"Math is Cool" Masters – 2008-09

Sponsored by: Basic American Foods
11th & 12th Grade – December 6, 2008
Individual Multiple Choice Contest

1	If the pattern below is extended to the fifth row, what number would fit for x?

$$1^2 + 2^2 + 2^2 = 9 = 3^2$$
$$2^2 + 3^2 + 6^2 = 49 = 7^2$$
$$3^2 + 4^2 + 12^2 = 169 = 13^2$$
$$4^2 + 5^2 + 20^2 = \underline{\ \ } = \underline{\ \ }$$
$$\underline{\ \ } + \underline{\ \ } + \underline{\ \ } = \underline{\ \ } = x^2$$

A) 29   B) 31   C) 33   D) 34   E) 35

2	How many isosceles triangles with integer side lengths and a perimeter of 2008 are there?

A) 167   B) 501   C) 667   D) 672   E) 834

3	What positive value of x makes this proportion true? $\dfrac{169}{x} = \dfrac{x}{289}$

A) 209   B) 214   C) 221   D) $100\sqrt{5}$ E) 225

4	Evaluate: $5\dfrac{2}{15} \times 6\dfrac{6}{7}$

A) $31\dfrac{17}{21}$ B) $33\dfrac{3}{5}$ C) $34\dfrac{3}{7}$ D) $34\dfrac{1}{21}$ E) $35\dfrac{1}{5}$

5	What is the probability that a family of 7 children has at least 2 girls? Assume that the probability of a child being a girl is equal to the probability of it being a boy, and that there are no twins, triplets, etc.

A) $\dfrac{121}{128}$ B) $\dfrac{15}{16}$ C) $\dfrac{119}{128}$ D) $\dfrac{61}{64}$ E) $\dfrac{117}{128}$

6	What is the sum of all values of L satisfying $2^{3L} - 3 \cdot 2^{2L} - 10 \cdot 2^{L} + 24 = 0$?

A) $-\dfrac{1}{4}$ B) $\dfrac{1}{2}$ C) $\dfrac{5}{2}$ D) 3   E) $\dfrac{15}{4}$

7
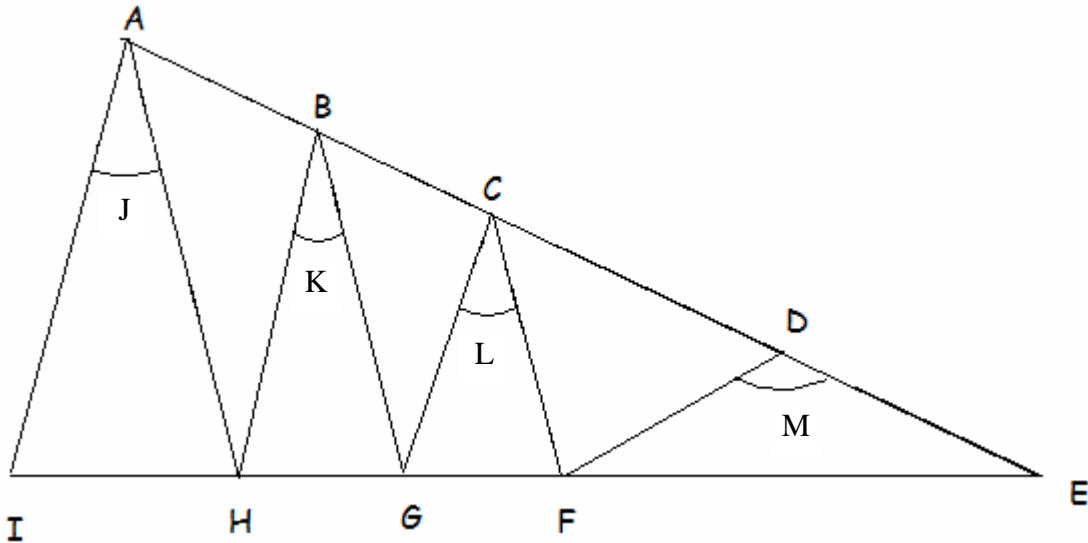
AE = EI, AI = AH = BH = BG = CG = CF = DF = DE. What is J + K + L + M in degrees?

A) 3690   B) 336   C) 410   D) 225   E) The answer not given.

8 What is the sum of the first 100 triangular numbers?

A) 5050   B) 10100   C) 171700   D) 338350   E) The answer not given.

9 In the game of "Dynamus", a person draws a card out of a standard 52 card deck and rolls a fair six-sided die. Face cards count as 10 and aces count as 1. If the number on the die is greater than the number on the card, the player gains $1.00. If the number on the card is greater than the number on the die, the player loses $0.10. If the numbers are equal, the player neither gains nor loses. If Dillon plays this game 100 times, what is his expected gain or loss? Gains are positive, losses are negative.

A) $-\dfrac{141}{78}$ B) -1   C) $\dfrac{5}{6}$ D) $\dfrac{155}{13}$ E) $\dfrac{2913}{156}$

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Team Contest

1	Write the letters A-F in order of ascending value. $A = 138 \times 142$ $\qquad$ $B = 9^2 \cdot 15^2$ $\qquad$ $C = 20,000$ $\qquad$ $D = 145 \times 135$ $E = 16^2 + 12^2$ $\qquad$ $F = 17^2 \cdot 7^2$
2	In a triangle with sides measuring 9, 11, and 12 cm, what is the length, in centimeters, of the shortest altitude?
3	Find the sum of all values of θ between 0 and 2π inclusive that satisfy $4\cos^2(2\theta) = 3$.
4	What is the largest value of n for which $\binom{n}{n-1} + \binom{n}{2n-10} = \binom{n+1}{2n-7}$?
5	How many four-digit positive integers have four distinct digits arranged so that of the three pairs of adjacent digits, exactly one pair has a right digit that is less than the left digit?
6	At Crusty Crumbs, patrons can have donuts made to order with their choice of toppings. They offer: $\qquad$ rainbow and chocolate sprinkles; $\qquad$ sugar, maple, and chocolate glazes; $\qquad$ cream and raspberry fillings; $\qquad$ raisins; $\qquad$ and chopped nuts, but you can have at most one glaze and at most one filling per donut. How many different donuts can be made?
7	In the game of Finnbiz, two players take turns rolling a fair six-sided die. The first player cannot win on his first turn. On all subsequent turns, if the rolling player rolls higher than the previous roll, he wins, otherwise the other player takes a turn. What is the probability that the first player wins on his second turn?
8	What is the sum of the terms of an infinite geometric sequence with a fifth term of 128 and a common ratio of 1/3 ?
9	A set of seven integers from 0 to 100 inclusive has a unique mode of 52, a median of 49, and a mean of 53. What is the maximum possible value of the range?
10	Use the digits 9, 8, 6, and 4 exactly once each and parentheses and the operations +, -, $\times$, and $\div$ as much or as little as desired to create an expression which evaluates to 28.

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Pressure Round Contest

1	What value(s) of z satisfy the following? $3(z-4)+2(4z+1)-(4z+8)-3(z-4)=5(2z-6)-4(z+2)+(8-3z)+2(4z+1)$
2	Express in simplest radical form: $\sqrt{33075}$
3	A triangle with sides measuring 3, 4, and 5 cm is inscribed in the smallest square possible. What is the area, in square centimeters, of the square?
4	A triangle with sides measuring 8, 10, and x cm has a $60°$ angle opposite the side of length x. What is the value of x?
5	The number $1A345BA8$ is divisible by 24. What is the largest possible sum of A and B?

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Mental Math Contest

PERSON 1		
1.1	What is the sum of the first eleven positive even numbers?	132
1.2	What is the surface area, in square centimeters, of a right circular cylinder with a diameter of eight centimeters and a height of three-over-pi centimeters?	24+32pi [cm²]
1.3	When the special number is tripled and the result is reduced by fourteen, the final answer is seventy. What is the special number?	28
1.4	What are the roots of the quadratic equation X-squared [PAUSE] minus five X [PAUSE] plus four [PAUSE] equals zero?	[x=] 1, 4 [any order]
PERSON 2		
2.1	What is the measure, in degrees, of each interior angle of a regular eighteen-gon?	160 [°]
2.2	What is the product of forty-eight and sixty-seven?	3216
2.3	A tank has three piranhas and seven stingrays. If I randomly choose two of these to eat, what is the probability that I pick one of each?	7/15
2.4	What is the sum of the nine smallest prime numbers?	100
PERSON 3		
3.1	What is the sum of the terms of an infinite geometric sequence with a first term of seven and a constant ratio of five over twenty-two?	154/17
3.2	In Guitar Hero Three, Jeremy just scored nearly three million points on "Hit Me With Your Best Shot". If he achieved this score by hitting ninety percent of the notes and there are nine-hundred ten notes, how many notes did he hit?	819 [notes]
3.3	What is the perimeter, in centimeters, of a rectangle with a dimension of fifteen centimeters and an area of forty-five centimeters?	36 [cm]
3.4	What is the sum of eighteen squared plus seven cubed?	667
PERSON 4		
4.1	A right rectangular prism has side lengths of seven, eight and nine units. Find the length of one of its space diagonals.	$\sqrt{194}$ [units]
4.2	When two fair six-sided dice are rolled, what is the probability that the product of the numbers shown is four?	1/12
4.3	What is the perimeter of a square with an area of thirty-six square centimeters?	24 [cm]
4.4	What is five factorial plus four factorial?	144

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COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	What is the log in base three of seven-hundred twenty-nine plus the log in base seven of three-hundred forty three?	9
2	What is the value of the cosine of fifteen degrees?	$\dfrac{\sqrt{6}+\sqrt{2}}{4}$
3	A corral contains both horses and humans. If there are forty-three heads and one-hundred twenty-eight feet, how many humans are in the corral?	22 [humans]
4	Three circles of radius twelve centimeters are mutually externally tangent, and a fourth circle is circumscribed around them, tangent to all three. What is the diameter, in centimeters, of the fourth circle?	$24+16\sqrt{3}$ [cm]
5	What is the secant of the smallest angle in a right triangle with legs measuring six and nine centimeters?	$\dfrac{\sqrt{13}}{3}$
6	T is inversely proportional to S, but proportional to the cube of R. If T is twenty when S is ten and R is thirty, what is T when S is five and R is fifteen?	5
7	What is the probability that when six fair coins are flipped at least two of them show tails?	57/64
	## Extra Problem – Only if Needed	
8	How many rectangles of any shape or size are formed by the gridlines in a three by four array of unit squares?	60 [rectangles]

COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	What is the smallest perfect number greater than one-hundred?	496
2	In the dataset three, five, eleven, thirteen, thirteen, what is the sum of the mean, mode, and median?	33
3	What is the sine of eight pi over three?	$\dfrac{\sqrt{3}}{2}$
4	How many integers are in the domain of the real-valued function Q of P equals the square root of the quantity [PAUSE] one-hundred twenty-five minus P squared?	23 [integers]
5	When the digits of a positive two-digit number are reversed, the result is sixteen less than three times the original number. What was the original number?	26
6	Convert the base six number three-four-five to a base ten number.	$137_{[10]}$
7	What is the volume, in cubic centimeters, of a right circular cylinder with a base radius of four centimeters whose surface area is equal to that of a sphere with a radius of eight centimeters?	448π [cm^3]
	Extra Problem – Only if Needed	
8	What is the sum of X and Y in the sequence fourteen, nineteen, X, thirty-two, forty, forty-nine, Y?	84

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COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	What is the area of the triangle with vertices at the points two COMMA three, four COMMA five, and six COMMA negative one?	8
2	What is the radius of the circle with equation X squared minus eight X plus Y squared plus four Y equals twenty-five?	$3\sqrt{5}$
3	What is the distance between the vertices of the hyperbola with equation thirty-six X-squared minus nine Y-squared equals one?	1/3
4	What is the smallest integer value of *n* such that a regular N-gon has more diagonals than it has degrees in one of its interior angles?	20
5	If the quantity M plus nine over the quantity M minus three is equal to the quantity M plus four over the quantity M plus two, what is the value of M?	-3
6	Evaluate the quantity negative one [PAUSE] to the negative third power [PAUSE] plus the quantity negative five [PAUSE] to the negative second power.	-24/25
7	What is the area, in square centimeters, of an equilateral triangle with a side measuring ten centimeters?	$25\sqrt{3}$ [cm^2]
	## Extra Problem – Only if Needed	
8	If Erik can build a wall in fifteen days and Tom can build a wall in sixty days, how many days would it take the two of them working together?	12 [days]

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School Name_____Team #_____

Proctor Name_____Room #_____

First Score

(out of 18)

STUDENT NAME_____

INDIVIDUAL MULTIPLE CHOICE - 15 minutes
This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. **Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet.** *No talking during the test.*

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	B		
2	B		
3	C		
4	E		
5	B		
6	D		
7	B		
8	C		
9	D		

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School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

First Score

(out of 20)

Team Contest – Score Sheet

TEAM TEST - 15 minutes

*When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as **2 or 0**. Record all answers on the colored answer sheet.*

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	E F B D A C [in that order]		
2	$\frac{4\sqrt{35}}{3}$ [cm]		
3	8π		
4	6		
5	1638		
6	192		
7	35/108		
8	15,552		
9	77		
10	$4 \times (9 + 6 - 8)$ [and perhaps others = 28]		

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School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

PRESSURE ROUND - 10 minutes

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

Pressure Round Answers

	Answer
1	22/7
2	$105\sqrt{3}$
3	256/17 [cm^2]
4	$2\sqrt{21}$ [cm^2]
5	16