

# "Math is Cool" Masters - 2009-10

Sponsored by: Takata

Pre-Algebra - December 5, 2009

Individual Contest

**Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.**

## **GENERAL INSTRUCTIONS applying to all tests:**

- *Good sportsmanship is expected throughout the competition by all involved. Bad sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise:*
  - *For problems dealing with money, a decimal answer should be given.*
  - *Express all rational, non-integer answers as reduced common fractions.*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary unless it is a problem that deals with time and in that case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of  $\pi$  where applicable.*
- *Do not round any answers unless stated otherwise.*
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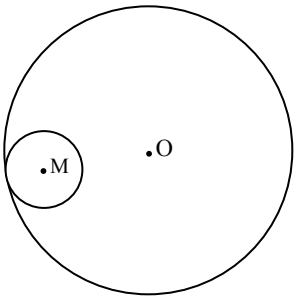
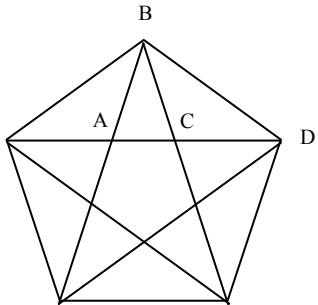
## **INDIVIDUAL TEST - 35 minutes**

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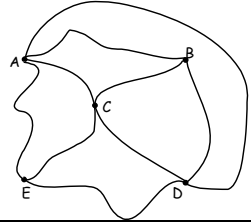
1	Calculate the mean of this set of numbers: {103, 4, 107, 6, 111, 8, 115, 10}
2	How many ways can you rearrange the letters in the word MONSTER?
3	How many sides are there on a heptagon?
4	How many distinct ordered pairs of whole numbers $(x, y)$ exist such that $xy = 16$ ?
5	When Snorlax is hungry, he can eat 4 cheeseburgers in 3 minutes. Snorlax wakes up at 1:00 pm and is hungry until 7:15 pm. How many cheeseburgers could Snorlax eat during the given time?
6	Evaluate $(2^2 + 7 - 3 \times 2 - 64 \div 16) \times (132 \div 12)$
7	What is the 15 <sup>th</sup> term of the Fibonacci Sequence? The Fibonacci Sequence begins with two 1s and each successive term is the sum of the two preceding terms.
8	Evaluate: $\frac{7!5!}{8!3!}$
9	In the town of Sadguppypuppy, there were 12 alien abductions in January, 14 alien abductions in February, 11 alien abductions in March and 19 alien abductions in April. How many alien abductions during the month of May would result in a mean of 15 abductions per month during this 5-month stretch?
10	Every day, Hal turns the ringer off on his phone during his 30 minute nap. The entire nap occurs at a random time between 4:00 pm and 6:00 pm. If Sherry calls him at 4:32 pm on Tuesday, what is the probability that the phone will not ring?
11	What is the probability of drawing an ace, then a red jack, without replacement, from a standard deck of 52 cards?
12	Trung went for a 6-mile run. Being the mathematician he is, he tried to calculate the number of feet in 6 miles. In his head he multiplied incorrectly and got a number that was three-quarters of the correct number. What is the positive difference between the number of feet Trung thought he ran and the actual number of feet he ran?
13	Mitchell arrived at the dentist's office at 8:28 am. If he had arrived 12 minutes earlier, he would have been 11 minutes late for his appointment. What time was Mitchell's appointment?
14	What number can be subtracted from both the numerator and the denominator of $\frac{17}{35}$ to make a fraction equal to $\frac{2}{5}$ ?
15	The points $(2, 2)$ , $(1, -1)$ , $(0, 0)$ , $(-1, -1)$ , $(-2, 2)$ and back to $(2, 2)$ are connected in the order given to form a concave pentagon. What is the number of square units in the area of this pentagon?
16	Solve for x: $2^{x-8} = 256$

17	<p>Circle O has radius 12 centimeters. Circle M is internally tangent to circle O as shown and has a radius of 2 centimeters. Circle M remains internally tangent to circle O while it rolls exactly once around the interior of circle O. What is the number of centimeters traveled by point M through this rotation?</p>	
18	<p>The diagonals of a regular pentagon form a five-pointed star. What is the ratio of the number of degrees in the measure of angle ABC to the number of degrees in the measure of angle BCD? Answer as a common fraction.</p>	
19	<p>Bill rolls two standard dice, one red and one white, and so does Bob. What is the probability that they roll the same thing?</p>	
20	<p>Express the base 4 number <math>213_4</math> as a number in base 2.</p>	
21	<p>When the simplified fraction <math>\frac{2}{7}</math> is expressed as a repeating decimal, what is the sum of the digits in the repeated sequence?</p>	
22	<p>Aladdin rides a rectangular magic carpet. If one side of the carpet is 3 feet less than <math>x</math>, and another side is 2 feet more than <math>x</math>, find a simplified algebraic expression in terms of <math>x</math> in expanded or factored form, giving the number of square feet in the area of the carpet.</p>	
23	<p>Solve for <math>x</math>: <math>7x + 11(x - 4) = 73</math></p>	
24	<p>Bertha buys ingredients for wasabi-chocolate muffins at the grocery store. As she walks the aisles in search of wasabi she wonders, "What 4-digit integer is a palindrome and a product of three consecutive prime numbers?" What is this number?</p>	
25	<p>A multiple of 37 between 800 and 900 can be expressed as the sum of three consecutive positive integers. What is the largest of these three integers?</p>	
26	<p>Suhmin wants to fold a sheet of paper into 1024 sections. How many times must she fold it in half?</p>	
27	<p>Find the value of <math>\frac{x+y}{xy}</math> if <math>3x + 5y = 31</math> and <math>y - 2x = 1</math>.</p>	
28	<p>I have an eight by eight black and white checkerboard. Trung takes my checkerboard and paints all of the squares on the edges white. What is the new ratio of black to white squares, expressed as a common fraction?</p>	
29	<p>How many total digits are used to number a 132-page book if every other page is numbered and the first page to be numbered is page 1?</p>	
30	<p>George is currently <math>\frac{3}{5}</math> the age of Manny. Five years ago, Manny was twice as old as George. How many years old is George now?</p>	

# Challenge Questions - Pre-Algebra

**31** In a geometric sequence, each term is multiplied by a constant factor to get the next term. What is the fifth term in a geometric sequence whose first term is  $\frac{1}{6}$  and whose second term is  $\frac{1}{\sqrt{2}}$ ?

**32** The figure at right shows a map of the riding trails at King City Stables, with letters indicating points at which trails intersect. Liz wants to ride on all the paths exactly once each, but doesn't mind going through an intersection point more than once. Name all points A through E at which she could start her ride. If it is impossible to ride on all paths exactly once each, answer "none".



**33** Maryann can paint a wall in 45 minutes. It takes her kid brother Junior 1 hour and 45 minutes to paint the same wall. How many minutes would it take Maryann and Junior to paint the wall, if they work together? Answer as a decimal to the nearest tenth.

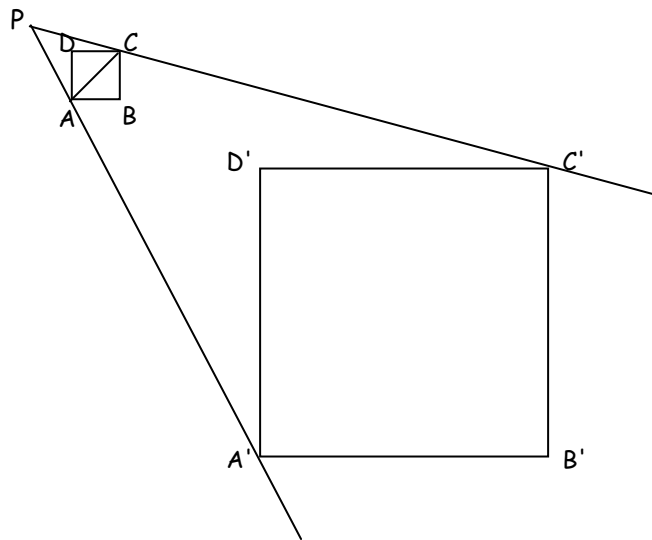
**34** A sprained ankle swells and doubles in size on the day it is sprained and the following day. Then, with proper icing, the ankle loses twenty-five percent of its size each day thereafter until it is back to its normal size. With proper icing, on which day after a sprain will the injured ankle be back to its normal size? (Note: If the ankle is sprained on Monday, then Tuesday is the 1<sup>st</sup> day after the sprain.)

**35** In how many distinct orders can you arrange three different keys on a plain key ring with no clasp?

**36** In the magic square at right, every row, column, and diagonal has the same sum. Five missing values are indicated by letters. Of the four values indicated by a, b, d, and e, list the letters of all that are less than the value of c.

a	175%	$\frac{1}{2}$
b	c	d
2	e	1

**37** In the drawing square  $A'B'C'D'$  is a dilation of square  $ABCD$  and point  $P$  is the center of dilation. (A dilation is an enlargement or a reduction of a shape by a scale factor resulting in an image that is similar to the original shape).  $PD = 3$  inches,  $DD' = 15$  inches and  $AC = 2\sqrt{2}$  inches. How many inches are in the length of  $B'D'$ ?

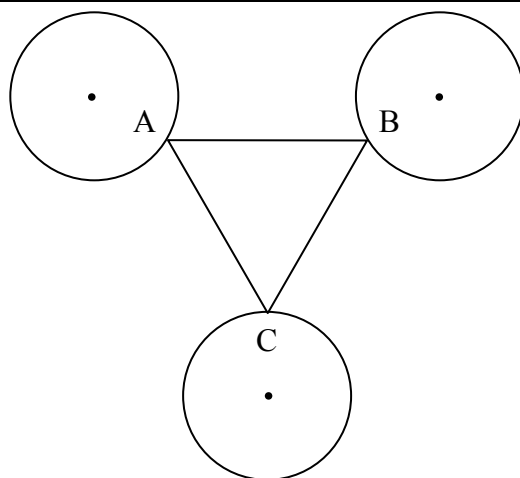


**38** A number  $n$ , not necessarily an integer, is selected at random such that  $-5 < n < 7$ . What is the probability that  $2 < n < 9$ ?

**39** One dot is added to one of the faces of a standard cubical die. When this die is rolled twice, the probability that the sum is 6 equals the probability that the sum is 7. What was the number of dots originally on the face to which the extra dot was added?

40

Three congruent circles, each with a radius of 6 inches, are drawn as shown such that the center of any one of the circles is 18 inches from the centers of each of the other two. Triangle  $ABC$  is the smallest possible equilateral triangle that can be drawn such that each of its three vertices is on one of the circles. What is the area of triangle  $ABC$ ?



Pre-Algebra

# "Math is Cool" Masters - 2009-10

Pre-Algebra - December 5, 2009

Final Score:

**KEY**

First Score

School Name \_\_\_\_\_ Team # \_\_\_\_\_

Proctor Name \_\_\_\_\_ Room # \_\_\_\_\_

**STUDENT NAME** \_\_\_\_\_

## Individual Contest - Score Sheet

### DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	58		
2	5040 [ways]		
3	7 [sides]		
4	5 [pairs]		
5	500 [cheeseburgers]		
6	11		
7	610		
8	5/2		
9	19 [abductions]		
10	1/3		
11	2/663		
12	7920 [feet]		
13	8:05 am		
14	5		
15	8 [un <sup>2</sup> ]		
16	[x =] 16		
17	20π [cm]		
18	1/3		
19	1/36		
20	100111 <sub>[2]</sub>		

	Answer	1 or 0	1 or 0
21	27		
22	x <sup>2</sup> -x-6 or (x+2)(x-3) or (x-3)(x+2) [ft <sup>2</sup> ]		
23	[x =] 13/2		
24	1001		
25	297		
26	10 [times]		
27	7/10		
28	9/23		
29	143 [digits]		
30	15 [yrs]		
31	54		
32	B, E [either order]		
33	31.5 [minutes]		
34	6 [th day]		
35	1 [order]		
36	b, e [either order]		
37	12√2 [in]		
38	5/12		
39	2 [dots]		
40	-162 + 108√3 [in <sup>2</sup> ]		

# "Math is Cool" Masters - 2009-10

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Algebra I - December 5, 2009  
Individual Contest

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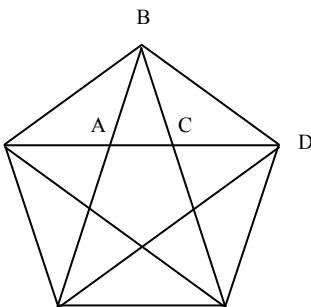
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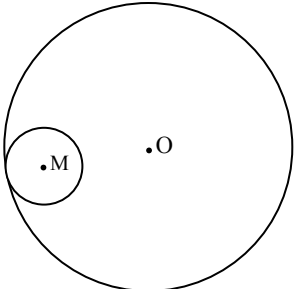
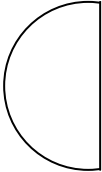
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Algebra I - December 5, 2009  
Individual Contest

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9	Evaluate: $\frac{7!5!}{8!3!}$
10	Solve for x: $2^{x-8} = 256$
11	<p>The diagonals of a regular pentagon form a five-pointed star. What is the ratio of the number of degrees in the measure of angle ABC to the number of degrees in the measure of angle BCD? Answer as a common fraction.</p> 
12	Express the base 4 number $213_4$ as a number in base 2.
13	If $\frac{2}{A} + \frac{B}{6} = \frac{1}{2}$ , where A and B are positive integers, what is the largest possible sum of A + B?
14	There are 23 girls standing in a circle passing a volleyball clockwise around the circle. Anna starts with the ball and passes it to Suzie, skipping the 4 people between Anna and Suzie. Suzie passes the ball to Jen, skipping the 4 people between Suzie and Jen, and so on. How many times will the ball go around the circle in this way before it gets back to Anna?



15	What is the remainder when $3^{2009}$ is divided by 5?	
16	Circle O has radius 12 centimeters. Circle M is internally tangent to circle O as shown and has a radius of 2 centimeters. Circle M remains internally tangent to circle O while it rolls exactly once around the interior of circle O. What is the number of centimeters traveled by point M through this rotation?	
17	Bill rolls two standard dice, one red and one white, and so does Bob. What is the probability that they roll the same thing?	
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22	How many total digits are used to number a 132-page book if every other page is numbered and the first page to be numbered is page 1?	
23	What is the number of inches in the perimeter of the given semicircle, if the radius is 8 inches?	
24	A piece of string is cut into two pieces, one of which is 25% longer than the other. If the longer piece is 90 cm long, how many centimeters long is the shorter piece? If your answer is not a whole number, give it as a decimal.	
25	As an ordered pair, what are the coordinates of the point (x, y) if it is as far to the right of the y-axis as the point (2, -4) is below the x-axis, and it is as far below the x-axis as the point (-6, 7) is to the left of the y-axis?	
26	Cindy has a jug that holds 2010 milliliters (mL). She buys juice in bottles holding $\frac{3}{8}$ of a liter. If she buys the smallest number of bottles of juice that will fill the jug, how many mL of juice will she have left over?	
27	When the simplified fraction $\frac{2}{7}$ is expressed as a repeating decimal, what is the sum of the digits in the repeated sequence?	
28	Aladdin rides a rectangular magic carpet. If one side of the carpet is 3 feet less than x, and another side is 2 feet more than x, find a simplified algebraic expression in terms of x in expanded or factored form, giving the number of square feet in the area of the carpet.	
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# Challenge Questions - Algebra 1

**31** Find the value of  $\frac{x+y}{xy}$  if  $3x + 5y = 31$  and  $y - 2x = 1$ .

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**33** Maryann can paint a wall in 45 minutes. It takes her kid brother Junior 1 hour and 45 minutes to paint the same wall. How many minutes would it take Maryann and Junior to paint the wall, if they work together? Answer as a decimal to the nearest tenth.

**34** A sprained ankle swells and doubles in size on the day it is sprained and the following day. Then, with proper icing, the ankle loses twenty-five percent of its size each day thereafter until it is back to its normal size. With proper icing, on which day after a sprain will the injured ankle be back to its normal size? (Note: If the ankle is sprained on Monday, then Tuesday is the 1<sup>st</sup> day after the sprain.)

**35** In how many distinct orders can you arrange three different keys on a plain key ring with no clasp?

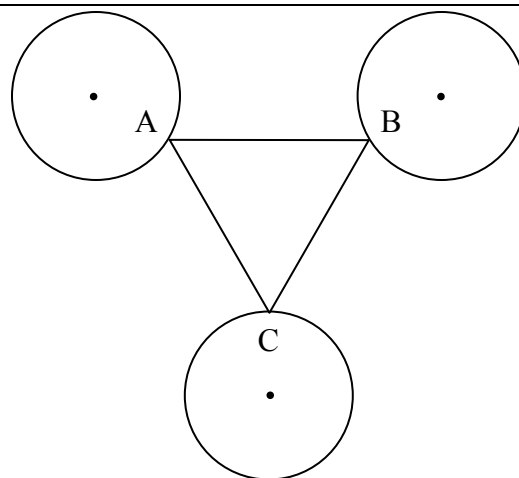
**36** In the magic square at right, every row, column, and diagonal has the same sum. Five missing values are indicated by letters. Of the four values indicated by a, b, d, and e, list the letters of all that are less than the value of c.

a	175%	$\frac{1}{2}$
b	c	d
2	e	1

**37** A number  $n$ , not necessarily an integer, is selected at random such that  $-5 < n < 7$ . What is the probability that  $2 < n < 9$ ?

**38** The total surface area of a cube is 18 square inches. What is the sum, in inches, of the lengths of all the space diagonals of that cube? (A space diagonal of a cube is a line segment from one corner of the cube to the farthest opposite corner).

**39** One dot is added to one of the faces of a standard cubical die. When this die is rolled twice, the probability that the sum is 6 equals the probability that the sum is 7. What was the number of dots originally on the face to which the extra dot was added?



**40** Three congruent circles, each with a radius of 6 inches, are drawn as shown such that the center of any one of the circles is 18 inches from the centers of each of the other two. Triangle ABC is the smallest possible equilateral triangle that can be drawn such that each of its three vertices is on one of the circles. What is the area of triangle ABC?



# "Math is Cool" Masters - 2009-10

Sponsored by: Takata  
Geometry - December 5, 2009  
Individual Contest

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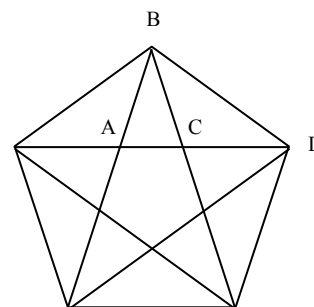
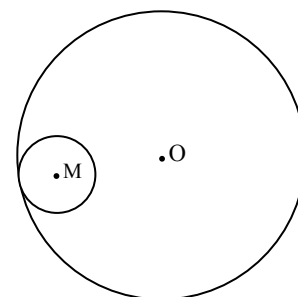
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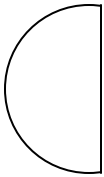
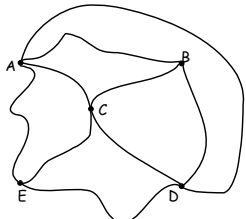
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 Geometry - December 5, 2009  
 Individual Contest

1	Calculate the mean of this set of numbers: {103, 4, 107, 6, 111, 8, 115, 10}
2	How many sides are there on a heptagon?
3	When Snorlax is hungry, he can eat 4 cheeseburgers in 3 minutes. Snorlax wakes up at 1:00 pm and is hungry until 7:15 pm. How many cheeseburgers could Snorlax eat during the given time?
4	Evaluate $(2^2 + 7 - 3 \times 2 - 64 \div 16) \times (132 \div 12)$
5	Evaluate: $\frac{7!5!}{8!3!}$
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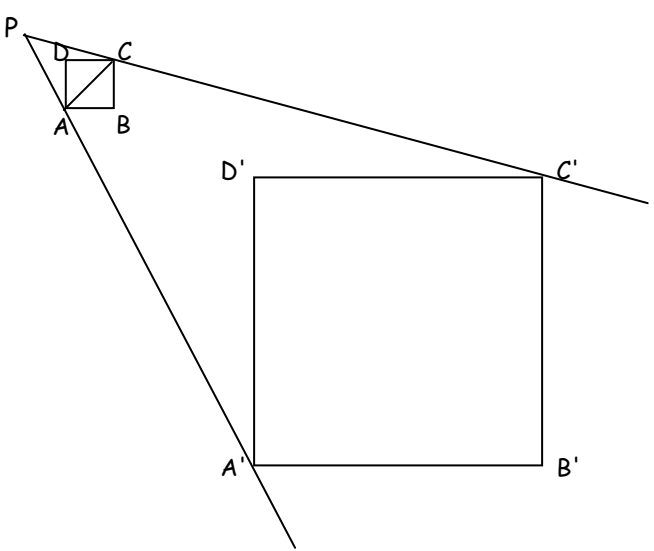
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18	A piece of string is cut into two pieces, one of which is 25% longer than the other. If the longer piece is 90 cm long, how many centimeters long is the shorter piece? If your answer is not a whole number, give it as a decimal.
19	As an ordered pair, what are the coordinates of the point (x, y) if it is as far to the right of the y-axis as the point (2, -4) is below the x-axis, and it is as far below the x-axis as the point (-6, 7) is to the left of the y-axis?
20	Cindy has a jug that holds 2010 milliliters (mL). She buys juice in bottles holding $\frac{3}{8}$ of a liter. If she buys the smallest number of bottles of juice that will fill the jug, how many mL of juice will she have left over?
21	When the simplified fraction $\frac{2}{7}$ is expressed as a repeating decimal, what is the sum of the digits in the repeated sequence?
22	Aladdin rides a rectangular magic carpet. If one side of the carpet is 3 feet less than x, and another side is 2 feet more than x, find a simplified algebraic expression in terms of x in expanded or factored form, giving the number of square feet in the area of the carpet.
23	Solve for x: $7x + 11(x - 4) = 73$
24	A multiple of 37 between 800 and 900 can be expressed as the sum of three consecutive positive integers. What is the largest of these three integers?
25	Find the value of $\frac{x+y}{xy}$ if $3x + 5y = 31$ and $y - 2x = 1$ .
26	How many total digits are used to number a 132-page book if every other page is numbered and the first page to be numbered is page 1?
27	In a geometric sequence, each term is multiplied by a constant factor to get the next term. What is the fifth term in a geometric sequence whose first term is $\frac{1}{6}$ and whose second term is $\frac{1}{\sqrt{2}}$ ?
28	The figure at right shows a map of the riding trails at King City Stables, with letters indicating points at which trails intersect. Liz wants to ride on all the paths exactly once each, but doesn't mind going through an intersection point more than once. Name all points A through E at which she could start her ride. If it is impossible to ride on all paths exactly once each, answer "none".
	

29	I have an eight by eight black and white checkerboard. Trung takes my checkerboard and paints all of the squares on the edges white. What is the new ratio of black to white squares, expressed as a common fraction?
30	Maryann can paint a wall in 45 minutes. It takes her kid brother Junior 1 hour and 45 minutes to paint the same wall. How many minutes would it take Maryann and Junior to paint the wall, if they work together? Answer as a decimal to the nearest tenth.

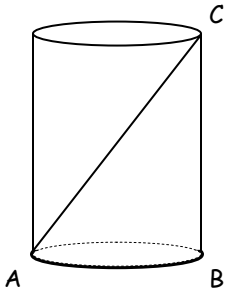
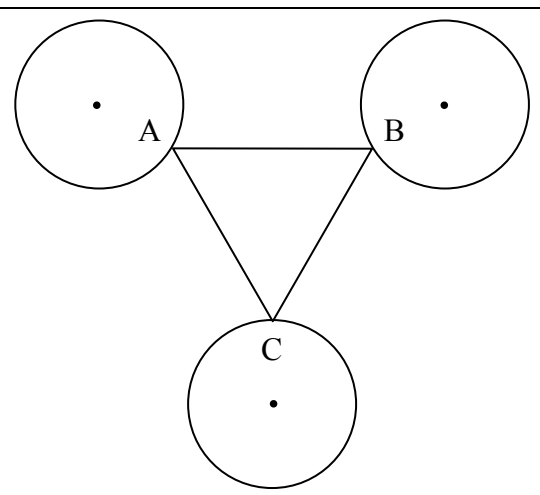
## Challenge Questions - Geometry

31	A sprained ankle swells and doubles in size on the day it is sprained and the following day. Then, with proper icing, the ankle loses twenty-five percent of its size each day thereafter until it is back to its normal size. With proper icing, on which day after a sprain will the injured ankle be back to its normal size? (Note: If the ankle is sprained on Monday, then Tuesday is the 1 <sup>st</sup> day after the sprain.)
32	In how many distinct orders can you arrange three different keys on a plain key ring with no clasp?
33	In the magic square at right, every row, column, and diagonal has the same sum. Five missing values are indicated by letters. Of the four values indicated by a, b, d, and e, list the letters of all that are less than the value of c.

a	175%	$\frac{1}{2}$
b	c	d
2	e	1

34	<p>In the drawing square <math>A'B'C'D'</math> is a dilation of square <math>ABCD</math> and point <math>P</math> is the center of dilation. (A dilation is an enlargement or a reduction of a shape by a scale factor resulting in an image that is similar to the original shape). <math>PD = 3</math> inches, <math>DD' = 15</math> inches and <math>AC = 2\sqrt{2}</math> inches. How many inches are in the length of <math>B'D'</math>?</p> 
----	--

35	A number $n$ , not necessarily an integer, is selected at random such that $-5 < n < 7$ . What is the probability that $2 < n < 9$ ?
36	A curve is described by the equation $x^2 + y^2 = 4$ . What is the slope of the line tangent to this curve at the point $(0, 2)$ ?

37	<p>A right cylindrical can has an 8-inch straw inside it as shown. One end of the straw is resting at point A. Point A and point B are endpoints of the diameter of the circular base of the cylinder. The other end of the straw rests on point C, which is directly above point B. What is the volume of the can if the radius of the can is 1.5 inches? Answer as a common fraction in terms of <math>\pi</math> and in simplest radical form.</p>	
38	<p>The total surface area of a cube is 18 square inches. What is the sum, in inches, of the lengths of all the space diagonals of that cube? (A space diagonal of a cube is a line segment from one corner of the cube to the farthest opposite corner).</p>	
39	<p>One dot is added to one of the faces of a standard cubical die. When this die is rolled twice, the probability that the sum is 6 equals the probability that the sum is 7. What was the number of dots originally on the face to which the extra dot was added?</p>	
40	<p>Three congruent circles, each with a radius of 6 inches, are drawn as shown such that the center of any one of the circles is 18 inches from the centers of each of the other two. Triangle ABC is the smallest possible equilateral triangle that can be drawn such that each of its three vertices is on one of the circles. What is the area of triangle ABC?</p>	

Geometry





# "Math is Cool" Masters - 2009-10

Sponsored by:

8th Grade - December 5, 2009

## Individual Multiple Choice Contest

Itchy Itchy Falls is plagued by a persistent giant mosquito population. Scientists there have classified the four types of mosquito and gathered the information shown below in an effort to understand this issue.

Mosquito Type	Maximum Volume of Blood Held in Gut (mL)	Average Number of Bites per Minute	Probability of disease transmission (after biting)
Hooverus Siphonus	9	5	0.4
Vladimus Rex	5	7	0.6
Hemosucculus	7	13	0.3
Draculus Maximus	3	16	0.2

1	Bug repellent has a 0.7 probability of blocking mosquitoes from biting. If you are attacked by a Hooverus Siphonus mosquito, while wearing bug repellent, what is the probability that it will transmit disease to you? A) 0.42      B) 0.28      C) 0.12      D) 0.18      E) Answer not given
2	Each mosquito collects 1.5 ml of blood with one bite. A Hoovers Siphonus mosquito and a Draculus Maximus mosquito each spend 40 minutes collecting blood. What is the positive difference in the expected number of mL of blood collected by the two mosquitoes? (Don't let the gut size be a limiting factor in this question.) A) 420      B) 90      C) 60      D) 30      E) Answer not given
3	If a Hooverus Siphonus mosquito bites 5 people, what is the probability that it transmits disease to exactly 3 of them? A) $288/3125$ B) $144/625$ C) $432/3125$ D) $216/625$ E) Answer not given
4	Assume that all Hooverus Siphonus mosquitoes and Hemosucculus mosquitoes bite exactly the average number of times per minute, and that you have an endless supply of both types. Then in 1 minute, what is the largest total number of bites that you cannot have? A) 59      B) 23      C) 61      D) 47      E) Answer not given
5	A mosquito is flying in a circle, and the center of Suman's head is the center of the circle. If the mosquito is 20 inches from the center of Suman's head and travels $5\pi$ inches, how many degrees does it travel? A) 90      B) 45      C) 60      D) 30      E) Answer not given
6	A mosquito begins at the top left corner of a $3 \times 7$ grid. It must move to the bottom right corner by moving only down or to the right along the lines of the grid. How many possible paths can the mosquito take? A) 120      B) 21      C) 35      D) 100      E) Answer not given

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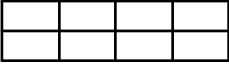
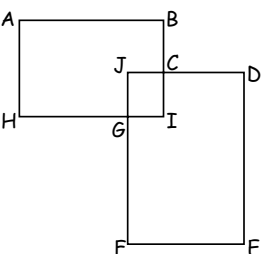
7	<p>A prehistoric mosquito collects 2 ml of blood in the first hour, <math>\frac{4}{3}</math> ml of blood in the second hour, <math>\frac{8}{9}</math> ml of blood in the third hour, and so on. If this process had continued infinitely, how much blood will the mosquito collect?</p> <p>A) 2.5 ml      B) 3 ml      C) 4 ml      D) 6 ml      E) Answer not given</p>
8	<p>The number of the three types of mosquito in a new environment can be expressed by the following system of equations where a, b, and c each represent the number of a certain type of mosquito:</p> $4a + 3b - 2c = 35$ $a - b - c = 1$ $a + 2b + 4c = 35$ <p>Find the total number of mosquitoes <math>a + b + c</math>.</p> <p>A) 34      B) 17      C) 20      D) 10      E) Answer not given</p>
9	<p>The mosquito population at Bug Bite Bog is defined by the equation <math>x^2 - 6x + 9 = 0</math> where x represents the time at any hour between 1 pm and 5 pm. At what hour is the mosquito population 0?</p> <p>A) 5      B) 1      C) 4      D) 3      E) Answer not given</p>

# "Math is Cool" Masters - 2009-10

Sponsored by:

8th Grade - December 5, 2009

Team Contest

1	Helen's last day of school is June 9th. Her birthday is June 25th. She randomly chooses a date in June after school is out. Find the probability (as a reduced fraction) that this date is after her birthday.
2	A pikachu has one head and two legs. A bulbasaur has one head and four legs. In a group of pickachus and bulbasaur having 23 heads and 70 legs, each pikachu greets each bulbasaur exactly once. How many greetings are there?
3	To play the game of Remaindering, divide a positive integer $n$ by each of the nine integers 1 through 9 (round 1). In each case with a non-zero remainder, divide the remainders into $n$ (round 2). Repeat until all remainders are 0, at which point the game is complete. What is the largest 2-digit value of $n$ for which it takes 6 rounds to complete a game of Remaindering?
4	A model of Saturn consists of two parts fitted together. Part A is a circular metal disk of diameter 8 inches, which has a concentric circular hole cut out of its center. Part B is a solid sphere that fits exactly into the cutout hole of part A. The outer diameter of part A is twice the diameter of its cutout hole. Find the number of square inches in the surface area of part B.
5	The average of $A$ and $B$ is $C$ . The average of $C$ and $D$ is $E$ . If $D$ is 3 times $C$ , by what factor should you multiply $E$ to get the sum of $A + B$ ?
6	My new Shrinking Budget Calculator (SBC) treats 1-digit and 2-digit input numbers normally. But if I input a 3-digit integer into the SBC, it randomly discards one of the digits before performing calculations. It then calculates correctly with the shrunken numbers and displays the answers correctly. I enter and add 3 positive integers, one of which is 8. The second number I enter has 2 digits, and the third has 3 digits. All the <u>digits</u> I enter are different. The sum displayed is 113. What is the largest possible number I could have entered?
7	A chocolate bar is scored to break into 8 individual pieces, which are arranged as indicated. If each break is along the lines and through one layer of attached pieces only, what is the smallest number of breaks it could take to break the bar into 8 pieces? 
8	Paul takes the first 10 counting numbers in order (1, 2, ..., 10) and adds the first two, then multiplies by the third, then adds the fourth, then multiplies by the fifth, and so on. Paula does the same thing, except she multiplies first, then adds, then multiplies, and so on. How much greater is Paula's result than Paul's?
9	Chan buys at least one notebook costing \$3.25 each and some pads of paper at \$1.75 each. His total bill (ignoring tax) is \$22.00. What is the largest number of pads of paper Chan could have bought?
10	Find the total number of square units in the area of figure ABCDEFGH, if ABIH, DEFJ, and CIGJ are all similar rectangles, and $CJ = 5$ , $CD = 15$ , $GF = 18$ , and $GH = 13$ units. The figure is not necessarily drawn to scale. 

# "Math is Cool" Masters - 2009-10

Sponsored by:

8th Grade - December 5, 2009

Pressure Round Contest

1	Sam has 6 cups and 6 coins. In how many ways can Sam distribute all his coins into one or more cups? Ignore any differences among cups or coins and consider only the distribution of coins into cups (eg, 5 coins in one cup and 1 in another counts as only one way to distribute the coins no matter which coins or which cups are used).
2	The $n$ students in Math Team could arrange themselves in groups of 4 with nobody left over, but could not arrange themselves in groups of 5, 6, or 7 with nobody left over. How many possible values of $n$ are between 50 and 100?
3	A 4-digit positive integer has all 4 digits the same. Find the average number of positive integer factors of the 9 numbers having this form, and give your answer rounded to the closest integer.
4	The sum of the degree measures of two interior angles of an isosceles trapezoid is $250^\circ$ . What is the least possible number of degrees in any angle of this trapezoid?
5	What is the sum of the solutions to the equation: $22x^2 + 67x - 35 = 0$ ?

# "Math is Cool" Masters - 2009-10

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8th Grade - December 5, 2009

Mental Math Contest

PERSON 1		
1.1	What is the length of the space diagonal of a cube with edges of length four?	$4\sqrt{3}$ [units]
1.2	Convert thirty-four yards to inches.	1224 [inches]
1.3	Determine the slope of the line perpendicular to the line: two X plus seven Y equals eleven.	$7/2$
1.4	What is the sum of three hundred seventy-six plus four hundred twenty-nine?	805
PERSON 2		
2.1	In simplest radical form, what is the geometric mean of three and eighteen?	$3\sqrt{6}$
2.2	Find the length of the altitude to the hypotenuse of a triangle with legs of length of six and eight. Express your answer as a decimal.	4.8
2.3	What is the sum of the first eleven positive integers?	66
2.4	Find the y-intercept of the line that is given by the equation: five X plus eight Y equals twenty-two.	$11/4$ or (0, $11/4$ )
PERSON 3		
3.1	Solve for X: five X minus seven equals two X plus eight.	[X=] 5
3.2	What is the probability of obtaining a sum of four when I roll two fair six-sided dice?	$1/12$
3.3	What is the maximum possible number of intersections that can be made from four distinct circles?	12 [intersections]
3.4	If four apples weigh the same as three bananas, and five bananas weigh the same as two pineapples, then how many apples weigh the same as six pineapples?	20 [apples]
PERSON 4		
4.1	Evaluate: square root of nine times five to the third power.	375
4.2	What is the sum, in degrees, of the interior angles of a pentagon in degrees?	540 [degrees]
4.3	How many different ways can the letters in the word "MATTER" be arranged?	360[ways]
4.4	What is the sum of the two smallest primes that are larger than fifty?	112

# "Math is Cool" Masters - 2009-10

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7<sup>th</sup> & 8<sup>th</sup> Grade - December 5, 2009

## COLLEGE KNOWLEDGE BOWL ROUND #1

#	Problem	Answer
1	What is the probability of rolling a sum of seven with two six-sided dice?	1/6
2	How many factors does 108 have?	12
3	Solve for x: Five times the quantity three x plus two equals twenty-five.	x=1
4	Danny can run twice as fast as Jill. He can run 4 miles in 20 minutes. How long, in minutes, will it take Jill to run 7 miles?	70 [min]
5	Evaluate: two to the tenth power.	1024
6	A coin is flipped 7 times. What is the probability of getting exactly 4 heads?	35/128
7	Jim gets a 68, 100, 83, and 92 on his first four tests. What score must he get on the next test to have an average of 88?	97 [%]
<b>Extra Problem - Only if Needed</b>		
8	If today is Wednesday, what day of the week will it be 22 days from now?	Thursday

# "Math is Cool" Masters - 2009-10

Sponsored by: Takata

7<sup>th</sup> & 8<sup>th</sup> Grade - December 5, 2009

## COLLEGE KNOWLEDGE BOWL ROUND #2

#	Problem	Answer
1	On the farm, Noel saw cows and chickens. If there were 25 heads and 90 feet, how many cows were there?	20 [cows]
2	A phone call cost 11 cents per minute for the first 7 minutes and 6 cents per minute for every minute after that. If Ashley spent one dollar and seven cents on a phone call, how many minutes was she on the phone?	12 [min]
3	The sum of two numbers is 44 and the difference is 12. What is the smaller of the two numbers?	16
4	What is the maximum number of times a triangle and a circle can intersect?	6 [times]
5	Find the mean of the following set: {14,36,33,0,202}.	57
6	The store marked AquaSocks from 30 dollars down to 27 dollars and 60 cents. What is the percent decrease?	8 [%]
7	If you went to sleep at 8:23 p.m. and woke up at 4:43 a.m., how many minutes did you sleep?	500 [min]
	<b>Extra Problem - Only if Needed</b>	
8	Lisa's 2208 quarters are worth how much in dollars?	[\$] 552



# "Math is Cool" Masters - 2009-10

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7<sup>th</sup> & 8<sup>th</sup> Grade - December 5, 2009

## COLLEGE KNOWLEDGE BOWL ROUND #3

#	Problem	Answer
1	If one clown can hold 19 balloons, how many balloons can 14 clowns hold?	266 [balloons]
2	The product of 5 odd numbers is odd, even, or cannot be determined?	Odd
3	What is the sum of the reduced numerator and denominator of seventy-eight over one hundred thirty-two?	35
4	Adam noted the average weight of the first 4 animals was 2000 pounds. The average weight of the next 96 animals was only 100 pounds. What was the average weight of all the animals?	176 [pounds]
5	From a standard deck of cards, what is the probability of drawing a red 8?	1/26
6	Bob's chickens lay 8 eggs a day. How many days will he need to wait in order to have 20 dozen eggs ready for market?	30 [days]
7	A tree has 2 branches when it is 1 year old and it doubles the number of branches it has each year thereafter. How many branches will the tree have when it's 8 years old?	256 [branches]
	<b>Extra Problem - Only if Needed</b>	
8	What is one 8 plus eight 5's?	48

# "Math is Cool" Masters - 2009-10

8th Grade - December 5, 2009

# KEY

School Name \_\_\_\_\_ Team # \_\_\_\_\_

Proctor Name \_\_\_\_\_ Room # \_\_\_\_\_

**STUDENT NAME** \_\_\_\_\_

First Score

(out of 18)

## INDIVIDUAL MULTIPLE CHOICE - 15 minutes

*This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. **Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet. No talking during the test.***

**DO NOT WRITE IN SHADED REGIONS**

	Answer	-1, 0 or 2	-1, 0 or 2
1	C		
2	E (660)		
3	B		
4	D		
5	B		
6	A		
7	D		
8	B		
9	D		

# "Math is Cool" Masters - 2009-10

8th Grade - December 5, 2009

**KEY**

School Name \_\_\_\_\_ Team # \_\_\_\_\_

Proctor Name \_\_\_\_\_ Room # \_\_\_\_\_

First Score

(out of 20)

**STUDENT NAME** \_\_\_\_\_

## Team Contest - Score Sheet

**TEAM TEST - 15 minutes**

*When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as 2 or 0. Record all answers on the colored answer sheet.*

**DO NOT WRITE IN SHADED REGIONS**

	Answer	2 or 0	2 or 0
1	5/21		
2	132 [greetings]		
3	95		
4	$16\pi$ [in <sup>2</sup> ]		
5	1		
6	975		
7	7 [breaks]		
8	8095		
9	7 [pads]		
10	720 [un <sup>2</sup> ]		

"Math is Cool" Masters - 2009-10  
8th Grade - December 5, 2009

KEY

School Name \_\_\_\_\_ Team # \_\_\_\_\_

Proctor Name \_\_\_\_\_ Room # \_\_\_\_\_

**STUDENT NAME** \_\_\_\_\_

**PRESSURE ROUND - 10 minutes**

*When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.*

**Pressure Round Answers**

Answer	
1	11 [ways]
2	6 [values]
3	10 [factors]
4	55 <sup>[°]</sup>
5	-67/22