus 3 x cubed plus 2 x squared minus 5 x minus 3 equals 0?	1
PERSON 4		
4.1	What is the sum of log base two of one fourth and log base two of eight?	1
4.2	What is the area of a triangle with side lengths of 9, 12 and 15?	54 [un^2]
4.3	What is the cube root of the product of the roots of the equation x-cubed plus 3 times x-squared minus 10 times x minus 27	3
4.4	What is the sum of the base three number two-two and the base five number four-four expressed in base two?	100000 [2]

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11th & 12th Grade – October 24, 2008

COLLEGE KNOWLEDGE BOWL ROUND #1 - SET A

#	Problem	Answer
1	A triangle is formed by connecting the points negative one comma negative one; one comma three; and zero comma negative two. Find the coordinates of the centroid.	Zero comma zero or Origin
2	What is the domain of the inverse sine function?	[-1,1]
3	What is the sum of the quantity twenty-seven over thirty six to the negative one power plus one-fourth?	19/12
4	Using a standard deck of fifty two cards what is the probability of drawing three jacks in a row one at a time without replacement?	1/5525
5	What is the cosine of an angle in the third quadrant if the tangent of that angle is the square root of eleven divided by 5?	-5/6
6	What is the lateral surface area of the cone formed by rotating the line segment from four comma zero to zero comma three around the y-axis?	20π
7	What is the positive difference between the mean and the median of the set of numbers: ten, thirty-six, three, seventy-five, eight, and forty-eight?	7
	Extra Problem – Only if Needed	
8	The graph of x squared minus eight x plus three passes through which quadrants?	One, two, and four

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COLLEGE KNOWLEDGE BOWL ROUND #2- SET A

#	Problem	Answer
1	Hannah rolls 2 similar fair n-sided dice. What is n, if the most probable sum of the numbers is 14?	13
2	What is the sine of forty five degrees plus the cosine of forty five degrees plus the tangent of forty five degrees?	$1+\sqrt{2}$
3	Evaluate nineteen thousand five hundred and thirty nine divided by thirteen then divided by 3.	501
4	What is the smallest positive abundant number, that is, the smallest number whose proper factors add to more than the number itself?	12
5	Evaluate negative one plus e raised to the quantity pi times i.	-2
6	Take the number one three nine base sixteen and change it to base seven.	$625_{[7]}$
7	What is the tangent of an angle in the first quadrant if the cosine of the same angle is 8 divided by 17?	15/8
	Extra Problem – Only if Needed	
8	Evaluate nine hundred sixty six times three hundred seventy two.	359352

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COLLEGE KNOWLEDGE BOWL ROUND #3- SET A

#	Problem	Answer
1	If Mulan wants to rearrange the letters in her name [M-U-L-A-N] what would the fifty-second permutation be if they were ordered alphabetically?	m-a-n-u-l
2	If a cone of height nine and radius five is filling at a rate of two pi per second, how long, in seconds, will it take to fill up the cone?	75/2 [seconds]
3	Assuming that an ace is higher than a seven in a standard deck of fifty two cards, what is the probability of drawing a pair of matching cards lower than seven?	5/221
4	A country has 4 cent coins and 7 cent coins. What is the largest number of cents that cannot be made with just this country's coins?	17 [cents]
5	If X is the set of all even numbers between zero and forty and Y is the set of all multiples of three between zero and forty, what is the sum of the elements in the intersection of sets X and Y?	126
6	Two circular pulleys each of radius 1 foot have centers 3 feet apart. What is the length, in feet, of a band wound tightly around the outside of the pulleys?	6 + 2π [feet]
7	What is the equation, in slope intercept form, of the slant as-sim-tote of the function g of "x" equals the quantity 2 "x" squared plus 7 "x" minus 6 divided by the quantity "x" minus 2?	y equals 2 x plus 11
	Extra Problem – Only if Needed	
8	If a set of numbers is one-half, one-third, one fourth and so on to one seventh, what is the product of the set?	1/5040

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COLLEGE KNOWLEDGE BOWL ROUND #1 - SET B

#	Problem	Answer
1	What is the space diagonal of a rectangular prism with side lengths of seventeen, thirty two, and forty six?	$3\sqrt{381}$
2	What is the area of a sector with a central angle of pi radians of a circle with radius 2?	2π [un^2]
3	How many dollars would Casey Jones have three years from now if his four hundred dollars was compounded yearly at a ten percent annual interest rate rounded to the nearest cent?	[$] 532.40
4	How many distinct prime factors does thirty two thousand three hundred have?	4
5	What is the log base 2 of 5, times the log base 25 of 4?	1
6	What is the sum of one two six in base 10 and three two one in base 4? Express your answer in base 16.	$B7_{[16]}$
7	If a series is one half plus two thirds plus three fourths plus four fifths plus five sixths, what is the sum?	71/20
	Extra Problem – Only if Needed	
8	The sum of the reciprocals of two numbers is two sevenths. The product of their reciprocals is six elevenths. What is the sum of the two numbers?	11/21

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COLLEGE KNOWLEDGE BOWL ROUND #2- SET B

#	Problem	Answer
1	If the day of the week of the day before yesterday begins with the letter "S" and the day after tomorrow begins with the letter "T," what day was it yesterday?	Monday
2	How many integers, x, are there such that the absolute value of the difference between x and 8 is between 3 and 10 inclusive?	16
3	If f of "x" equals 3 "x" plus 6 then what is f inverse of "x"?	[y=] (x-6)/3
4	What is the positive difference between the volumes of one sphere of radius six and another with a radius of three?	252π
5	How many integer side lengths are possible for the third side of a triangle with the first two sides with lengths seven and nine?	13
6	What is the perimeter of a triangle, in centimeters, with an area of one hundred ten square centimeters and whose inscribed circle has radius five centimeters?	44 [cm]
7	What is the sum of the number of faces, vertices, and edges of a cube?	26
	Extra Problem – Only if Needed	
8	My friend Andrea has 5 children with at least one girl. What is the probability that at least 4 of the children are girls?	6/31

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COLLEGE KNOWLEDGE BOWL ROUND #3- SET B

#	Problem	Answer
1	What is the lateral surface area, in inches squared, of a cone with a base radius of five inches and a height of twelve inches?	65π [in^2]
2	Is the following an odd function, even function or neither: f of x equals x times the sine of x?	even
3	What is the remainder when one thousand seven hundred thirty-two is divided by thirty-three?	16
4	The sum 1 plus r plus r-squared plus r-cubed and so on equals two-thirds. What is r?	-1 / 2
5	What is the equation of the line, in standard form, that is tangent to the curve x times y equal 1 at the point 1 comma 1?	X + Y = 2
6	What is the log of one thousand plus the log base 2 of sixteen?	7
7	Amy, Bob and six of their friends sit in a row at the movies. How many ways can this be done so that Amy and Bob do not sit next to each other?	30240 [ways]
	Extra Problem – Only if Needed	
8	Find the eleventh term of the following sequence: one, one, two, three, five …	89

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PreCalculus & Calculus – October 24, 2008

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	1728 [in^3]		
2	3 [numbers]		
3	540 [°]		
4	120		
5	x^2		
6	253		
7	90		
8	1/6		
9	[x=] 14		
10	-4		
11	-8		
12	1/5		
13	9		
14	$27\sqrt{3}$ [un^2]		
15	5		
16	0		
17	59		
18	40 [un]		
19	8 [ways]		
20	17/18		

	Answer	1 or 0	1 or 0
21	$5566_{[7]}$		
22	1/4		
23	12		
24	246 [cm^2]		
25	-1024		
26	$6\sqrt{6}$		
27	3 [units]		
28	1/2		
29	$\frac{\sqrt{2}-\sqrt{6}}{4}$		
30	3/32		
31	6		
32	44 [ways]		
33	625		
34	$2\sqrt{10}$ [units]		
35	143/4 [units]		
36	72		
37	5:12 PM		
38	1		
39	7/15		
40	605 [meters]		

"Math is Cool" Championships – 2008-09

11th & 12th Grade – October 24, 2008

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Final Score:
KEY

First Score

(out of 18)

INDIVIDUAL MULTIPLE CHOICE - 15 minutes

This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. ***Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet.*** *No talking during the test.*

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	C		
2	B		
3	C		
4	B		
5	D		
6	E (2:36pm)		
7	B		
8	C		
9	E (21/10)		

"Math is Cool" Championships – 2008-09

11th & 12th Grade – October 24, 2008

Final Score:
KEY

School Name_____Team #_____

Proctor Name_____Room #_____

First Score

(out of 20)

STUDENT NAME_____

Team Contest – Score Sheet

TEAM TEST - 15 minutes

When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as ***2 or 0****. Record all answers on the colored answer sheet.*

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	$27/2$ [un^2]		
2	4 [integers]		
3	9 [times]		
4	15		
5	4		
6	$\frac{\sqrt{3}}{3}$		
7	9/14		
8	$\frac{-8 \pm \sqrt{17}}{9}$		
9	4.8 [units]		
10	100/301		

"Math is Cool" Championships – 2008-09

9th, 10th, 11th & 12th Grade – October 24, 2008

Final Score:
KEY

First Score

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

PRESSURE ROUND - 10 minutes

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

Pressure Round Answers

	Answer
1	4/3
2	$\frac{1}{4}x^3 - \frac{7}{4}x^2 + \frac{7}{4}x + \frac{15}{4}$
3	$2+\sqrt{2}$
4	4/9
5	-3, 3, 9, -9 [in any order]