Sponsored by: Pre-Algebra – November 6, 2009 Individual Contest

### Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- Good sportsmanship is expected throughout the competition by <u>all</u> involved.
  Bad sportsmanship may result in disqualification.
- Calculators or any other aids may not be used on any portion of this contest.
- Unless stated otherwise:
   For problems dealing with money, a decimal answer should be given.
   Express all rational, non-integer answers as reduced common fractions.
- All radicals must be simplified and all denominators must be rationalized.
- Units are not necessary unless it is a problem that deals with time and in that
- case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.
  - Leave all answers in terms of π where applicable.
  - Do not round any answers unless stated otherwise.
  - Record all answers on the colored cover sheets in the answer column only.
  - Make sure all answer sheets have all the information at the top of the sheet filled out.
  - Tests will be scored as a 0 if answers are not recorded on the answer sheets.
  - Blank answer sheets and answer sheets with no name will also be scored as a 0.

#### **INDIVIDUAL TEST - 35 minutes**

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

### "Math is Cool" Championships – 2009–10 Sponsored by: Pre-Algebra – November 6, 2009 Individual Contest

| 1  | What is the smallest three-digit number in which the hundred's digit is equal t tens digit and the ones digit?   | o the                   | produc                   | t of t                    | he              |
|----|--|-------------------------|--------------------------|---------------------------|-----------------|
| 2  | What is the side length of the smallest square with integer side lengths in v<br>square units in its area is greater than the number of units in its perimeter?  | vhich                   | the nu                   | mber                      | of              |
| 3  | Evaluate: 231 + 899  |                         |                          |                           |                 |
| 4  | Let A be the number of digits in the number one billion and let B be the number of zeros in the expansion of 10 <sup>11</sup> . What is the value of AB?   |                         |                          |                           |                 |
| 5  | What is the sum of the least common multiple of 8 and 9 and the second least of and 9?   | commo                   | on mult                  | iple of                   | 8               |
| 6  | The array of numbers is almost a magic square, where the sum of the numbers in every row, column and diagonal is 15. What is the sum of the sums   | 9                       | 6                        | 3                         |                 |
|    | of any row, column or diagonal having a sum other than 15?   | 5                       | 2                        | 8                         |                 |
|    |  | 1                       | 7                        | 4                         |                 |
| 7  | It takes Wayne 28 spoonfuls to eat a bowl of cereal topped with almonds<br>almonds on his cereal, what is the average number of almonds per spoonful? Ex<br>a decimal to the nearest tenth.  | . If V<br>press         | Vayne<br>your a          | puts 9<br>nswer           | 98<br>as        |
| 8  | Jenna reads at a rate of $\frac{2}{3}$ page per minute. How many minutes will it take her to read a 288-page   |                         |                          |                           |                 |
| 9  | How many minutes are there in three days and three hours?  |                         |                          |                           |                 |
| 10 | The date 01/04/09 is made of three consecutive perfect squares in ascending right. What is the next date satisfying this pattern? Answer in the form <i>dd/m</i>   | g orde<br>m/yy.         | r fron                   | n left                    | to              |
| 11 | In the number expression to the right, the "x" or a " $\div$ ". What is the positive difference 12 $\Box$ 4 - 10 box can be filled with either a between the two resulting simplifications?  |                         |                          |                           |                 |
| 12 | Simplify: $3^4 \cdot 2^2 \cdot 5^3$  |                         |                          |                           |                 |
| 13 | In the following number sequence, the first number is divided by two to get<br>The second number is multiplied by three to get the third number. In thi<br>continues by alternately dividing by two and then multiplying by three. Wh<br>integer number in this sequence? Answer as a decimal to the nearest tenth.<br>40, 20, 60, 30, 90, | the s<br>s way<br>at is | econd<br>the s<br>the fi | numbe<br>sequen<br>rst no | er.<br>ce<br>n- |
| 14 | Jenny was looking under the cushions of her couch and altogether she for<br>ballpoint pen, a tube of lip balm, a golf tee, and 37 cents. What is the maximum<br>items that she found?  | ound a<br>n poss        | i tech<br>ible nu        | deck,<br>Imber            | a<br>of         |
| 15 | Evaluate: $\frac{3}{4} \cdot \frac{2}{11} \cdot \frac{5}{9} \cdot \frac{33}{75} \cdot \frac{10}{17}$   |                         |                          |                           |                 |

| 16       | In the given trapezoid, the length of side AB is             | 16 cm and<br>the distance A B  |
|----------|--|--|
|          | from side AB to side DC is 8 cm what is the                  | number of  |
|          | square centimeters in the area of trapezoid ABC              | D3   |
|          |  | C D  |
|          |  |  |
| 17       | A line segment is drawn between the points (-13,             | -5) and (8, 1) on a coordinate plane. This segment                               |
|          | v-coordinates of these additional points?                    | coordinates are integers. What is the sum of the                                 |
| 10       | What is the median of the set of numbers which               | form the following arithmetic sequence:  |
| 18       | 8, 15, 22, , 218, 225, 232                                   | <u> </u>   |
| 19       | If 22 shakes = 30 florgs and 18 florgs = 60 riddle           | 25, how many riddles equal one shake?  |
| 20       | Figure 1 Figure 2 Figure 3 Figure 4 Figure                   | <u>; 5</u>   |
|          |  | 7  |
|          |  | N  |
|          |  |  |
|          |  | ,<br>,   |
|          |  | $\backslash$   |
|          |  | )  |
|          | All sincles and semisingles in the pattern have the          | a come nodius. If the area of the comisingle in                                  |
|          | Figure 1 is $\pi$ square units, what is the number of s      | square units in the combined area of all of the                                  |
|          | circles in Figure 100?                                       | ·  |
| 21       | What is the median of the first ten prime numbe              | rs?  |
| <u> </u> | Amy writer a list of the first ten positive ave              | n integens. Timmy whites a new list in which he                                  |
| 22       | doubles each of the numbers in Amy's list. If t              | he sum of the numbers in Amy's list is X and the                                 |
|          | sum of the numbers in Jimmy's list is Y, what is >           | ( + Y?   |
| 23       | Evaluate: 3 <sup>8</sup> - 6 <sup>4</sup>                    |  |
| 21       | In the diagram, what is the ratio of AB to $\Box$            |  |
| 24       | AC? Assume the lines in the grid are evenly                  |  |
|          | spaced and answer as a common fraction.                      |  |
|          |  |  |
|          |  |  |
|          | -  |  |
|          | -  | +  |
|          |  |  |
| ~~       | a ab a b a b a b a b a b a b a b a b a                       |  |
| 25       | Given that $\frac{1}{11} = 6$ , and a and b are integers and | 1 <a<b, <math="" is="" of="" the="" then="" value="" what="">\frac{-}{b}?</a<b,> |
| 26       | Circle A has radius 4 inches and circle B has ra             | dius 7 inches. Their centers are 20 inches apart.                                |
| 20       | What is the number of inches in the longest po               | ussible segment that can be drawn such that one                                  |
|          | enapoint is on circle A and one enapoint is on circ          |  |

| 27 | Annie walks to school at a rate of 4 miles per hour and her brother Raymond walks at a rate of 3   |
|----|--|
| 61 | miles per hour. The distance from home to school along their route is $\frac{3}{4}$ mile. What is the number of  |
|    | seconds that Annie can wait to leave after Raymond so that they arrive at school at the same time?   |
| 28 | What is the positive difference between the greatest and least values in the list below?   |
| 20 | $\left(\frac{\sqrt{400}}{5}\right)^2$ 2 <sup>4</sup> $\sqrt[3]{3375}$ $\frac{85}{5}$ $\frac{33}{2}$  |
|    |  |
| 29 | The first term of a geometric sequence is 375 and the fourth term is 24. What is the third term?   |
| 30 | Given regular pentagon ABCDE, how many triangles can be<br>drawn such that each triangle shares a distance set of three<br>vertices with the pentagon? |

|    | Challenge Questions   |
|----|---|
| 31 | Let x represent any positive integer. Answer A if the following expression will always be even, B if it will always be odd, or C if it will sometimes be even and sometimes odd.  |
|    | $\frac{x^2+x}{2}+2$   |
| 32 | Bobby runs $\frac{2}{3}$ as fast as Billy. If it takes Billy 8 minutes to run a mile, how many minutes does it  |
|    | take Bobby to run a mile?   |
| 33 | Flaky Jake's calculator follows standard order of operations, but Sneaky Pete played a trick on him<br>by switching the + and the ÷ buttons. When Flaky Jake did the calculations 80 + 120 ÷ 4, the<br>calculator showed a number that did not seem correct. So Flaky Jake did the calculations by hand.<br>Assuming he completed the calculations correctly, what is the positive difference between the<br>number he got doing the problem by hand and the number shown on the calculator? Express your<br>answer as a mixed number |
| 34 | The phrase "Bee's knees" consists of a four-letter word and a five letter word. If you ignore the apostrophe, how many new distinct two word phrases can be created where the first word has four letters and the second word has five letters by exchanging one letter in the first word with one letter in the second word? The words do not need to make sense.  |
| 35 | In the equation $A + 45 + B + 21 + C + 37 = 334$ , the letters A, B, and C stand for positive integers that form an arithmetic sequence (that is, $A + d = B$ and $B + d = C$ , where d may be positive, negative, or 0). What is the largest possible value for C?   |
| 36 | There are four numbers whose prime factorization has the form $a \cdot b \cdot c$ where a, b, and c are distinct members of the set {2, 3, 5, 7}. What is the sum of these four numbers?  |
| 37 | Evaluate: ∜12·75·50·45·12   |
| 38 | Maryann can paint a wall in 45 minutes. It takes her kid brother Junior 1 hour and 45 minutes to paint the same wall. How many minutes would it take Maryann and Junior to paint the wall, if they work together? Answer as a decimal to the nearest tenth.   |
| 39 | Thirty percent of the girls and forty percent of the boys in a certain math class are going to miss class on a Friday because of a band trip. If thirty-two percent of the students in the class will be on the band trip, what is the least possible number of students remaining in the class?  |
| 40 | On Saturday afternoon six friends play doubles tennis. Partners change after every game such that<br>during any given game two players sit out and no two games involve the same four people paired<br>against each other the same way. For example, for the set of four players Arthur, Beth, Charlie,   |
|    | and Denise, there are three possible pairings: AB vs CD, AC vs BD and AD vs BC. What is the maximum number of games that can be played under these conditions?  |

#### Sponsored by: Beginning Algebra – November 6, 2009 Individual Contest

### Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- Good sportsmanship is expected throughout the competition by <u>all</u> involved.
  Bad sportsmanship may result in disqualification.
- Calculators or any other aids may not be used on any portion of this contest.
- Unless stated otherwise:
   For problems dealing with money, a decimal answer should be given.
   Express all rational, non-integer answers as reduced common fractions.
- All radicals must be simplified and all denominators must be rationalized.
- Units are not necessary unless it is a problem that deals with time and in that
- case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.
  - Leave all answers in terms of  $\pi$  where applicable.
  - Do not round any answers unless stated otherwise.
  - Record all answers on the colored cover sheets in the answer column only.
  - Make sure all answer sheets have all the information at the top of the sheet filled out.
  - Tests will be scored as a 0 if answers are not recorded on the answer sheets.
  - Blank answer sheets and answer sheets with no name will also be scored as a 0.

#### **INDIVIDUAL TEST - 35 minutes**

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. Each problem is scored as 1 or 0. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

### "Math is Cool" Championships – 2009–10 Sponsored by: Beginning Algebra – November 6, 2009 Individual Contest

| 1  | What is the smallest three-digit number in which the hundred's digit is equal to the product of the tens digit and the ones digit?   |                  |                    |                  |        |
|----|--|------------------|--------------------|------------------|--------|
| 2  | What is the side length of the smallest square with integer side lengths in which the number of square units in its area is greater than the number of units in its perimeter?   |                  |                    |                  |        |
| 3  | Evaluate: 231 + 899  |                  |                    |                  |        |
| 4  | What is the sum of the least common multiple of 8 and 9 and the second least of and 9?   | commo            | n mult             | iple of a        | 8      |
| 5  | The array of numbers is almost a magic square, where the sum of the numbers in every row, column and diagonal is 15. What is the sum of the sums   | 9                | 6                  | 3                |        |
|    | of any row, column or diagonal having a sum other than 15?   | 5                | 2                  | 8                |        |
|    |  | 1                | 7                  | 4                |        |
| 6  | Jenna reads at a rate of $\frac{2}{3}$ page per minute. How many minutes will it take her  | ' to re          | ad a 2             | :88-pag          | e      |
| 7  | The date 01/04/09 is made of three consecutive perfect squares in ascending right. What is the next date satisfying this pattern? Answer in the form <i>dd/m</i>   | g orde<br>m/yy.  | r fron             | n left t         | Ō      |
| 8  | Simplify: $3^4 \cdot 2^2 \cdot 5^3$  |                  |                    |                  |        |
| 9  | In the following number sequence, the first number is divided by two to get the second number.<br>The second number is multiplied by three to get the third number. In this way the sequence<br>continues by alternately dividing by two and then multiplying by three. What is the first non-<br>integer number in this sequence? Answer as a decimal to the nearest tenth.<br>40, 20, 60, 30, 90 |                  |                    |                  |        |
| 10 | Jenny was looking under the cushions of her couch and altogether she for<br>ballpoint pen, a tube of lip balm, a golf tee, and 37 cents. What is the maximum<br>items that she found?  | ound a<br>n poss | tech<br>ible nu    | deck,<br>Imber o | a<br>f |
| 11 | Evaluate: $\frac{3}{4} \cdot \frac{2}{11} \cdot \frac{5}{9} \cdot \frac{33}{75} \cdot \frac{10}{17}$   |                  |                    |                  |        |
| 12 | In the given trapezoid, M and N are midpoints of sides AC<br>and BD respectively. If MN is 13 cm and the shortest<br>distance from side AB to side DC is 8 cm, what is the number<br>of square centimeters in the area of trapezoid ABCD?<br>C   |                  | /                  | E<br>N<br>D      | 3      |
| 13 | Raymond multiplies 7 twos together to get a number. Betty wants to multiply and she wants to get a number that is larger than Raymond's number. What is of fours she can multiply together to do this?   | some †<br>the sr | fours t<br>nallest | rogethe<br>numbe | r<br>r |

| 14 | Figure 1 Figure 2 Figure 3 Figure 4 Figure 5   |   |  |  |  |
|----|--|---|--|--|--|
|    |  |   |  |  |  |
|    |  |   |  |  |  |
|    |  |   |  |  |  |
|    |  |   |  |  |  |
|    |  |   |  |  |  |
|    |  |   |  |  |  |
|    | All circles and semicircles in the pattern have the same radius. If the                      | area of the semicircle in                                     |  |  |  |
|    | Figure 1 is $\pi$ square units, what is the number of square units in the contract $\pi$     | mbined area of all of the                                     |  |  |  |
|    | circles in Figure 100?<br>Amy writes a list of the first ten positive even integers. Timmy w | urites a new list in which he                                 |  |  |  |
| 15 | doubles each of the numbers in Amy's list. If the sum of the numbe                           | ers in Amy's list is X and the                                |  |  |  |
|    | sum of the numbers in Jimmy's list is Y, what is X + Y?                                      |   |  |  |  |
| 16 | A team has won x% of its first 20 games of the season. In order to v                         | win at least 60% of its games<br>wes. What is the value of x2 |  |  |  |
| 17 | In the diagram, what is the ratio of AB to   |   |  |  |  |
| 1/ | AC? Assume the lines in the grid are evenly  |   |  |  |  |
|    | spaced and answer as a common fraction.  |   |  |  |  |
|    |  |   |  |  |  |
|    |  |   |  |  |  |
|    |  |   |  |  |  |
|    |  |   |  |  |  |
|    |  |   |  |  |  |
| 10 | If 22 shakes = 30 florgs and 18 florgs = 60 riddles, how many riddles e                      | equal one shake?  |  |  |  |
| 10 |  |   |  |  |  |
| 19 | What is the median of the first ten prime numbers?   |   |  |  |  |
| 20 | Evaluate: 3 <sup>8</sup> - 6 <sup>4</sup>  |   |  |  |  |
| 21 | Given regular pentagon ABCDE, how many triangles can be                                      | В   |  |  |  |
| 21 | drawn such that each triangle shares a distance set of three                                 | $\frown$  |  |  |  |
|    | vertices with the pentagon?  |   |  |  |  |
|    | A  | $\sum C$  |  |  |  |
|    |  | $\setminus$ /   |  |  |  |
|    |  |   |  |  |  |
|    |  |   |  |  |  |
|    | 2  |   |  |  |  |
| 22 | Bobby runs $\frac{L}{3}$ as fast as Billy. If it takes Billy 8 minutes to run a mi           | le, how many minutes does it                                  |  |  |  |
|    | take Bobby to run a mile?  |   |  |  |  |
| 23 | The given table of values could be extended to show solutions to                             | x 1 2 ····  |  |  |  |
|    | What is the positive difference between the v-values of each $\int_{-\infty}^{\infty}$       | V 1 2   |  |  |  |
|    | equation when x = 5?   | y 1 3 ····  |  |  |  |

| 24 | Given that $\frac{3ab}{11} = 6$ , and a and b are integers and 1 <a<b, <math="" is="" of="" the="" then="" value="" what="">\frac{a}{b}?</a<b,>   |  |  |  |  |
|----|---|--|--|--|--|
| 25 | Circle A has radius 4 inches and circle B has radius 7 inches. Their centers are 20 inches apart.<br>What is the number of inches in the longest possible segment that can be drawn such that one<br>endpoint is on circle A and one endpoint is on circle B? |  |  |  |  |
| 26 | The first term of a geometric sequence is 375 and the fourth term is 24. What is the third term?  |  |  |  |  |
| 27 | What is the positive difference between the greatest and least values in the list below?  |  |  |  |  |
| 61 | $\left(\frac{\sqrt{400}}{5}\right)^2$ 2 <sup>4</sup> $\sqrt[3]{3375}$ $\frac{85}{5}$ $\frac{33}{2}$   |  |  |  |  |
| 20 | $1 \times 7^3 + 3 \times 7^2 + 5 \times 7^1 + 6 \times 7^0$ can be expressed as the base 7 number 13567. What is the sum of   |  |  |  |  |
| 20 | 13567 and 117 expressed as a base 7 number?   |  |  |  |  |
| 29 | Flaky Jake's calculator follows standard order of operations, but Sneaky Pete played a trick on him   |  |  |  |  |
| 27 | by switching the + and the ÷ buttons. When Flaky Jake did the calculations 80 + 120 ÷ 4, the  |  |  |  |  |
|    | calculator showed a number that did not seem correct. So Flaky Jake did the calculations by hand.   |  |  |  |  |
|    | Assuming he completed the calculations correctly, what is the positive difference between the   |  |  |  |  |
|    | answer as a mixed number  |  |  |  |  |
| 20 | Let x represent any positive integer. Answer A if the following expression will always be even, B if  |  |  |  |  |
| 30 | it will always be odd, or C if it will sometimes be even and sometimes odd.   |  |  |  |  |
|    | $\frac{x^2+x}{2}+2$   |  |  |  |  |

| Challenge Questions |  |  |  |  |
|---------------------|--|--|--|--|
| 31                  | Evaluate: <sup>5</sup> √12 · 75 · 50 · 45 · 12   |  |  |  |
| 32                  | What is the median of all three-digit palindromes?   |  |  |  |
| 33                  | In the equation $A + 45 + B + 21 + C + 37 = 334$ , the letters A, B, and C stand for positive integers that form an arithmetic sequence (that is, $A + d = B$ and $B + d = C$ , where d may be positive, negative, or 0). What is the largest possible value for C?  |  |  |  |
| 34                  | There are four numbers whose prime factorization has the form $a \cdot b \cdot c$ where a, b, and c are distinct members of the set {2, 3, 5, 7}. What is the sum of these four numbers?   |  |  |  |
| 35                  | Thirty percent of the girls and forty percent of the boys in a certain math class are going to miss class on a Friday because of a band trip. If thirty-two percent of the students in the class will be on the band trip, what is the least possible number of students remaining in the class?   |  |  |  |
| 36                  | A cylindrical piece of cheese has a volume of $81\pi$ cubic inches and a radius of 3 inches. This piece<br>of cheese is sliced into x number of cylindrical pieces, each with a height of $\frac{1}{8}$ inch and with the<br>same base area as the cylinder. What is the value of x2   |  |  |  |
| 37                  | The phrase "Bee's knees" consists of a four-letter word and a five letter word. If you ignore the apostrophe, how many new distinct two word phrases can be created where the first word has four letters and the second word has five letters by exchanging one letter in the first word with one letter in the second word? The words do not need to make sense.   |  |  |  |
| 38                  | From a bag containing 9 pennies and 6 dimes, some coins are stolen. After the theft, the probability is $\frac{1}{15}$ that if two coins are removed from the bag, without replacement, they will both be dimes. Find the total value, in cents, of the coins that were stolen from the bag. Give all possible answers.  |  |  |  |
| 39                  | On Saturday afternoon six friends play doubles tennis. Partners change after every game such that during any given game two players sit out and no two games involve the same four people paired against each other the same way. For example, for the set of four players Arthur, Beth, Charlie, and Denise, there are three possible pairings: AB vs CD, AC vs BD and AD vs BC. What is the maximum number of games that can be played under these conditions? |  |  |  |
| 40                  | In the figure shown, angle C and angle BDA are<br>complementary. AB = 24 inches and BD = 30<br>inches. What is the number of inches in the<br>length of $\overline{BC}$ ?<br>24 in.<br>30 in.  |  |  |  |
|                     | A D C  |  |  |  |

### "Math is Cool" Championships – 2009–10 Sponsored by: 7<sup>th</sup> Grade – November 6, 2009 Individual Multiple Choice Contest

Street musicians are modern day bards, milling about the people and weaving mystical songs. Their main source of income consists of money offered by passers-by. A diligent street musician named Cannonball can perform at locations A, B, or C. On average, he earns 50 cents for every 2 pedestrians that pass by.





### "Math is Cool" Championships – 2009–10 Sponsored by: 7th Grade – November 6, 2009 Team Contest

| 1  | While visiting Yellowstone National Park, Mike hiked 3 miles on Thursday, 5 miles on Friday, and 6 miles on Saturday. He spent a total of 1 hr 10 min hiking on Thursday and 2 hr 9 min |
|----|---|
|    | hiking on Saturday. His average speed for all three days was $2\frac{1}{2}$ miles per hour. How many  |
|    | <u>minutes</u> did Mike hike on Friday?   |
| 2  | My number is a positive integer with units digit 8. If $\frac{3}{4}$ of my number is a positive integer   |
|    | between 100 and 200, how many values of my number are possible?   |
| 3  | In the money system of Arboria, there are 56 lifs in one tuig, and 84 tuigs are equal in value  |
|    | to 3 lims. Holly has one lim and 14 tuigs. Each package of nerts costs 6 lifs. Find the largest   |
|    | number of packages of nerts that Holly could buy.   |
| 4  | How many prime numbers less than 100 are 1 more than a multiple of 4?   |
| 5  | Find the next term in the following sequence: 1, 5, 12, 22, 35,   |
| 6  | Rhombus MNOP is inscribed in isosceles A O B  |
|    | trapezoid ABCD as shown. The length of side   |
|    | AB is 14 cm, the length of side CD is 8 cm, and   |
|    | the height of trapezoid ABCD is 6 cm. What is   |
|    | the ratio of the area of triangle PDM to the  |
|    | drea of rhombus MINOP? Answer as a common D M C   |
|    |   |
| 7  | A population of bats in a cave decreased by 20% in 2008 and by 15% in 2009. By what   |
| -  | percent would the population need to increase in 2010 to return to the number of bats in the  |
|    | cave before the 2008 decrease? Give the closest of the following answers: 15%, 20%, 35%,  |
|    | 50%, 65%  |
| 8  | Find the number of 3-millimeter pieces that can be cut from a string of length 891 meters,  |
|    | and express your answer in scientific notation with the decimal part given to the nearest   |
|    | hundredth.  |
| 9  | Sanjukta plays a game in which she tosses 6 coins (2 dimes and 4 nickels). For each dime that   |
|    | lands heads up, she scores 10 points, and for each nickel that lands heads up, she scores 5   |
|    | points. A coin that lands tails up scores 0 points. As a reduced fraction, what is the  |
|    | probability that Sanjukta's score is 20?  |
| 10 | At a sale of used books to support the public library, the prices are \$5 each for the first 5  |
|    | books, \$4 each for the next 4 books, and so on. After 15 books are paid for, the customer  |
|    | gets 5 tree books. Justin gets a total of 20 books at the sale. What is the average cost in   |
|    | dollars of Justin's books?  |

### "Math is Cool" Championships – 2009–10 Sponsored by: 7th Grade – November 6, 2009 Pressure Round Contest

| 1 | What is the number of inches in the diameter of the largest sphere that can<br>be placed in a box with integer side lengths and a volume of 240 cubic<br>inches?   |  |  |
|---|--|--|--|
| 2 | Assume that the probability that a randomly selected person is left-handed is $\frac{3}{10}$ . In a team of four mathletes, what is the probability that no one is left-handed? Answer as a common fraction.   |  |  |
| 3 | Let A, B, and C be different numbers chosen from the 5 smallest positive perfect square numbers. What is the smallest possible value for $\frac{A}{B-C}$ ? Simplify your answer.   |  |  |
| 4 | Assume the number pattern shown at the right<br>continues indefinitely and that row one consists of the<br>number 1, row two consists of the numbers 2 and 3,<br>etc. What is the sum of the middle numbers in the $1^{st}$ ,<br>$3^{rd}$ , $5^{th}$ , $7^{th}$ , $9^{th}$ , $11^{th}$ , $13^{th}$ , $15^{th}$ , $17^{th}$ , and $19^{th}$ rows? |  |  |
| 5 | How many integers from 1 to 500 are multiples of 17?   |  |  |

### "Math is Cool" Championships – 2009–10 Sponsored by: 7th Grade – November 6, 2009 Mental Math Contest

| PERSO | DN 1  |                 |                                       |
|-------|---|-----------------|---------------------------------------|
| 1.1   | On any given day, the probability that Paige will eat ice cream is one-eighth,<br>and the probability that she will get hit by lightning is one-fourth. If these<br>two events are independent, find the probability that Paige will eat ice cream<br>and get hit by lightning on the same day. Give your answer as a fraction. | 1/32            | 2                                     |
| 1.2   | The product of two whole numbers is 96, while their sum is 22. What is the  | 10              |                                       |
| 1.0   | positive difference between these two numbers?  | (2)             |                                       |
| 1.3   | When the point negative two comma three is reflected over the line y equals   | juals (3,-2) or |                                       |
|       | x, what are the coordinates of the new point?   | 10 5            | · · · · · · · · · · · · · · · · · · · |
| 1.4   | How many lines of symmetry does a regular decagon have?   | 10 [1           | ines                                  |
| PERSO | DN 2  |                 | ſ                                     |
| 2.1   | When one-sixth of the sum of 34 and 20 is multiplied by one half of the sum of and 5, what is the product?  | 3               | 36                                    |
| 2.2   | Disregarding units, what is the ratio of the surface area to the volume of a cube   | z of            | 3/2                                   |
|       | edge length 4? Express your answer as a reduced common fraction.  |                 |                                       |
| 2.3   | The greatest common factor of my number and 36 is 6. The greatest common  |                 | 30                                    |
|       | factor of my number and 20 is 10. What is the smallest my number can be if it is  |                 |                                       |
|       | positive?   |                 |                                       |
| 2.4   | .4 Shane writes all the prime numbers less than 100. How many times does he write   |                 | 3 [times]                             |
|       | the digit "5"?  |                 |                                       |
| PERSO | 2N 3  |                 |                                       |
| 3.1   | What is the number of square inches in the area of a triangle with side lengths   | of              | 30 [in²]                              |
|       | 5, 12, and 13 inches?   |                 |                                       |
| 3.2   | What is the reciprocal of the sum of one-third and one-sixth?   |                 | 2                                     |
| 3.3   | A single fair octahedral die is rolled, with sides numbered 1 through 8. What is  | the             | 5/8                                   |
|       | probability that the result is either not prime or not odd?   |                 |                                       |
| 3.4   | The hypotenuse of a right triangle is used as one side of a square. What is the   |                 | 289 [un <sup>2</sup> ]                |
|       | number of square units in the area of the square if the legs of the triangle have   | 2               |                                       |
|       | length 15 and 8 units?  |                 |                                       |
| PERSO | DN 4  |                 | ſ                                     |
| 4.1   | If the sum of two angles in a triangle is 132 degrees, what is the degree measur  | 'e of           | 48 [deg]                              |
|       | the third angle?  |                 |                                       |
| 4.2   | In how many ways can 4 identical cookies be distributed to David and Linda if ec  | ıch             | 3 [ways]                              |
|       | of them must get at least 1 cookie?   |                 |                                       |
| 4.3   | What is the height of a cylinder with volume 75 pi cubic units and radius 5 units   | ?               | 3 [units]                             |
| 4.4   | Evaluate: 3 raised to the fourth power, divided by 6 raised to the third power.   |                 | 3/8                                   |
|       | Express your answer as a reduced common fraction.   |                 |                                       |

# "Math is Cool" Championships – 2009-10 Sponsored by: 7<sup>th</sup> & 8<sup>th</sup> Grade - November 6, 2009

## COLLEGE KNOWLEDGE BOWL ROUND #1 - SET A

| Ħ | Problem  | Answer          |
|---|--|-----------------|
| 1 | What is the sum of the first thirteen odd positive integens?     | 160             |
| 1 | The sum of the first thirteen oud positive integers?             | 109             |
| 2 | If the letters J, U, and E are arranged in every possible way,   | OEJ             |
|   | and listed in alphabetical order, what is the fifth arrangement? |                 |
| 3 | Six people are in a room. If each person bumps fists with every  | 15 [fist-bumps] |
|   | other person once, how many fist-bumps occur?                    |                 |
| 4 | Farmer Sampson raises ostriches and llamas in his backyard. If   | 9 [llamas]      |
|   | there are twenty-one heads and sixty feet, how many llamas       |                 |
|   | does Farmer Sampson have?  |                 |
| Б | What is the smallest positive integer that has a remainder of A  | 30              |
| 5 | when divided by 7 and a nemainder of 7 when divided by 82        | 59              |
|   | when divided by 7 and a remainder of 7 when divided by 8?        |                 |
| 6 | What is the probability of drawing, without replacement, a King  | 4/663           |
|   | and then a two from a standard deck of fifty-two cards?          |                 |
| 7 | Given the numbers between ten and forty-nine, inclusive, what is | 280             |
|   | the sum of all the digits?                                       |                 |
|   |  |                 |
|   | Extra Problem – Only if Needed                                   |                 |
| 8 | Alex the koala bear is climbing up a three-hundred foot cliff-   | 74 [nights]     |
|   | face. Every day, he goes up six feet, and every night he slides  |                 |
|   | down two feet. How many nights does he spend on the cliff-       |                 |
|   | face?  |                 |
|   | Tucer  |                 |

# "Math is Cool" Championships – 2009–10 Sponsored by:

7<sup>th</sup> & 8<sup>th</sup> Grade - November 6, 2009

# COLLEGE KNOWLEDGE BOWL ROUND #2- SET A

| # | Problem  | Answer             |
|---|--|--------------------|
| 1 | What is the slope of the line given by the equation three X plus     | -3/5               |
|   | five Y equals thirty-nine?   |                    |
| 2 | What percent of five to the second power is two to the fourth        | 64 [%]             |
|   | power?   |                    |
| 3 | What is the average of the mean, median, and mode of the set         | 6.5                |
|   | {seven, six, six, nine, four, ten}? If you answer is not an integer, |                    |
|   | answer as a decimal.   |                    |
| 4 | A dartboard consists of two concentric circles. One is of radius     | 16/25              |
|   | five, and the other is of radius three. What is the probability      |                    |
|   | that a thrown dart that lands inside the larger circle is outside    |                    |
|   | the smaller circle?  |                    |
| 5 | How many positive integers X satisfy the following inequality:       | 3 [integers]       |
|   | the square-root of one-hundred thirteen is less than X is less       |                    |
|   | than the square-root of one-hundred ninety-three?                    |                    |
| 6 | A set of eight positive single-digit integers has a mean of seven.   | 6/7                |
|   | When one of the integers is removed from the set, the mean           |                    |
|   | increases by x. What is the largest possible value of x?             |                    |
| 7 | If a ten foot ladder leans against a wall with the top of the        | $5\sqrt{3}$ [feet] |
|   | ladder reaching a height of five feet above the ground, what is      |                    |
|   | the distance, in feet, between the base of the ladder and the        |                    |
|   | base of the wall?  |                    |
|   |  |                    |
|   | Extra Problem - Unly it Needed                                       |                    |
| 8 | How many distinct prime factors does the number one-hundred          | 1 [prime factor]   |
|   | seven have?  |                    |

# "Math is Cool" Championships – 2009-10 Sponsored by: 7<sup>th</sup> & 8<sup>th</sup> Grade - November 6, 2009

# COLLEGE KNOWLEDGE BOWL ROUND #3- SET A

| # | Problem   | Answer        |
|---|---|---------------|
| 1 | If the point seven comma thirteen is reflected over the y-axis,   | (-7, -13)     |
|   | and then the x-axis, what are the coordinates of the resulting    |               |
|   | point?  |               |
| 2 | How many diagonals can be drawn in a regular hexagon?             | 9 [diagonals] |
| 3 | David drops a ball that bounces back to half the height it fell.  | 33 [feet]     |
|   | If David drops the ball from a height of twelve feet, how many    |               |
|   | feet has the ball traveled when it hits the ground for the        |               |
|   | fourth time?  |               |
| 4 | Angle A is supplementary to angle B. If angle A is five times     | 150 [°]       |
|   | larger than angle B, what is the measure, in degrees, of angle A? |               |
| 5 | Benjamin has six identical ten-pound weights and three identical  | 2 [ways]      |
|   | twenty-pound weights for a total of nine weights altogether. In   |               |
|   | how many distinct ways can he divide his set of weights into two  |               |
|   | groups of equal weight?   |               |
| 6 | What is the product of eighty-nine and forty-five?                | 4005          |
| 7 | If Annie can smack four mosquitoes per minute and Tom can         | 24 [minutes]  |
|   | smack seven mosquitoes per minute, how many more minutes          |               |
|   | than Tom will Annie take to smack two-hundred twenty-four         |               |
|   | mosquitoes?   |               |
|   |   |               |
|   | Extra Problem - Only it Needed                                    |               |
| 8 | What is the sum of the distinct prime factors of 2009?            | 48            |

# "Math is Cool" Championships - 2009-10 Sponsored by:

7<sup>th</sup> & 8<sup>th</sup> Grade - November 6, 2009

# COLLEGE KNOWLEDGE BOWL ROUND #1 - SET B

| # | Problem  | Answer                                     |
|---|--|--|
| 1 | What is the product of the largest two-digit integer and the   | 98901                                      |
| 2 | A square sheet of paper with side length eight inches is folded<br>exactly in half to form a rectangle. What is the ratio of the<br>number of inches in the perimeter of the rectangle to the<br>number of square inches in the area of the original square? | 3/8 or<br>3 to 8                           |
| 3 | Six friends are living in an apartment together and they evenly<br>split the monthly rent of three thousand dollars. By how many<br>dollars does each person's share of the monthly rent increase if<br>one person moves out?                                | [\$]100                                    |
| 4 | What is the probability that a randomly chosen number from<br>the set of two-digit positive integers will have at least one digit<br>that is either a six or a seven?  | 17/45                                      |
| 5 | Find the distance between the points three comma five and eight comma seventeen.   | 13   |
| 6 | In a set of three circles the first one has a radius of 3 inches,<br>the second one has a radius of 5 inches and the third one has a<br>radius of 7 inches. What is the number of square inches in the<br>average area of the three circles?                 | 83pi/3 (83 "pie"<br>over 3) [sq<br>inches] |
| 7 | If a stack of five CDs is six millimeters tall, how many meters<br>would be in the height of a stack of six hundred CDs? If your<br>answer is not an integer, give it as a decimal to the nearest<br>hundredth.  | .72 [meters]                               |
|   | Extra Problem - Only if Needed   |  |
| 8 | At a camp of two-hundred fifty students, one-hundred students<br>devour dried mangoes, seventy-two students devour Red Vines,<br>thirty-five students devour both. How many students devour<br>neither Red Vines nor dried mangoes?                          | 113 [students]                             |

# "Math is Cool" Championships - 2009-10 Sponsored by:

7<sup>th</sup> & 8<sup>th</sup> Grade - November 6, 2009

# COLLEGE KNOWLEDGE BOWL ROUND #2- SET B

| # | Problem  | Answer            |
|---|--|-------------------|
| 1 | What is the value of three factorial times four factorial divided          | 6/5               |
|   | by five factorial?   |                   |
| 2 | What is the largest two-digit integer that is the product of a             | 95                |
|   | prime number and 19?   |                   |
| 3 | What is the slope of the line containing the points two comma              | -2/5 or 2/-5 or - |
|   | three and seven comma one?   | (2/5)             |
| 4 | Let $\mathcal{S}$ be the set of integers that are between one and ten, not | 2/9               |
|   | including one and ten. What is the smallest possible quotient of           |                   |
|   | two distinct members of <i>S</i> ?   |                   |
| 5 | A set of numbers consists of all composite integers from one to            | 12                |
|   | twenty, including one and twenty. What is the median of the                |                   |
|   | set?   |                   |
| 6 | Betsy was born on October 13, 1961. What was the number of                 | 576 [months]      |
|   | months in her age on October 13, 2009?                                     |                   |
| 7 | At the annual dog and penguin owner's convention, Miriam saw a             | 7 [penguins]      |
|   | room full of dogs and penguins. She counted twenty heads and               |                   |
|   | sixty-six feet. How many penguins were there?                              |                   |
|   |  |                   |
|   | Extra Problem - Only It Needed   |                   |
| 8 | Susie's arm extends two feet from her body, and the javelin she            | 42π [feet]        |
|   | is holding onto extends five feet beyond that. If she spins one-           |                   |
|   | thousand eighty degrees, how many feet has the end of the                  |                   |
|   | javelin traveled?  |                   |

### "Math is Cool" Championships – 2009-10 Sponsored by: 7<sup>th</sup> & 8<sup>th</sup> Grade - November 6, 2009

# COLLEGE KNOWLEDGE BOWL ROUND #3- SET B

| # | Problem  | Answer  |
|---|--|---|
| 1 | What is the number of distinct ways that the letters of the word COOKBOOK, spelled C-O-O-K-B-O-O-K, can be arranged?   | 840 [ways]  |
| 2 | What is six to the second power divided by two to the fourth power?  | 9/4   |
| 3 | A regular hexagon has nine diagonals. How many of these diagonals intersect each other at the exact center of the hexagon?   | 3 [diagonals]   |
| 4 | What is the sum of the three smallest positive integers and the two largest negative integers?   | 3   |
| 5 | An octagon and a pentagon are drawn on the same sheet of<br>paper. How many distinct segments can be drawn connecting the<br>vertices of the octagon to the vertices of the pentagon?  | 40 [segments]   |
| 6 | A cylindrical glass has height seven inches and diameter four<br>inches. What is the number of inches in the longest straight<br>straw that will fit inside the glass without extending above the<br>rim? Assume the straw has no thickness. | $\sqrt{65}$ [inches]<br>(square root of 65,<br>radical 65, or root<br>65) |
| 7 | Robby deals himself two aces and two queens from a well-<br>shuffled standard deck of cards. What is the probability that<br>the next card dealt is either an ace or a queen?  | 1/12  |
|   | Extra Problem - Only if Needed   |   |
| 8 | What is the smaller angle, in degrees, formed by the minute and<br>hour hands of an analog clock at 3:15 PM? If you answer is not<br>an integer, answer as a decimal.  | 7.5 [°]   |

Pre-Algebra - November 6, 2009

Final Score:

KEY

School Name\_\_\_\_\_ Proctor Name\_\_\_\_\_

\_Team #\_\_\_\_\_ \_\_Room #\_\_\_\_\_

First Score

#### STUDENT NAME\_\_\_\_\_

#### Individual Contest – Score Sheet DO NOT WRITE IN SHADED REGIONS

|    | Answer                 | 1 or 0 | 1 or 0 |    | Answer           | 1 or 0 | 1 or 0 |
|----|------------------------|--------|--------|----|------------------|--------|--------|
| 1  | 111                    |        |        | 21 | 12               |        |        |
| 2  | 5 [units]              |        |        | 22 | 330              |        |        |
| 3  | 1130                   |        |        | 23 | 5265             |        |        |
| 4  | 110                    |        |        | 24 | 13/10            |        |        |
| 5  | 216                    |        |        | 25 | 2/11             |        |        |
| 6  | 36                     |        |        | 26 | 31 [inches]      |        |        |
| 7  | 3.5 [almonds]          |        |        | 27 | 225 [seconds]    |        |        |
| 8  | 432 [minutes]          |        |        | 28 | 2                |        |        |
| 9  | 4500 [minutes]         |        |        | 29 | 60               |        |        |
| 10 | 04/09/16               |        |        | 30 | 10 [triangles]   |        |        |
| 11 | 45                     |        |        | 31 | С                |        |        |
| 12 | 40500                  |        |        | 32 | 12 [minutes]     |        |        |
| 13 | 67.5                   |        |        | 33 | $105\frac{1}{3}$ |        |        |
| 14 | 41 [items]             |        |        | 34 | 15 [phrases]     |        |        |
| 15 | 1/51                   |        |        | 35 | 153              |        |        |
| 16 | 104 [cm <sup>2</sup> ] |        |        | 36 | 247              |        |        |
| 17 | -4                     |        |        | 37 | 30               |        |        |
| 18 | 120                    |        |        | 38 | 31.5 [minutes]   |        |        |
| 19 | 50/11 [riddles]        |        |        | 39 | 17 [students]    |        |        |
| 20 | 100π [units²]          |        |        | 40 | 45 [games]       |        |        |
|    |                        |        |        |    |                  |        |        |
|    |                        |        |        |    |                  |        |        |

Beginning Algebra - November 6, 2009

School Name\_\_\_\_\_Team #\_\_\_\_\_

Final Score:

First Score

KEY

Proctor Name\_\_\_\_\_

Room #

STUDENT NAME\_\_\_\_\_

#### Individual Contest – Score Sheet DO NOT WRITE IN SHADED REGIONS

|    | Answer                 | 1 or 0 | 1 or 0 |    | Answer                           | 1 or 0 | 1 or 0 |
|----|------------------------|--------|--------|----|----------------------------------|--------|--------|
| 1  | 111                    |        |        | 21 | 10 [triangles]                   |        |        |
| 2  | 5 [units]              |        |        | 22 | 12 [minutes]                     |        |        |
| 3  | 1130                   |        |        | 23 | 72                               |        |        |
| 4  | 216                    |        |        | 24 | 2/11                             |        |        |
| 5  | 36                     |        |        | 25 | 31 [inches]                      |        |        |
| 6  | 432 [minutes]          |        | ·      | 26 | 60                               |        |        |
| 7  | 04/09/16               |        |        | 27 | 2                                |        |        |
| 8  | 40500                  |        |        | 28 | 1400[7]                          |        |        |
| 9  | 67.5                   |        |        | 29 | $105\frac{1}{3}$                 |        |        |
| 10 | 41 [items]             |        |        | 30 | С                                |        |        |
| 11 | 1/51                   |        |        | 31 | 30                               |        |        |
| 12 | 104 [cm <sup>2</sup> ] |        |        | 32 | 550                              |        |        |
| 13 | 4 [fours]              |        |        | 33 | 153                              |        |        |
| 14 | 100π [units²]          |        |        | 34 | 247                              |        |        |
| 15 | 330                    |        |        | 35 | 17 [students]                    |        |        |
| 16 | 65                     |        |        | 36 | 72                               |        |        |
| 17 | 13/10                  |        |        | 37 | 15 [phrases]                     |        |        |
| 18 | 50/11 [riddles]        |        |        | 38 | 32, 45 [cents]<br>[either order] | ·      |        |
| 19 | 12                     |        |        | 39 | 45 [games]                       |        |        |
| 20 | 5265                   |        |        | 40 | 40 [inches]                      |        |        |
|    |                        |        |        |    |                                  |        |        |
|    |                        |        |        |    |                                  |        |        |

| "Math is Cool" Championships – 2009–10<br>7th Grade – November 6, 2009 |             |  |
|--|-------------|--|
| School NameTeam #  |             |  |
| Proctor NameRoom #   | First Score |  |

(out of 18)

#### STUDENT NAME

#### INDIVIDUAL MULTIPLE CHOICE - 15 minutes

This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet. No talking during the test.

|   | Answer | -1, 0 or 2 | -1, 0 or 2 |
|---|--------|------------|------------|
| 1 | A      |            |            |
| 2 | С      |            |            |
| 3 | В      |            |            |
| 4 | E      |            |            |
| 5 | A      |            |            |
| 6 | D      |            |            |
| 7 | A      |            |            |
| 8 | В      |            |            |
| 9 | С      |            |            |
|   |        |            |            |

#### DO NOT WRITE IN SHADED REGIONS

7th Grade - November 6, 2009

School Name\_\_\_\_\_Team #\_\_\_\_\_ Proctor Name\_\_\_\_\_

\_Room #\_\_\_\_\_

Final Score:

First Score

(out of 20)

STUDENT NAME\_\_\_\_\_

#### Team Contest - Score Sheet

#### TEAM TEST - 15 minutes

When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as 2 or 0. Record all answers on the colored answer sheet.

#### DO NOT WRITE IN SHADED REGIONS

| Answer |                    | 2 or 0 | 2 or 0 |
|--------|--------------------|--------|--------|
| 1      | 137 [min]          |        |        |
| 2      | 6 [values]         |        |        |
| 3      | 392 [packages]     |        |        |
| 4      | 11 [primes]        |        |        |
| 5      | 51                 |        |        |
| 6      | 2/11               |        |        |
| 7      | 50 [%]             |        |        |
| 8      | $2.97 \times 10^5$ |        |        |
| 9      | 7/32               |        |        |
| 10     | [\$]2.75           |        |        |
|        |                    |        |        |

| "Math is Cool" Ch<br>7th Grade - | n <mark>ampionships - 2009-10</mark><br>- November 6, 2009 | Final Score:<br>KEY |
|----------------------------------|--|---------------------|
| School Name                      | Team #   | First Score         |
| Proctor Name                     | Room #   |                     |
| STUDENT NAME                     |  |                     |

#### PRESSURE ROUND - 10 minutes

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

**Pressure Round Answers** 

| Answer |                      |
|--------|----------------------|
| 1      | 5 [inches]           |
| 2      | <u>2401</u><br>10000 |
| 3      | -25/3                |
| 4      | 670                  |
| 5      | 29 [integers]        |