"Math is Cool" Masters – 2008-09

Sponsored by: EKA Chemicals

Pre-Algebra – December 6, 2008

Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by all involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
  - *For problems dealing with money, a decimal answer should be given.*
  - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of π where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

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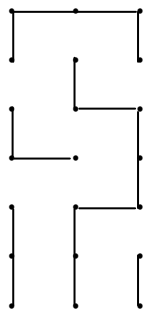
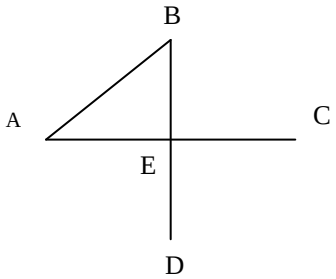
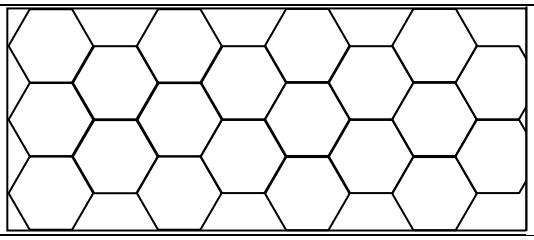
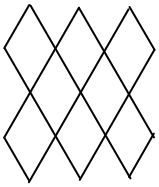
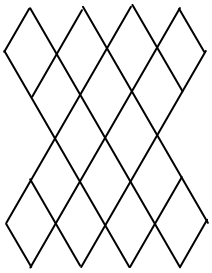
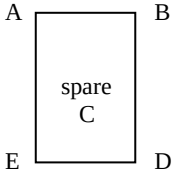
Pre-Algebra – December 6, 2008

Individual Contest

1	What is the sum of the numbers of edges and the number of vertices of ten decagons?
2	Evaluate: $\frac{1}{3}\cdot\frac{3}{6}\cdot\frac{6}{10}\cdot\frac{10}{15}\cdot\frac{15}{21}\cdot\frac{21}{28}\cdot\frac{28}{36}\cdot\frac{36}{45}\cdot\frac{45}{55}\cdot\frac{55}{66}$
3	The state fish of Hawaii is the humuhumunukunukuapua'a. What is the ratio of vowels to consonants in this word?
4	A square with edge length 6 inches is made into a rectangle by doubling one edge and multiplying the other edge by five. What is the number of inches in the perimeter of the new rectangle?
5	The letter W consists of four noncollinear segments. How many other letters of the given alphabet consist of four non-collinear segments? A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
6	What is the sum of the smallest prime number, the largest two-digit multiple of three, and the smallest four-digit perfect square?
7	In the following number, give the product of the digit in the hundreds place and the digit in the ten-thousandths place: 7,104,521.9683362
8	1.215 is equivalent to the quotient of 12150 divided by 10^a. What is the value of a?
9	A certain bird chirps 27 times every minute. How many times will the bird chirp in 9 minutes?
10	Andy and Betty are playing a game. The players take turns connecting any two horizontally or vertically adjacent dots by drawing a segment. If a player draws a segment that is the fourth side of a square, s/he may draw another segment. If a segment does not complete a square, the turn is over. Players score 1 point for each completed square. It is Andy's turn and the game grid is as shown. How many choices does Andy have of where to draw a segment that will not give Betty an opportunity to complete a square on her next turn? No segment can be drawn where one already exists. 
11	E is the midpoint of both BD and AC. AB = 6 inches. How many inches are in the length of CD? 
12	Evaluate: $\frac{2}{3}[5\,(\frac{6!}{8}) + 3\,(\frac{4!}{8})]$

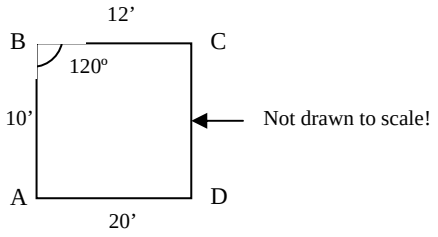
13	A circle has a radius of 5 cm. The circle is to be placed on a grid of 1 cm by 1 cm squares such that the center of the circle coincides with an intersection of four grid squares. Through how many grid squares does the circumference of the circle pass? (The circumference of the circle is not considered to be passing through a grid square when it shares only a single point with the perimeter of a grid square.)
14	Let A be the sum of $\frac{1}{3}$ and $\frac{3}{4}$. Let B be the positive difference between $\frac{1}{3}$ and $\frac{3}{4}$. What is AB?
15	The product of 3, 5, 7, 11, 13 and 37 is a six-digit number in the form aaa,aaa. What is the value of a?
16	Penny has $203.76 in her checking account. She has to pay her phone bill, which includes a fixed monthly charge of $15 plus 8¢ per minute. If she has talked on her phone for 503 minutes this month, how much money, in dollars, will be left in her checking account after she pays this bill?
17	Solve for x: $2^{x-8} = 256$
18	A jar has eleven black marbles and fourteen white marbles. If Suzy pulls two marbles from the jar at the same time, what is the probability that they are the same color?
19	How many multiples of 7 are also multiples of 13 from 1 to 500?
20	A plane is covered with a tessellation of regular hexagons as shown. What is the minimum number of colors needed to color in each hexagon so that no two adjacent hexagons have the same color? 
21	Figure 1  Figure 2  Figure 3  Figure 1 is made of six line segments. What is the minimum number of line segments needed to draw the 100^{th} figure in the sequence?
22	A drawer has 5 blue socks, 11 green socks, 3 blue mittens and 7 green mittens all made of the same material. Lily cannot tell the difference between the feel of a sock or a mitten and the mittens look the same regardless of whether they are worn on left or right hands. Socks also look the same regardless of whether they are worn on the left or right foot. If she randomly pulls items from the drawer, how many items must she take in order to guarantee one matching pair of socks and one matching pair of mittens of the same color as the socks?
23	Evaluate: $3^8 - 6^4$
24	Edna's car has four tires plus a spare. In an attempt to allow all five tires to wear out evenly, she rotates them according to the following scheme. Every 5,000 miles she rotates them clockwise so that the tire in position A goes to position B, the tire in position B goes to position C, etc. If the tires are meant to last 60,000 miles and she begins this rotation plan when the tires are new, how many miles will she be able to drive her car before she needs to replace the tires? 

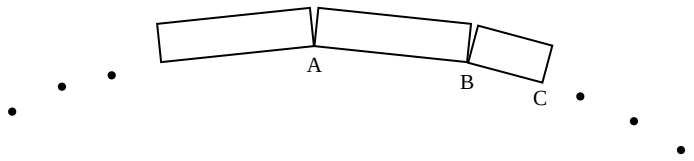
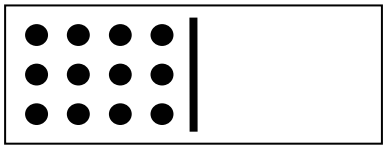
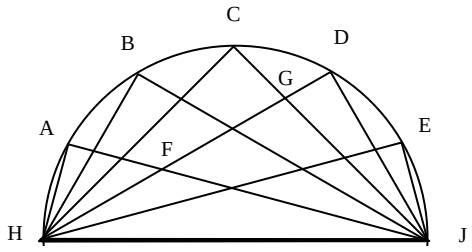
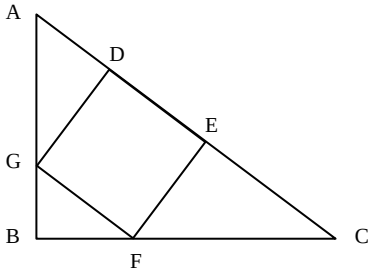
25	Let A and B be any 4-digit integers. Integers C and D are formed by swapping the tens and hundreds digits of A with the tens and hundreds digits of B. For example, if A = 2468 and B = 1357 then C = 2358 and D = 1467. What is the maximum possible value of C – D?
26	What is 472_{10} equivalent to in base 6?
27	Jeff and Sabina are playing "Guess my number". Jeff has secretly written a three-digit number in which all three digits are different. Sabina has made four guesses and Jeff has correctly indicated for each guess the number of correct digits and the number of correct digits in the correct place. What is Jeff's number?
28	Perfect numbers are numbers that are equal to the sum of their proper factors. For example, 6 = 1 + 2 + 3. Even perfect numbers can be generated by the formula $(2^P - 1)(2^{P-1})$, when $2^P - 1$ is a prime number. What is the fourth largest even perfect number?
29	There are several three digit palindromes. If they all were listed in order from least to greatest, what would be the median of the list?
30	Audrey got her learner's permit for driving an automobile on Saturday, June 21. She can get her driver's license in as little as six months after she gets her learner's permit. In other words, she could get it on the 21st of the sixth month after June. If the Department of Licensing is the only place she can get her driver's license and they are closed on Sundays and Mondays, what is the earliest date that she can get her driver's license? Give the month and day.

Sabina's guess	Number of correct digits	Number of correct digits in the correct place
789	1	0
456	1	1
123	0	0
854	2	2
?		

Challenge Questions

31	ABCD is a trapezoid with side BC parallel to side AD. AD is 20 feet, AB is 10 feet and BC is 12 feet in length. What is the area of ABCD in feet squared?
32	Three couples are to be seated at a round table at a party. If each of the six people must sit next to his/her partner, in how many distinct ways can they be seated at the table? Seating arrangements that are simply rotations of other arrangements are not considered distinct.
33	The first term of a geometric sequence is 375 and the fourth term is 24. What is the third term of the sequence?



34	James is creating a border for his garden in the shape of a polygon with 36 sides. There are twenty-four 2′ by 6″ blocks and twelve 1′ by 6″ blocks. He lays the blocks nearly end to end, but rotates them slightly so that they just touch at a corner. As he lays the blocks in a clockwise direction around the garden he starts with two 2′ by 6″ blocks and then one 1′ by 6″ block. He then repeats this three-block pattern all the way around the garden. Everywhere two 2′ by 6″ blocks meet the angle formed by them is the same and everywhere one 2′ by 6″ block meets one 1′ by 6″ block, the angle formed is the same. What is the number of degrees in the angle formed by one 2′ by 6″ block and one 1′ by 6″ block? This is angle ABC in the drawing. 
35	In dominoes, each piece has two groups of dots showing, which represent numbers. Each of the two numbers can be any integer from 0 through 12 inclusive. Every integer is matched up exactly once with each of the integers 0 through 12 inclusive. An example is shown representing (12, 0). What fraction of the number of pieces in the set has exactly one seven showing? 
36	A set of six numbers has a range of 13 and a mean of 20. What is the largest possible value of the median? Answer as a mixed number.
37	When a number N is divided by another number D, Q is the integer part of the quotient and R is the remainder, where R < D. If both D and N are randomly chosen 1-digit positive integers, not necessarily different, what is the probability that R > Q?
38	Points A, B, C, D and E are evenly spaced along the circumference of the semicircle shown. What is the ratio of angle CGD to angle AFD? 
39	The singing group Craig and the Croatians has eight members: two high tenors, two second tenors, two baritones and two basses. A concert line-up must have at least one high tenor, one second-tenor, one baritone and one bass. How many possible concert line-ups are possible for this singing group?
40	Triangle ABC has side lengths 3 cm, 4 cm and 5 cm as shown. Quadrilateral DEFG is a square. What is the number of square centimeters in the area of DEFG? 

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Pre-Algebra – December 6, 2008

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	200		
2	1/66		
3	4/3		
4	84 [in]		
5	2 [letters]		
6	1125		
7	15		
8	4		
9	243 [chirps]		
10	2 [choices]		
11	6 [in]		
12	306		
13	28 [squares]		
14	65/144		
15	5		
16	[$] 148.52		
17	[x=] 16		
18	73/150		
19	5 [multiples]		
20	3 [colors]		

	Answer	1 or 0	1 or 0
21	402 [segments]		
22	19 [items]		
23	5265		
24	75,000 [miles]		
25	8999		
26	$2104_{[6]}$		
27	850		
28	8128		
29	550		
30	December 23		
31	$80\sqrt{3}$ [ft^2]		
32	16 [ways]		
33	60		
34	171 [°]		
35	12/91		
36	$24\frac{1}{3}$		
37	5/9		
38	7/9		
39	81 [line-ups]		
40	3600/1369 [cm^2]		

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	Answer	1 or 0	1 or 0
1			
2			
3			
4			
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20			

	Answer	1 or 0	1 or 0
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"Math is Cool" Masters – 2008-09

Sponsored by: REC Silicon

Algebra I – December 6, 2008

Individual Contest

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Algebra I – December 6, 2008

Individual Contest

1	The letter W consists of four noncollinear segments. How many other letters of the given alphabet consist of four non-collinear segments? A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
2	What is the sum of the smallest prime number, the largest two-digit multiple of three, and the smallest four-digit perfect square?
3	In the following number, give the product of the digit in the hundreds place and the digit in the ten-thousandths place: 7,104,521.9683362
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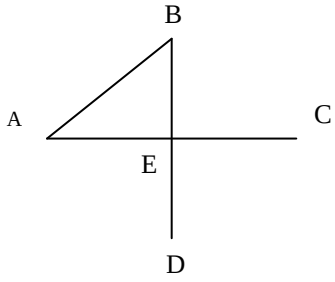
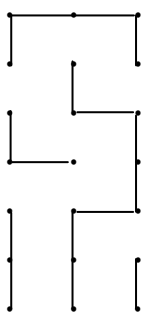
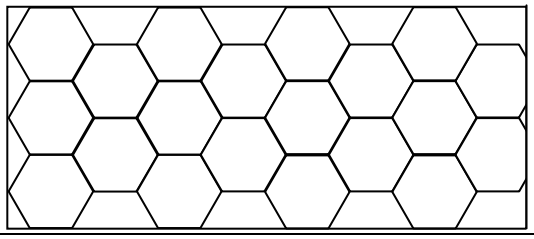
7	E is the midpoint of both BD and AC. AB = 6 inches. How many inches are in the length of CD? 
8	Evaluate: $\frac{2}{3}[5(\frac{6!}{8}) + 3(\frac{4!}{8})]$
9	The graph of y = sin x passes through the points $\left(\frac{\pi}{2},1\right)$ and $\left(\frac{3\pi}{2},-1\right)$. What is the slope of the line through these two points? Answer in terms of π.
10	The product of 3, 5, 7, 11, 13 and 37 is a six-digit number in the form aaa,aaa. What is the value of a?
11	Andy and Betty are playing a game. The players take turns connecting any two horizontally or vertically adjacent dots by drawing a segment. If a player draws a segment that is the fourth side of a square, s/he may draw another segment. If a segment does not complete a square, the turn is over. Players score 1 point for each completed square. It is Andy's turn and the game grid is as shown. How many choices does Andy have of where to draw a segment that will not give Betty an opportunity to complete a square on her next turn? No segment can be drawn where one already exists. 

12	A plane is covered with a tessellation of regular hexagons as shown. What is the minimum number of colors needed to color in each hexagon so that no two adjacent hexagons have the same color? 
13	Solve for x: $2^{x-8} = 256$
14	Penny has $203.76 in her checking account. She has to pay her phone bill, which includes a fixed monthly charge of $15 plus 8¢ per minute. If she has talked on her phone for 503 minutes this month, how much money, in dollars, will be left in her checking account after she pays this bill?
15	How many multiples of 7 are also multiples of 13 from 1 to 500?
16	Let A and B be any 4-digit integers. Integers C and D are formed by swapping the tens and hundreds digits of A with the tens and hundreds digits of B. For example, if A = 2468 and B = 1357 then C = 2358 and D = 1467. What is the maximum possible value of C – D?
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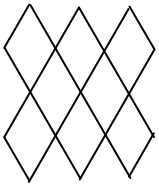
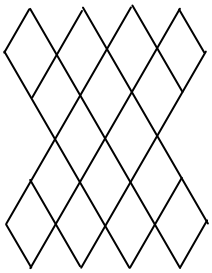
19	Figure 1  Figure 2  Figure 3  Figure 1 is made of six line segments. What is the minimum number of line segments needed to draw the 100^{th} figure in the sequence?
20	A pencil has the shape of a regular hexagonal prism before it is sharpened and not including the eraser. If the pencil is 8 inches long and one side of the hexagonal base is $\frac{1}{8}$ inch, what is the number of cubic inches in the volume of the hexagonal prism portion of the pencil? Answer as a common fraction in simplest radical form.
21	What is 472_{10} equivalent to in base 6?

22 Jeff and Sabina are playing "Guess my number". Jeff has secretly written a three-digit number in which all three digits are different. Sabina has made four guesses and Jeff has correctly indicated for each guess the number of correct digits and the number of correct digits in the correct place. What is Jeff's number?

Sabina's guess	Number of correct digits	Number of correct digits in the correct place
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854	2	2
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23 Perfect numbers are numbers that are equal to the sum of their proper factors. For example, 6 = 1 + 2 + 3. Even perfect numbers can be generated by the formula $(2^P - 1)(2^{P-1})$, when $2^P - 1$ is a prime number. What is the fourth largest even perfect number?

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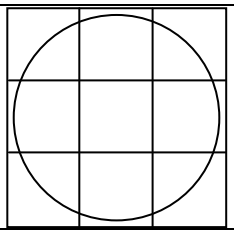
26 What is the smallest possible integer solution to the following inequality:
$-|x - 3| > -12$

27 Nine congruent squares are drawn as shown. A circle is drawn which contains the center point of the four corner squares. What is the ratio of the area of one of the nine squares to the area of the circle? Answer as a common fraction in terms of π.



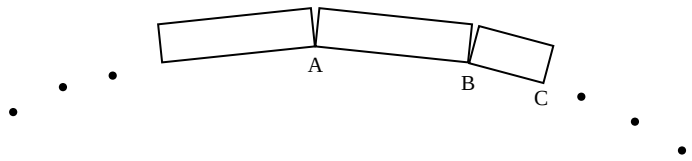
28 A fly walks only along the edges of a cube with volume 27 cubic inches. What is the number of inches in the shortest possible distance the fly must walk in order to have walked along every edge of the cube at least once?

29 There are several three digit palindromes. If they all were listed in order from least to greatest, what would be the median of the list?

30 The first term of a geometric sequence is 375 and the fourth term is 24. What is the third term of the sequence?

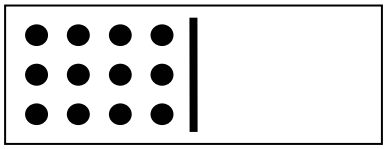
Challenge Questions

31 James is creating a border for his garden in the shape of a polygon with 36 sides. There are twenty-four 2′ by 6″ blocks and twelve 1′ by 6″ blocks. He lays the blocks nearly end to end, but rotates them slightly so that they just touch at a corner. As he lays the blocks in a clockwise direction around the garden he starts with two 2′ by 6″ blocks and then one 1′ by 6″ block. He then repeats this three-block pattern all the way around the garden. Everywhere two 2′ by 6″ blocks meet the angle formed by them is the same and everywhere one 2′ by 6″ block meets one 1′ by 6″ block, the angle formed is the same. What is the number of degrees in the angle formed by one 2′ by 6″ block and one 1′ by 6″ block? This is angle ABC in the drawing.



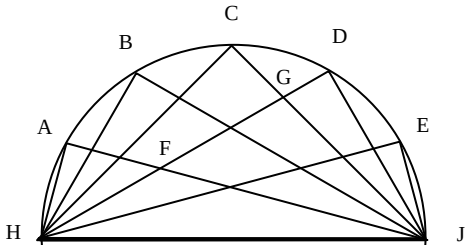
32 For how many positive three-digit integers can the product of their digits be expressed as a^4, where a is a positive integer?

33 In dominoes, each piece has two groups of dots showing, which represent numbers. Each of the two numbers can be any integer from 0 through 12 inclusive. Every integer is matched up exactly once with each of the integers 0 through 12 inclusive. An example is shown representing (12, 0). What fraction of the number of pieces in the set has exactly one seven showing?

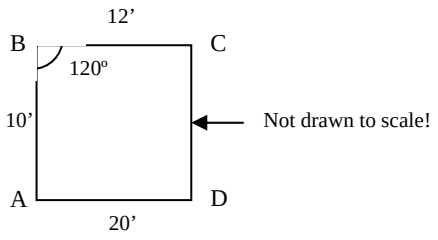
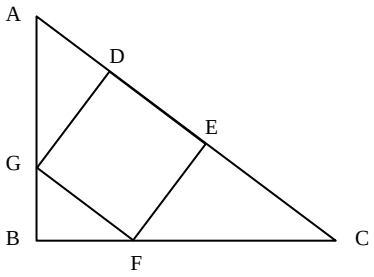


34 A cylindrical racquetball container has a radius of three inches and a height of $5\frac{1}{2}$ inches. If racquetballs have a radius of one inch, what is the maximum number of racquetballs that will fit in the container? Assume none of the racquetballs are compressed.

35 Points A, B, C, D and E are evenly spaced along the circumference of the semicircle shown. What is the ratio of angle CGD to angle AFD?



36 A set of six numbers has a range of 13 and a mean of 20. What is the largest possible value of the median? Answer as a mixed number.

37	ABCD is a trapezoid with side BC parallel to side AD. AD is 20 feet, AB is 10 feet and BC is 12 feet in length. What is the area of ABCD in feet squared? 
38	When a number N is divided by another number D, Q is the integer part of the quotient and R is the remainder, where R < D. If both D and N are randomly chosen 1-digit positive integers, not necessarily different, what is the probability that R > Q?
39	The singing group Craig and the Croatians has eight members: two high tenors, two second tenors, two baritones and two basses. A concert line-up must have at least one high tenor, one second-tenor, one baritone and one bass. How many possible concert line-ups are possible for this singing group?
40	Triangle ABC has side lengths 3 cm, 4 cm and 5 cm as shown. Quadrilateral DEFG is a square. What is the number of square centimeters in the area of DEFG? 

"Math is Cool" Masters – 2008-09

Algebra I – December 6, 2008

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:
KEY

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	2 [letters]		
2	1125		
3	15		
4	4		
5	243 [chirps]		
6	4/9		
7	6 [in]		
8	306		
9	$-2/\pi$		
10	5		
11	2 [choices]		
12	3 [colors]		
13	[x=] 16		
14	[$] 148.52		
15	5 [multiples]		
16	8999		
17	28 [squares]		
18	19 [items]		
19	402 [segments]		
20	$\frac{3\sqrt{3}}{16}$ [in^3]		

	Answer	1 or 0	1 or 0
21	$2104_{[6]}$		
22	850		
23	8128		
24	73/150		
25	December 23		
26	-8		
27	$\frac{1}{2\pi}$		
28	45 [in]		
29	550		
30	60		
31	171 [°]		
32	22 [integers]		
33	12/91		
34	20 [racquetballs]		
35	7/9		
36	$24\frac{1}{3}$		
37	$80\sqrt{3}$ [ft^2]		
38	5/9		
39	81 [line-ups]		
40	3600/1369 [cm^2]		

"Math is Cool" Masters – 2008-09

Algebra I – December 6, 2008

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
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	Answer	1 or 0	1 or 0
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"Math is Cool" Masters – 2008-09

Sponsored by: REC Silicon

Geometry – December 6, 2008

Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by <u>all</u> involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
  - *For problems dealing with money, a decimal answer should be given.*
  - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of π where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

INDIVIDUAL TEST - 35 minutes

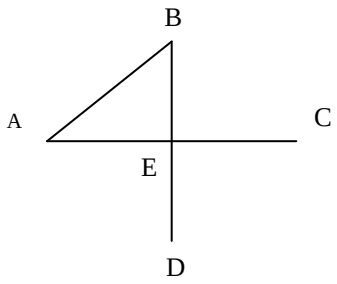
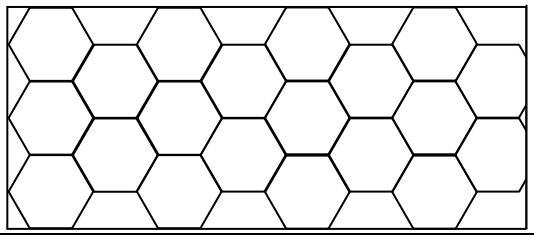
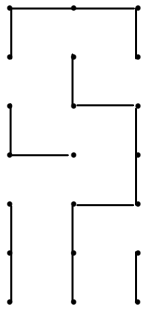
When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

"Math is Cool" Masters – 2008-09

Sponsored by: REC Silicon

Geometry – December 6, 2008

Individual Contest

1	1.215 is equivalent to the quotient of 12150 divided by 10^a. What is the value of a?
2	E is the midpoint of both BD and AC. AB = 6 inches. How many inches are in the length of CD? 
3	Evaluate: $\frac{2}{3}\left[5\left(\frac{6!}{8}\right) + 3\left(\frac{4!}{8}\right)\right]$
4	The state fish of Hawaii is the humuhumunukunukuapua'a. What is the ratio of vowels to consonants in this word?
5	The product of 3, 5, 7, 11, 13 and 37 is a six-digit number in the form aaa,aaa. What is the value of a?
6	A plane is covered with a tessellation of regular hexagons as shown. What is the minimum number of colors needed to color in each hexagon so that no two adjacent hexagons have the same color? 
7	The product of 6 and $\frac{5}{7}$ is a. The product of 5 and $\frac{6}{7}$ is b. What is the value of a – b?
8	How many multiples of 7 are also multiples of 13 from 1 to 500?
9	Penny has $203.76 in her checking account. She has to pay her phone bill, which includes a fixed monthly charge of $15 plus 8¢ per minute. If she has talked on her phone for 503 minutes this month, how much money, in dollars, will be left in her checking account after she pays this bill?
10	Andy and Betty are playing a game. The players take turns connecting any two horizontally or vertically adjacent dots by drawing a segment. If a player draws a segment that is the fourth side of a square, s/he may draw another segment. If a segment does not complete a square, the turn is over. Players score 1 point for each completed square. It is Andy's turn and the game grid is as shown. How many choices does Andy have of where to draw a segment that will not give Betty an opportunity to complete a square on her next turn? No segment can be drawn where one already exists. 

11 What is 472_{10} equivalent to in base 6?

12 The graph of y = sin x passes through the points $\left(\frac{\pi}{2},1\right)$ and $\left(\frac{3\pi}{2},-1\right)$. What is the slope of the line through these two points? Answer in terms of π.

13

Figure 1 Figure 2 Figure 3

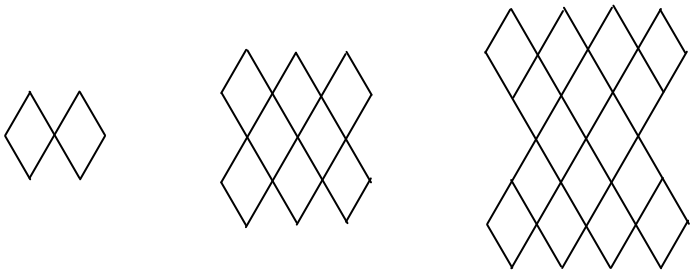


Figure 1 is made of six line segments. What is the minimum number of line segments needed to draw the 100th figure in the sequence?

14 A drawer has 5 blue socks, 11 green socks, 3 blue mittens and 7 green mittens all made of the same material. Lily cannot tell the difference between the feel of a sock or a mitten and the mittens look the same regardless of whether they are worn on left or right hands. Socks also look the same regardless of whether they are worn on the left or right foot. If she randomly pulls items from the drawer, how many items must she take in order to guarantee one matching pair of socks and one matching pair of mittens of the same color as the socks?

15 Audrey got her learner's permit for driving an automobile on Saturday, June 21. She can get her driver's license in as little as six months after she gets her learner's permit. In other words, she could get it on the 21st of the sixth month after June. If the Department of Licensing is the only place she can get her driver's license and they are closed on Sundays and Mondays, what is the earliest date that she can get her driver's license? Give the month and day.

16 Evaluate: $3^8 - 6^4$

17 A pencil has the shape of a regular hexagonal prism before it is sharpened and not including the eraser. If the pencil is 8 inches long and one side of the hexagonal base is $\frac{1}{8}$ inch, what is the number of cubic inches in the volume of the hexagonal prism portion of the pencil? Answer as a common fraction in simplest radical form.

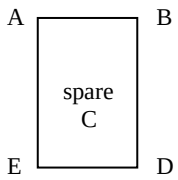
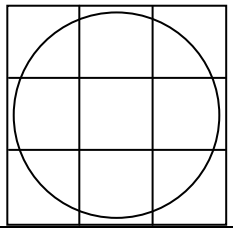
18 A circle has a radius of 5 cm. The circle is to be placed on a grid of 1 cm by 1 cm squares such that the center of the circle coincides with an intersection of four grid squares. Through how many grid squares does the circumference of the circle pass? (The circumference of the circle is not considered to be passing through a grid square when it shares only a single point with the perimeter of a grid square.)

19 Jeff and Sabina are playing "Guess my number". Jeff has secretly written a three-digit number in which all three digits are different. Sabina has made four guesses and Jeff has correctly indicated for each guess the number of correct digits and the number of correct digits in the correct place. What is Jeff's number?

Sabina's guess	Number of correct digits	Number of correct digits in the correct place
789	1	0
456	1	1
123	0	0
854	2	2
?		

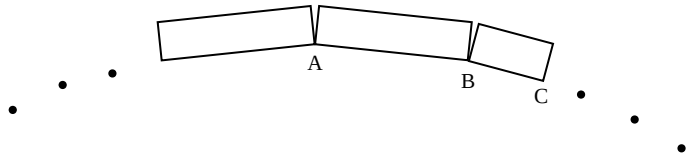
20 There are several three digit palindromes. If they all were listed in order from least to greatest, what would be the median of the list?

21 Let A and B be any 4-digit integers. Integers C and D are formed by swapping the tens and hundreds digits of A with the tens and hundreds digits of B. For example, if A = 2468 and B = 1357

	then C = 2358 and D = 1467. What is the maximum possible value of C – D?
22	A jar has eleven black marbles and fourteen white marbles. If Suzy pulls two marbles from the jar at the same time, what is the probability that they are the same color?
23	An isosceles triangle has two distinct angle measures. For isosceles ΔABC these two distinct angle measures add up to 108°. What is the number of degrees in the smaller of the two distinct angle measures?
24	Edna's car has four tires plus a spare. In an attempt to allow all five tires to wear out evenly, she rotates them according to the following scheme. Every 5,000 miles she rotates them clockwise so that the tire in position A goes to position B, the tire in position B goes to position C, etc. If the tires are meant to last 60,000 miles and she begins this rotation plan when the tires are new, how many miles will she be able to drive her car before she needs to replace the tires? 
25	Perfect numbers are numbers that are equal to the sum of their proper factors. For example, 6 = 1 + 2 + 3. Even perfect numbers can be generated by the formula $(2^P - 1)(2^{P-1})$, when $2^P - 1$ is a prime number. What is the fourth largest even perfect number?
26	A fly walks only along the edges of a cube with volume 27 cubic inches. What is the number of inches in the shortest possible distance the fly must walk in order to have walked along every edge of the cube at least once?
27	The first term of a geometric sequence is 375 and the fourth term is 24. What is the third term of the sequence?
28	What is the smallest possible integer solution to the following inequality: $-\|x - 3\| > -12$ 
29	Nine congruent squares are drawn as shown. A circle is drawn which contains the center point of the four corner squares. What is the ratio of the area of one of the nine squares to the area of the circle? Answer as a common fraction in terms of π.
30	For how many positive three-digit integers can the product of their digits be expressed as a^4, where a is a positive integer?

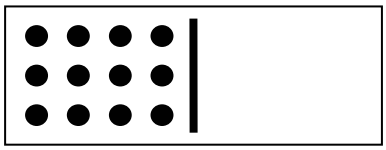
Challenge Questions

31 James is creating a border for his garden in the shape of a polygon with 36 sides. There are twenty-four 2′ by 6″ blocks and twelve 1′ by 6″ blocks. He lays the blocks nearly end to end, but rotates them slightly so that they just touch at a corner. As he lays the blocks in a clockwise direction around the garden he starts with two 2′ by 6″ blocks and then one 1′ by 6″ block. He then repeats this three-block pattern all the way around the garden. Everywhere two 2′ by 6″ blocks meet the angle formed by them is the same and everywhere one 2′ by 6″ block meets one 1′ by 6″ block, the angle formed is the same. What is the number of degrees in the angle formed by one 2′ by 6″ block and one 1′ by 6″ block? This is angle ABC in the drawing.



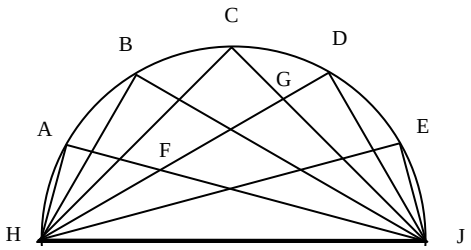
32 Three couples are to be seated at a round table at a party. If each of the six people must sit next to his/her partner, in how many distinct ways can they be seated at the table? Seating arrangements that are simply rotations of other arrangements are not considered distinct.

33 In dominoes, each piece has two groups of dots showing, which represent numbers. Each of the two numbers can be any integer from 0 through 12 inclusive. Every integer is matched up exactly once with each of the integers 0 through 12 inclusive. An example is shown representing (12, 0). What fraction of the number of pieces in the set has exactly one seven showing?

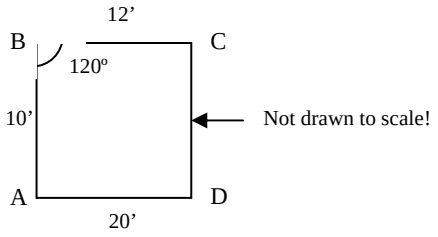
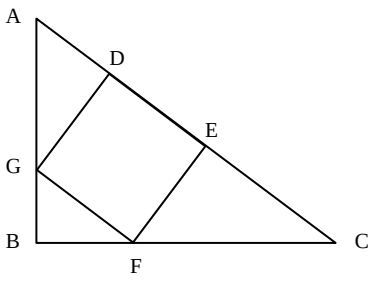


34 A cylindrical racquetball container has a radius of three inches and a height of $5\frac{1}{2}$ inches. If racquetballs have a radius of one inch, what is the maximum number of racquetballs that will fit in the container? Assume none of the racquetballs are compressed.

35 Points A, B, C, D and E are evenly spaced along the circumference of the semicircle shown. What is the ratio of angle CGD to angle AFD?



36 A set of six numbers has a range of 13 and a mean of 20. What is the largest possible value of the median? Answer as a mixed number.

37	ABCD is a trapezoid with side BC parallel to side AD. AD is 20 feet, AB is 10 feet and BC is 12 feet in length. What is the area of ABCD in feet squared? 
38	When a number N is divided by another number D, Q is the integer part of the quotient and R is the remainder, where R < D. If both D and N are randomly chosen 1-digit positive integers, not necessarily different, what is the probability that R > Q?
39	The singing group Craig and the Croatians has eight members: two high tenors, two second tenors, two baritones and two basses. A concert line-up must have at least one high tenor, one second-tenor, one baritone and one bass. How many possible concert line-ups are possible for this singing group?
40	Triangle ABC has side lengths 3 cm, 4 cm and 5 cm as shown. Quadrilateral DEFG is a square. What is the number of square centimeters in the area of DEFG? 

"Math is Cool" Masters – 2008-09

Geometry – December 6, 2008

Final Score: KEY

First Score

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	4		
2	6 [in]		
3	306		
4	4/3		
5	5		
6	3 [colors]		
7	0		
8	5 [multiples]		
9	[$] 148.52		
10	2 [choices]		
11	$2104_{[6]}$		
12	$-2/\pi$		
13	402 [segments]		
14	19 [items]		
15	December 23		
16	5265		
17	$\frac{3\sqrt{3}}{16}$ [in^3]		
18	28 [squares]		
19	850		
20	550		

	Answer	1 or 0	1 or 0
21	8999		
22	73/150		
23	36 [°]		
24	75,000 [miles]		
25	8128		
26	45 [in]		
27	60		
28	-8		
29	$\frac{1}{2\pi}$		
30	22 [integers]		
31	171 [°]		
32	16 [ways]		
33	12/91		
34	20 [racquetballs]		
35	7/9		
36	$24\frac{1}{3}$		
37	$80\sqrt{3}$ [ft^2]		
38	5/9		
39	81 [line-ups]		
40	3600/1369 [cm^2]		

"Math is Cool" Masters – 2008-09

Geometry – December 6, 2008

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1			
2			
3			
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	Answer	1 or 0	1 or 0
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"Math is Cool" Masters – 2008-09

Sponsored by: Western Polymer Corporation

Algebra II – December 6, 2008

Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by <u>all</u> involved. Bad sportsmanship may result in disqualification.*
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  - *For problems dealing with money, a decimal answer should be given.*
  - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of π where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

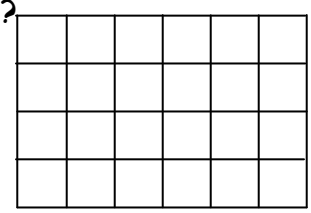
"Math is Cool" Masters – 2008-09

Sponsored by: Western Polymer Corporation

Algebra II – December 6, 2008

Individual Contest

1	Evaluate: $7419 + 6085$
2	Evaluate: $26\overline{)3042}$
3	What percent of 240 is 36?
4	Evaluate: 6^4
5	What is the difference between the square of 25 and the square of 20?
6	What value(s) of a satisfy $4a + 9 = 93$?
7	What value(s) of b satisfy $b^2 + 3b - 8 = 0$?
8	What are the coordinates, in the form (x, y), of the point of intersection of the lines $2x - y = -8$ and $x + 3y = 3$?
9	How many liters of water (0% acid) should be added to twelve liters of a 40% acid solution to create a 30% acid solution?
10	What is the equation, in slope-intercept form, of the line through the points $(1,4)$ and $(3,-2)$?
11	What is the prime factorization, in exponential form, of 135?
12	What are the coordinates, in the form (x, y), of the x-intercept(s) of the parabola $y = 2x^2 - 11x + 12$?
13	What is the median of the data set 2, 2, 2, 2, 3, 3, 4, 4, 4, 5, 7, 7, 8, 8, 8, 8, 9?
14	What is the measure, in degrees, of the smaller angle between the hands of a standard 12-hour analog clock at 11:40 PM?
15	If Tom is 35 and Katie is 3, how old, in years, will Tom be when his age is three times Katie's age?
16	What is the perimeter, in centimeters, of a right triangle with legs measuring 6 cm and 8 cm?
17	How many integers are possible for the length, in centimeters, of the third side of a triangle with two sides measuring 14 cm and 31 cm?
18	What is the area, in square centimeters, of a rectangle with a length of 22 cm and a perimeter of 57 cm?

19	What is the volume, in cubic centimeters, of a right circular cone with a height of 8 cm and a base radius of 6 cm?
20	What is the surface area, in square centimeters, of a right rectangular prism with dimensions of 9, 10, and 12 cm?
21	What is the least common multiple of 210 and 165?
22	What is the area, in square centimeters, of a triangle with sides measuring 6, 8, and 10 cm?
23	What is the first term of an arithmetic sequence with a tenth term of 100 and a fifteenth term of 80?
24	How many squares appear in the array of unit squares shown? 
25	What is the sum of the 19 smallest even numbers?
26	Evaluate: $\frac{(4-5i)(-1+2i)}{2+i}$
27	A bag contains four red marbles, eight white marbles, and nine blue marbles. What is the probability that a marble drawn from this bag is not blue?
28	A round dartboard has a radius of 20 cm and a bullseye with a diameter of 2 cm. What is the probability that a dart that hits the dartboard hits the bullseye?
29	If I invest one-thousand dollars in an account receiving eight percent annual interest compounded quarterly, how much money will be in the account after one year? Express your answer in dollars rounded to the nearest hundredth (cent).
30	A bus driver leaves the bus barn and picks up four passengers at the first stop. At the next stop, two passengers get off and five get on. At the third stop, three passengers get on and one gets off. How many people are on the bus after the third stop?

Challenge Questions

31	Convert the base ten number 395_{10} to a base six number.
32	What is the shortest distance from the point $(-1,6)$ to the line $3x-4y=13$?
33	A group of friends pools their money to buy a car. If there had been one fewer friend, each friend would have spent \$72 more than they did. If there had been two more friends, each friend would have spent \$72 less than they did. How many dollars did the car cost?
34	What is the radius, in centimeters, of circle O if the tangent line segment from point A measures 16 cm and the secant line segment from point A through O measures 32 cm?
35	How many positive five-digit integers are palindromes?
36	In the cryptarithm shown, where each instance of a letter represents the same digit (0-9) and no two different letters represent the same digit, what is the largest possible value of the four-digit number ABCD? $\begin{array}{r} AB \\ \times\, C \\ \hline CD \end{array}$
37	What is the next term of the sequence shown? 13, 27, 16, 33, 22, 45, 34, …
38	How many ways can seven people be seated relative to one another at a round table?
39	If $c(d)=4d-7$, evaluate $c^{-1}(17)$.
40	Abby, Sam, and Irina own a cat, a dog, and a hamster that are black, brown, and white (not necessarily in those orders). The cat is not white, nor is it Sam's. Irina does not own a hamster or a black pet. The brown pet is not Sam's, nor is it a dog. Abby does not own a cat, nor does she own a white animal. Who owns the cat, and what color is it?

"Math is Cool" Masters – 2008-09

Algebra II – December 6, 2008

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

KEY

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	13,504		
2	117		
3	15 [%]		
4	1296		
5	225		
6	21		
7	$\frac{-3 \pm \sqrt{41}}{2}$		
8	$(-3,2)$		
9	4 [L]		
10	$y = -3x + 7$		
11	$3^3 \cdot 5$		
12	(3/2,0) & $(4,0)$		
13	4		
14	110 [°]		
15	48 [years old]		
16	24 [cm]		
17	27 [cm]		
18	143 [cm^2]		
19	96π [cm^3]		
20	636 [cm^2]		

	Answer	1 or 0	1 or 0
21	2310		
22	24 [cm^2]		
23	136		
24	50 [squares]		
25	380		
26	$5 + 4i$		
27	4/7		
28	1/400		
29	[$] 1082.43		
30	10 [people]		
31	$1455_{[6]}$		
32	8		
33	[$] 864 [.00]		
34	12 [cm]		
35	900 [integers]		
36	1428		
37	69		
38	720 [ways]		
39	6		
40	Irina, Brown		

"Math is Cool" Masters – 2008-09

Algebra II – December 6, 2008

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1			
2			
3			
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	Answer	1 or 0	1 or 0
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"Math is Cool" Masters – 2008-09

Sponsored by: Basic American Foods

PreCalculus – December 6, 2008

Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by all involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
  - *For problems dealing with money, a decimal answer should be given.*
  - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of π where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

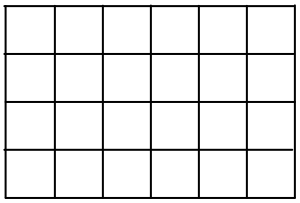
"Math is Cool" Masters – 2008-09

Sponsored by: Basic American Foods

PreCalculus – December 6, 2008

Individual Contest

1	How many ways can seven people be seated relative to one another at a round table?
2	Evaluate: $26\overline{)3042}$
3	What percent of 240 is 36?
4	Evaluate: 6^4
5	What is the difference between the square of 25 and the square of 20?
6	A bag contains four red marbles, eight white marbles, and nine blue marbles. What is the probability that a marble drawn from this bag is not blue?
7	What value(s) of b satisfy $b^2 + 3b - 8 = 0$?
8	What are the coordinates, in the form (x, y), of the point of intersection of the lines $2x - y = -8$ and $x + 3y = 3$?
9	How many liters of water (0% acid) should be added to twelve liters of a 40% acid solution to create a 30% acid solution?
10	What is the sum of the positive integer factors of 1200?
11	What is the area of the ellipse with equation $9x^2 + 4y^2 - 54x + 8y + 49 = 0$?
12	What are the coordinates, in the form (x, y), of the x-intercept(s) of the parabola $y = 2x^2 - 11x + 12$?
13	A round dartboard has a radius of 20 cm and a bullseye with a diameter of 2 cm. What is the probability that a dart that hits the dartboard hits the bullseye?
14	What is the measure, in degrees, of the smaller angle between the hands of a standard 12-hour analog clock at 11:40 PM?
15	If Tom is 35 and Katie is 3, how old, in years, will Tom be when his age is three times Katie's age?
16	A bus driver leaves the bus barn and picks up four passengers at the first stop. At the next stop, two passengers get off and five get on. At the third stop, three passengers get on and one gets off. How many people are on the bus after the third stop?
17	How many integers are possible for the length, in centimeters, of the third side of a triangle with two sides measuring 14 cm and 31 cm?

18	If $c(d) = 4d - 7$, evaluate $c^{-1}(17)$.
19	What is the volume, in cubic centimeters, of a right circular cone with a height of 8 cm and a base radius of 6 cm?
20	What is the surface area, in square centimeters, of a right rectangular prism with dimensions of 9, 10, and 12 cm?
21	What is the radius, in centimeters, of circle O if the tangent line segment from point A measures 16 cm and the secant line segment from point A through O measures 32 cm?
22	What is the area, in square centimeters, of a triangle with sides measuring 6, 8, and 10 cm?
23	Convert the base ten number 395_{10} to a base six number.
24	How many squares appear in the array of unit squares shown? 
25	How many positive five-digit integers are palindromes?
26	Evaluate: $\dfrac{(4-5i)(-1+2i)}{2+i}$
27	What is the prime factorization, in exponential form, of 135?
28	An ant is on an edge of a right rectangular prism one centimeter from a vertex. If the prism has edges measuring 9, 10, and 12 cm, what is the smallest number of centimeters the ant can crawl to reach a point one centimeter from the opposite (furthest) corner?
29	If I invest one-thousand dollars in an account receiving eight percent annual interest compounded quarterly, how much money will be in the account after one year? Express your answer in dollars rounded to the nearest hundredth (cent).
30	In a new game of chance, players pay \$40 to roll a single die. If the player rolls a 1, 2, 3, 4, or 5, they "win" \$1, \$2, \$3, \$4, or \$5, respectively. What should the payoff be for a roll of 6 if the casino wants to keep an average of 10% of the money players pay?

Challenge Questions

31	Cherie has two rectangular rugs, one of which measures 2x3 m and the other measures 1x4 m. What is the area, in square meters, of the smallest circular rug she can have made that can have both of these rugs lying on top of it without them overlapping?
32	What is the shortest distance from the point $(-1,6)$ to the line $3x-4y=13$?
33	Abby, Sam, and Irina own a cat, a dog, and a hamster that are black, brown, and white (not necessarily in those orders). The cat is not white, nor is it Sam's. Irina does not own a hamster or a black pet. The brown pet is not Sam's, nor is it a dog. Abby does not own a cat, nor does she own a white animal. Who owns the cat, and what color is it?
34	In a $45^\circ-60^\circ-75^\circ$ triangle, the shortest side measures 12 cm. What is the length, in centimeters, of the next shorter side?
35	When each vertex of an icosahedron is cut off, a truncated icosahedron is formed with faces that are each either a regular pentagon or a regular hexagon. Each pentagon is surrounded by hexagons, while each hexagon borders exactly three pentagons. How many edges does this solid have?
36	In the cryptarithm shown, where each instance of a letter represents the same digit (0-9) and no two different letters represent the same digit, what is the largest possible value of the four-digit number ABCD? $\begin{array}{r} AB \\ \times\, C \\ \hline CD \end{array}$
37	What is the next term of the sequence shown? 13, 27, 16, 33, 22, 45, 34, …
38	A friend you trust draws two cards from a standard 52-card deck and tells you that they did not draw a pair. What is the probability that the two cards are from the same suit?
39	A group of friends pools their money to buy a car. If there had been one fewer friend, each friend would have spent \$72 more than they did. If there had been two more friends, each friend would have spent \$72 less than they did. How many dollars did the car cost?
40	What is the largest real value of f satisfying the equation $5f^2+6fg+3g^2=48$ with a real value of g?

"Math is Cool" Masters – 2008-09

PreCalculus – December 6, 2008

Final Score:

KEY

First Score

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	720 [ways]		
2	117		
3	15 [%]		
4	1296		
5	225		
6	4/7		
7	$\frac{-3 \pm \sqrt{41}}{2}$		
8	$(-3,2)$		
9	4 [L]		
10	3844		
11	6π [un^2]		
12	(3/2,0) & $(4,0)$		
13	1/400		
14	110 [$^\circ$]		
15	48 [years old]		
16	10 [people]		
17	27 [cm]		
18	6		
19	96π [cm^3]		
20	636 [cm^2]		

	Answer	1 or 0	1 or 0
21	12 [cm]		
22	24 [cm^2]		
23	$1455_{[6]}$		
24	50 [squares]		
25	900 [integers]		
26	$5+4i$		
27	$3^3 \cdot 5$		
28	$\sqrt{433}$ [cm]		
29	[$] 1082.43		
30	[$]201[.00]		
31	$\frac{3145\pi}{576}$ [m^2]		
32	8		
33	Irina, Brown		
34	$6\sqrt{6}$ [cm]		
35	90 [edges]		
36	1428		
37	69		
38	1/4		
39	[$] 864 [.00]		
40	$2\sqrt{6}$		

"Math is Cool" Masters – 2008-09

PreCalculus – December 6, 2008

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1			
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	Answer	1 or 0	1 or 0
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"Math is Cool" Masters – 2008-09

Sponsored by: Basic American Foods

Calculus – December 6, 2008

Individual Contest

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INDIVIDUAL TEST - 35 minutes

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"Math is Cool" Masters – 2008-09

Sponsored by: Basic American Foods

Calculus – December 6, 2008

Individual Contest

1	How many ways can seven people be seated relative to one another at a round table?
2	Evaluate: $26\overline{)3042}$
3	What percent of 240 is 36?
4	Evaluate: $\begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$
5	What is the difference between the square of 25 and the square of 20?
6	What is the sum of the positive integer factors of 1200?
7	Evaluate the dot product: $\langle 3,5\rangle \cdot \langle -2,1\rangle$
8	What are the coordinates, in the form (x,y), of the point of intersection of the lines $2x - y = -8$ and $x + 3y = 3$?
9	How many liters of water (0% acid) should be added to twelve liters of a 40% acid solution to create a 30% acid solution?
10	In a $45° - 60° - 75°$ triangle, the shortest side measures 12 cm. What is the length, in centimeters, of the next shorter side?
11	Evaluate: $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}$ Express answer in reduced exponential form.
12	What are the coordinates, in the form (x,y), of the x-intercept(s) of the parabola $y = 2x^2 - 11x + 12$?
13	A round dartboard has a radius of 20 cm and a bullseye with a diameter of 2 cm. What is the probability that a dart that hits the dartboard hits the bullseye?
14	What is the measure, in degrees, of the smaller angle between the hands of a standard 12-hour analog clock at 11:40 PM?
15	If Tom is 35 and Katie is 3, how old, in years, will Tom be when his age is three times Katie's age?
16	Set J is the set of all positive multiples of three less than 100. Set K is the set of all positive multiples of six less than 80. How many elements are in the set $J \cup K$?
17	How many integers are possible for the length, in centimeters, of the third side of a triangle with two sides measuring 14 cm and 31 cm?

18	If $c(d) = 4d - 7$, evaluate $c^{-1}(17)$.
19	What is the volume, in cubic centimeters, of a right circular cone with a height of 8 cm and a base radius of 6 cm?
20	What is the third term of a harmonic sequence with a first term of 8 and a second term of 4?
21	What is the radius, in centimeters, of circle O if the tangent line segment from point A measures 16 cm and the secant line segment from point A through O measures 32 cm?
22	What is the area, in square centimeters, of a triangle with sides measuring 6, 8, and 10 cm?
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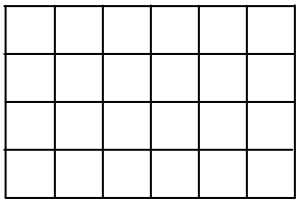
24	How many squares appear in the array of unit squares shown? 
25	How many positive five-digit integers are palindromes?
26	Evaluate: $\dfrac{(4-5i)(-1+2i)}{2+i}$
27	What is the area of the ellipse with equation $9x^2 + 4y^2 - 54x + 8y + 49 = 0$?
28	An ant is on an edge of a right rectangular prism one centimeter from a vertex. If the prism has edges measuring 9, 10, and 12 cm, what is the smallest number of centimeters the ant can crawl to reach a point one centimeter from the opposite (furthest) corner?
29	If I invest one-thousand dollars in an account receiving eight percent annual interest compounded quarterly, how much money will be in the account after one year? Express your answer in dollars rounded to the nearest hundredth (cent).
30	In a new game of chance, players pay \$40 to roll a single die. If the player rolls a 1, 2, 3, 4, or 5, they "win" \$1, \$2, \$3, \$4, or \$5, respectively. What should the payoff be for a roll of 6 if the casino wants to keep an average of 10% of the money players pay?

Challenge Questions

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36	In the cryptarithm shown, where each instance of a letter represents the same digit (0-9) and no two different letters represent the same digit, what is the largest possible value of the four-digit number ABCD? $\begin{array}{r} AB \\ \times\, C \\ \hline CD \end{array}$
37	What is the next term of the sequence shown? 13, 27, 16, 33, 22, 45, 34, …
38	Evaluate: $\sum_{h=1}^{8}\left(\frac{4}{h}-\frac{4}{h+1}\right)$
39	A group of friends pools their money to buy a car. If there had been one fewer friend, each friend would have spent \$72 more than they did. If there had been two more friends, each friend would have spent \$72 less than they did. How many dollars did the car cost?
40	What is the largest real value of f satisfying the equation $5f^2+6fg+3g^2=48$ with a real value of g?

"Math is Cool" Masters – 2008-09

Calculus – December 6, 2008

Final Score:

KEY

First Score

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	720 [ways]		
2	117		
3	15 [%]		
4	$\begin{bmatrix} 4 & 5 \\ -8 & 2 \end{bmatrix}$		
5	225		
6	3844		
7	-1		
8	$(-3,2)$		
9	4 [L]		
10	$6\sqrt{6}$ [cm]		
11	$2^{\frac{31}{32}}$		
12	(3/2,0) & $(4,0)$		
13	1/400		
14	110 [°]		
15	48 [years old]		
16	33 [elements]		
17	27 [cm]		
18	6		
19	96π [cm^3]		
20	8/3		

	Answer	1 or 0	1 or 0
21	12 [cm]		
22	24 [cm^2]		
23	$1455_{[6]}$		
24	50 [squares]		
25	900 [integers]		
26	$5+4i$		
27	6π [un^2]		
28	$\sqrt{433}$ [cm]		
29	[$] 1082.43		
30	[$]201[.00]		
31	$\frac{3145\pi}{576}$ [m^2]		
32	8		
33	Irina, Brown		
34	1/4		
35	90 [edges]		
36	1428		
37	69		
38	32/9		
39	[$] 864 [.00]		
40	$2\sqrt{6}$		

"Math is Cool" Masters – 2008-09

Calculus – December 6, 2008

Final Score:

First Score

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Proctor Name_____Room #_____

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
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	Answer	1 or 0	1 or 0
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