"Math is Cool" Masters – 2009-10

Sponsored by: Basic American Foods & EKA Chemicals

December 5, 2009

Geometry, Algebra 2, PreCalculus, Calculus - Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by all involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
  - *For problems dealing with money, a decimal answer should be given.*
  - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of π where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will also be scored as a 0.*

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

"Math is Cool" Masters – 2009-10

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Geometry, Algebra 2, PreCalculus, Calculus – December 5, 2009

Individual Contest

1	Evaluate: 987+789
2	What digit is in the thousandths place of the number 1,234.56789?
3	What value(s) of a satisfy $9a+8=71$?
4	What is the area, in square centimeters, of a circle with a diameter measuring 32 cm?
5	What is the volume, in cubic centimeters, of a right circular cylinder with a base radius of 7 cm and a height of 4 cm?
6	What is the perimeter, in centimeters, of a rhombus with sides measuring 28 cm?
7	What is the name for a polygon with five sides?
8	What is the seventh term of an arithmetic sequence with a first term of nine and a common difference of twelve?
9	How many prime numbers are between 6^2 and 7^2?
10	The probability that it rains tomorrow is one-third, while the probability that Tom will win a game of Mexica tonight is four-fifths. If Tom's win has no effect on the weather, what is the probability that Tom will win but it will not rain tomorrow?
11	What is the median of the data set {1, 3, 6, 21, 24, 25, 25}?
12	A goat is tethered to an external corner of a rectangular barn. If the rope is 40 m long and the barn measures 30 m by 50 m, what is the total area the goat can graze, in square meters?
13	Evaluate: $\log_4 512$
14	A bag contains four triangular tiles and six square tiles. When two tiles are drawn at random, what is the probability they are both triangular?
15	What is the next term of the sequence 1, 3, 2, 5, 4, 7, 8, 9, 16, 11, …?
16	Alex and Becky are racing in a peculiar manner. They both begin at the same spot on a 1 km circular track and move in opposite directions. Alex runs at 4 m/s and Becky runs at 5 m/s. How many times would they have passed each other if the race stops after 1 hour?
17	Evaluate: $\frac{2}{3}+\frac{4}{5}$

18	How many liters of 2% chlorine solution should be added to 6000 liters of a 3% chlorine solution to cause the combined solution to be 2.4% chlorine?
19	What are the coordinates, in the form (x, y), of the vertex of the parabola with equation $y = 2x^2 - 8x - 9$?
20	If Z zippers can be purchased for D dollars, how many quarters would be needed to buy six zippers in terms of D and Z?
21	What value(s) of z satisfy $\frac{z+4}{2z-1} = \frac{3z-8}{6z+3}$?
22	Evaluate: $\cfrac{3}{2+\cfrac{3}{2+\cfrac{3}{2+\cdots}}}$
23	Using the numbers 2, 3, 5, and 9 exactly once each, and the operations +, -, $\times$, and $\div$ (and parentheses) as much as you like, what is the closest number to 115 that can be generated?
24	The product of two positive integers is one hundred thousand and neither is a multiple of 10. What is the larger of the two numbers?
25	f(x) is a quadratic function with zeroes at x=2 and x=4. If f(0)=1, what is f(-1)?
26	What is the area of the pentagon with vertices at (3,5), (-6,-2), (2,-4), (-3,8), and (6,1)?
27	Evaluate: $\frac{10+4i}{1-i}$
28	Express the range of $s(t) = 4\log_3 t - 1$ in interval notation if the domain and range are both subsets of the real numbers.
29	Express the product of the base 7 numbers 64_7 and 56_7 in base 7.
30	How many positive multiples of 18 less than 1000 are also multiples of 24?

Challenge Questions

31	How many permutations of the word SEATTLE have the 'A' between the two 'T's?
32	A fair coin is repeatedly flipped. What is the probability that the first time heads is flipped twice in a row is on the seventh and eighth flips?
33	Solve for all values of x so that: $\log\left(x^2\right)+(\log x)^2=3$
34	What are the last 3 digits of 111^{89}?
35	What is the smallest positive integer that has exactly 18 positive integer factors and leaves a remainder of 9 when divided by 31?
36	Evaluate: $\sum_{a=1}^{5}\sum_{b=0}^{3}(a+b)^2$
37	In the data set {4, 16, 7, *x*, 19, 12, 13, *y*, 12, 16, 17, *z*, 13}, $0\le x<y<z\le 20$ and the mean is less than the unique mode, which is less than the median. What is the maximum possible value of $x+y+z$?
38	The length of a rectangle is cos(x) and the width is sin(x) for some value of x. What is the maximum perimeter of the rectangle?
39	Given the cryptarithm below, in which each instance of a given letter represents the same digit (0-9) and no two different letters represent the same digit, what is the maximum possible value of the five-digit number ABCDE? $\begin{array}{r} ABC \\ +CAD \\ \hline BEDA \end{array}$
40	A sphere is inscribed in a regular tetrahedron with edges measuring 12 cm. What is the radius of a smaller sphere that can be inscribed touching the original sphere and three faces of the tetrahedron?

"Math is Cool" Masters – 2009-10

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11^{th} & 12^{th} Grade – December 5, 2009

Individual Multiple Choice Contest

1	When $0.25\overline{4}$ is expressed as a proper fraction in the form $\frac{n}{m}$, where n and m are relatively prime, what is $n+m$? A) 925 B) 929 C) 935 D) 937 E) 941
2	What is the minimum distance from the point $(-2,-5)$ to the line $x+2y=13$? A) $\sqrt{5}$ B) 3 C) 5 D) $3\sqrt{7}$ E) $5\sqrt{5}$
3	What is the volume, in cubic centimeters, of a right circular cone with a base radius of 3 cm and a height of 10 cm? A) 30π B) 45π C) 60π D) 75π E) 90π
4	Which of the following describes the locus of points whose minimum distance from the line $y=x+1$ is twice their minimum distance from the line $y=2x-3$? A) Two intersecting lines B) ellipse C) four points D) hyperbolaE) parabola
5	What is the sine of 15 degrees times the cosine of 15 degrees? A) $\frac{\sqrt{3}}{16}$ B) $\sqrt{3}-1$ C) $\frac{1}{4}$ D) $\frac{\sqrt{3}-1}{4}$ E) $\frac{\sqrt{3}}{4}$
6	When two fair six-sided dice are rolled, what is the probability the numbers shown on their upper faces are relatively prime to one another? Note: consider one to be relatively prime to all counting numbers, including one. A) $\frac{19}{36}$ B) $\frac{5}{9}$ C) $\frac{7}{12}$ D) $\frac{11}{18}$ E) $\frac{23}{36}$
7	What is the range of the data set {3, 5, 8, 12, 13, 14, 31}? A) 3 B) 12 C) 28 D) 31 E) 34
8	What is the next term of the sequence 3, 8, 16, 27, 41, …? A) 58 B) 60 C) 62 D) 64 E) 66
9	Doug's sister is taller than Jim and Darla is shorter than Jim's daughter. Also, Doug is shorter than Sally. If there are only four people discussed in this problem, what is the tallest person's name? A) Darla B) Doug C) Jim D) Sally E) Answer not given

"Math is Cool" Masters – 2009-10

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11^{th} & 12^{th} Grade – December 5, 2009

Team Contest

1	Evaluate: $16^3 + 24^3$
2	Dennis wants to pour a four-inch-deep rectangular concrete walkway around his rectangular garden that measures six feet by eight feet. If he wants to use approximately three cubic yards of concrete, how many feet wide should he make his walkway, rounded down to the nearest foot?
3	When Tom and Lori stand four feet apart during a break from hiking through a flat plain, Lori realizes that the top of Tom's head lines up exactly with the top of a sixty-foot-tall tree. If Lori's eye is exactly five feet above the ground and the top of Tom's head is exactly six feet above the ground, how many feet is Lori from the tree?
4	In the cryptarithm MATH+IS=COOL, each instance of a letter represents the same digit 0-9 and different letters represent different digits. What is the maximum possible value of the ten-digit number MATHISCOOL?
5	How many positive three-digit multiples of 13 contain at least two different digits?
6	In how many ways can a five-member interview committee be selected from three administrators and seven teachers if the committee must contain at least one administrator?
7	In a seven-element set of integer test scores from 0 to 100 inclusive, the range is 72, the mean is 41, and the unique mode is 63. What is the lowest possible value of the median?
8	Someone hacked my calculator! They somehow have switched numbers on my keypad. If I enter $15 + 35$, the screen shows 110. If I enter $15 + 95$, the screen shows 130. If I enter $35 + 95$, the screen shows 50. What would happen if I entered $193 + 319$?
9	Using the numbers 1, 4, 6, and 7 exactly once each, and the operations +, -, $\times$, and $\div$ (and parentheses) as much as you like, write an expression that evaluates to 53.
10	The function $p(n)$ outputs the number of distinct prime factors n has, while the function $d(n)$ outputs the number of digits n has. For example $p(16) = 1$ and $d(16) = 2$. For how many positive integers of $n \le 50$ does $p(n) = d(n)$?

"Math is Cool" Masters – 2009-10

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11^{th} & 12^{th} Grade – December 5, 2009

Pressure Round Contest

1	A sequence of positive numbers is formed so that each term is proportional to the product of the two previous terms. If $a_3 = 2, a_4 = 6, a_6 = 8$, what is the value of a_2?
2	What is the largest number of distinct regions that can be formed by three congruent circles and two lines?
3	Express the base 9 number 678_9 in base 3.
4	What is the next term of the sequence 2, 9, 23, 51, 107, …?
5	Evaluate: $\sqrt[3]{504+\sqrt[3]{504+\sqrt[3]{504+\cdots}}}$

"Math is Cool" Masters – 2009-10

Sponsored by: Basic American Foods & EKA Chemicals

9th, 10th, 11th, 12th Grade – December 5, 2009

Mental Math Contest

PERSON 1		
1.1	What is the product of eight and twelve?	96
1.2	Half of my special number is eighteen more than thirty-one. What is my special number?	98
1.3	What is this number if the square root of this number is three more than the square root of 64?	121
1.4	If Mathiscoolium has a half-life of one day, how many grams of a one-thousand twenty-four gram sample will remain after two weeks?	1/16 [g]
PERSON 2		
2.1	What is the mode of the data set four, seven, ten, ten, eleven, thirteen, sixteen, forty-two, eighty-five?	10
2.2	Two numbers sum to one-hundred eighty-six and differ by forty-four. What is the larger of the two numbers?	115
2.3	What is the sixth term of a geometric sequence with first term three and common ratio two?	96
2.4	If D is congruent to nine modulo (PRONOUNCED "MOD-U-LOW") thirteen, what is the sum of all possible values of D between one and forty inclusive?	66
PERSON 3		
3.1	What is the area, in square centimeters, of a square with sides measuring nine centimeters?	81 [cm^2]
3.2	If three less than five times my age is sixty-two, how old am I, in years?	13 [yrs]
3.3	What are the odds against drawing a queen when one card is drawn from a standard fifty-two-card deck? Express your answer as a reduced ratio of the form a colon b.	12:1
3.4	What is the volume, in cubic centimeters, of an equilateral triangular prism with base edges measuring twelve centimeters and a height of seven centimeters?	$252\sqrt{3}$ [cm^3]
PERSON 4		
4.1	When two fair six-sided dice are rolled, what is the probability the numbers shown on their upper faces sum to four?	1/12
4.2	What is the sum, in degrees, of the measures of the interior angles of a regular octagon?	1080 [degrees]
4.3	Express the square root of one-hundred twenty-eight in simplest radical form.	$8\sqrt{2}$
4.4	A bucket weighs 25 pounds when it is 40% full and 40 pounds when it is 70% full. How many pounds does the bucket weigh when empty?	5 [pounds]

"Math is Cool" Masters – 2009-10

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9^{th}, 10^{th}, 11^{th} & 12^{th} Grade – December 5, 2009

COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	What is the sum, in degrees, of the interior angles, of a convex polygon that has twenty-seven diagonals?	1260 [degrees]
2	What is the surface area of a sphere with a volume of two hundred eighty-eight pi?	144π
3	There are five bacteria in a petri dish. If the bacteria population triples every half hour. How many hours have elapsed when the bacteria population exceeds three thousand?	3 [hours]
4	What is the maximum number of regions into which ten lines can divide a plane?	56 [regions]
5	What is the positive difference between the area and perimeter of a triangle with side lengths of five, seven, and eight?	$20-10\sqrt{3}$
6	What is the difference between the first perfect number and the number of regular polygons that can tessellate a plane?	3
7	How many ways can I divide ten apples among six people such that everyone receives at least one apple?	126 [ways]
	Extra Problem – Only if Needed	
8	What is the positive difference between the sum of the first ten positive odd numbers and the first nine positive even numbers?	10

"Math is Cool" Masters – 2009-10

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9th, 10th, 11th & 12th Grade – December 5, 2009

COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	What is the sum of the next two terms of the following sequence: eighty-one fifths, twenty-seven, forty-five, seventy-five, and so on…	1000/3
2	How many integers are possible lengths for the third side of a triangle with the first two sides of lengths five and seven?	9 [side lengths]
3	For what values of X is the following expression undefined? X over the quantity X squared plus five X minus fourteen.	2, -7
4	Find the coefficient of the X cubed term when the following expression is fully expanded: quantity two X plus three raised to the fifth power.	720
5	Abby chooses at random a positive integer less than thirty. What is the probability that it is neither prime nor divisible by five?	15/29
6	A ball is dropped from a height of twenty meters. Each time it hits the ground, it rebounds one fourth the distance it has fallen. What is the total distance the ball has traveled before it comes to rest?	100/3 meters
7	What is the volume, in cubic inches, of an octahedron with an edge length of three inches?	$9\sqrt{2}/2$ [in^3]
	Extra Problem – Only if Needed	
8	If F of X equals four X squared plus seventeen, what is the positive difference between F of five and F of six?	44

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9^{th}, 10^{th}, 11^{th} & 12^{th} Grade – December 5, 2009

COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	Biff flips five fair coins. What is the probability that there were more heads than tails?	1/2
2	What is the sum of the prime numbers greater than fifty and less than eighty?	463
3	A bowl contains five red marbles and eight green marbles. What is the probability a green marble is drawn on the second draw given a red marble is drawn and removed on the first draw?	2/3
4	A company sent a shipment of 8 sewing machines which included three faulty sewing machines. If Caleb chose three sewing machines at random, what is the probability that exactly two of the sewing machines were faulty?	15/56
5	Evaluate, and give your answer in base ten: one, zero, one, one base two plus one, zero, one base two.	16
6	The side lengths of a triangle are doubled. How much bigger is the area of the new triangle if the original triangle had an area of fifteen square inches?	45 [in^2]
7	What is the positive difference between the lengths of the longer and shorter legs of a right triangle whose perimeter and area both equal thirty?	7
	Extra Problem – Only if Needed	
8	What is the positive difference between the sum of the first five positive square numbers and the sum of the first four positive cubes?	45

"Math is Cool" Masters – 2009-10

Geometry, Algebra 2, PreCalculus, Calculus - - December 5, 2009

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

KEY

First Score

STUDENT NAME_____

Individual Contest – Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	1776		
2	7		
3	7		
4	256π [cm^2]		
5	196π [cm^3]		
6	112 [cm]		
7	Pentagon		
8	81		
9	4		
10	8/15		
11	21		
12	1225π [m^2]		
13	9/2		
14	2/15		
15	32		
16	32 [times]		
17	22/15		
18	9000 [L]		
19	(2,-17)		
20	24D/Z [quarters]		

	Answer	1 or 0	1 or 0
21	-2/23		
22	1		
23	117		
24	3125		
25	15/8		
26	87		
27	3+7i		
28	$(-\infty, \infty)$		
29	$5333_{[7]}$		
30	13		
31	420		
32	13/256		
33	10, 1/1000		
34	391		
35	288		
36	470		
37	26		
38	$2\sqrt{2}$		
39	21840		
40	$\frac{\sqrt{6}}{2}$		

"Math is Cool" Masters – 2009-10

11^{th} & 12^{th} Grade – December 5, 2009

KEY

School Name_____Team #_____

Proctor Name_____Room #_____

First Score

(out of 18)

STUDENT NAME_____

INDIVIDUAL MULTIPLE CHOICE - 15 minutes

This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. ***Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet.*** *No talking during the test.*

DO NOT WRITE IN SHADED REGIONS

Answer		-1, 0 or 2	-1, 0 or 2
1	B		
2	E		
3	A		
4	A		
5	C		
6	E		
7	C		
8	A		
9	D		

"Math is Cool" Masters – 2009-10

KEY

11th & 12th Grade – December 5, 2009

School Name_____Team #_____

Proctor Name_____Room #_____

First Score

(out of 20)

STUDENT NAME_____

Team Contest – Score Sheet

TEAM TEST - 15 minutes

When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as ***2 or 0****. Record all answers on the colored answer sheet.*

DO NOT WRITE IN SHADED REGIONS

	Answer	2 or 0	2 or 0
1	17920		
2	5 [ft]		
3	220 [ft]		
4	7963428005		
5	69		
6	231		
7	20		
8	1124		
9	4*(6+7)+1 [may be others]		
10	30		

"Math is Cool" Masters – 2009-10

11th & 12th Grade – December 5, 2009

KEY

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

PRESSURE ROUND - 10 minutes

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

Pressure Round Answers

	Answer
1	9
2	22 [regions]
3	$202122_{[3]}$
4	219
5	8