"Math is Cool" Championships – 1010-11

Sponsored by:

11th & 12th Grade – October 20, 2010

Individual Multiple Choice Contest

1	Evaluate: $2+3\times5-8\div4$. A) $\frac{17}{4}$ B) 15 C) $\frac{-15}{4}$ D) -28 E) Answer not given.
2	What is the arithmetic mean of 3, 5, 9, 12, and 21? A) 3 B) 5 C) 9 D) 10 E) 18
3	How many diagonals can be drawn in a pentagon? A) 5 B) 6 C) 7 D) 10 E) Answer not given.
4	If two sides of a triangle and the angle between them are 4 cm, 5cm, and 60 degrees, respectively, then what is the length of the third side? A) $\sqrt{41}$ B) $\sqrt{21}$ C) 6 D) $\sqrt{5}$ E) Answer not given.
5	How many positive factors does the number 1152 have? A) 18 B) 21 C) 24 D) 27 E) Answer not given.
6	Stacey has a 5-ounce cup, a 3-ounce cup, and a 1 quart jug of water. What is the least number of pours Stacey needs to perform in order to obtain exactly 4 ounces of water in a cup? A) 4 B) 5 C) 6 D) 7 E) 8
7	Four couples are seated around a round table with 9 seats for dinner. If each couple has to sit next to one another, then how many ways are there to arrange the couples? A) 15120 B) 384 C) 120 D) 24 E) Answer not given.
8	What is the thousands digit of 11^{11}? A) 0 B) 2 C) 3 D) 5 E) Answer not given.
9	Three standard six-sided dice are rolled. What is the probability that a sum of 12 is obtained? A) $\frac{1}{126}$ B) $\frac{2}{21}$ C) $\frac{4}{21}$ D) $\frac{25}{216}$ E) Answer not given.
10	Evaluate: $1+\sin(2\theta)$, when $\cos(\theta)=\frac{3}{5}$, θ is an angle in the first quadrant. A) $\frac{7}{5}$ B) 1 C) $\frac{13}{5}$ D) $\frac{11}{5}$ E) Answer not given.

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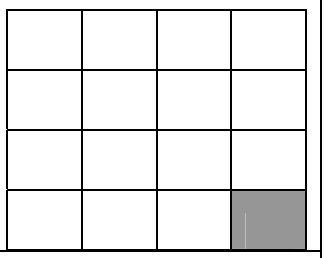
Team Contest

1	In how many ways can the letters in the word "WOOHOO" be arranged?
2	If sin x = 5/13 and x is in the second quadrant, what is tan x?
3	Express the base-six number 543_6 as a base-ten number.
4	What is the sum of the first 34 terms of an arithmetic sequence with a first term of 41 and a common difference of –3?
5	When five lines are drawn in a plane, what is the smallest number of regions that can be created if there must be at least one 90° angle and at least two 40° angles?
6	What is the remainder when 17^{17} is divided by 9?
7	What are the coordinates, in the form (x, y), of the center of the hyperbola with equation $4x^2 - 2y^2 - 16x + 20y = 0$?
8	What is the sum of all positive integer value(s) of n less than 100 such that $\sqrt{n + \sqrt{n + \sqrt{n + \cdots}}}$ will evaluate to an integer?
9	In the following figure, how many rectangles can be drawn that do not contain the shaded square? 
10	If $\sin x + \cos x = a$ then what is $\sin^3 x + \cos^3 x$ in terms of a?

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Pressure Round Contest

1	How many natural numbers less than 388080 are relatively prime to 388080?
2	Your trusted friend is dealt two cards from a standard 52-card deck. He looks at them and tells you that they are not of the same suit. What is the probability that they are a pair?
3	If $257 \equiv j \bmod 34$, what is the largest possible two-digit number that j could be?
4	The low tide levels at my beach are peculiar. On the first day, the low tide was 4 feet and on the second it was 6 feet. After that, each day's low tide was the average of the two previous days. If this continues indefinitely, what will be the level of the low tide?
5	What is the product of the real value(s) of g that satisfy the equation $3^{2g} - 36 \cdot 3^{g} + 243 = 0$?

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Mental Math Contest

PERSON 1		
1.1	Solve for x if four x plus thirteen equals fifty-seven.	11
1.2	What is the perimeter of a rectangle with length four and width seven halves?	15
1.3	What is the log base eight of thirty-two?	5/3
1.4	What is eighty-six times one hundred fourteen?	9804
PERSON 2		
2.1	What is the probability of getting all heads on seven flips of a fair coin?	1/128
2.2	What is twenty-six squared?	676
2.3	If the sine of x equals one-third, what is the sine of the angle x plus one-hundred eighty degrees?	-1/3
2.4	What is the sum of the arithmetic sequence, forty, thirty-six, thirty-two, and so on down to eight?	216
PERSON 3		
3.1	Is it possible to add two prime numbers and have the sum also be a prime number – yes or no?	Yes
3.2	What is the area of a rectangle with length four and one diagonal equal to five?	12
3.3	What is the sum of the numbers in the geometric series: one three, nine, twenty-seven and eighty-one?	121
3.4	In how many ways can three different math books, a physics book, a history book and a french book be arranged on a shelf so that the math books are kept together?	144
PERSON 4		
4.1	What is the slope of the line connecting the point four comma two and the point ten comma four?	1/3
4.2	How many ways can I interchange the letters in the word comma, C-O-M-M-A?	60
4.3	The log base 4 of x is equal to negative one-half, what is x?	1/2
4.4	Twin primes are two prime numbers separated by two such as seventy-one and seventy-three. What is the next set of twin primes larger than seventy-one and seventy-three?	101, 103

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COLLEGE KNOWLEDGE BOWL ROUND #1 - SET 1

#	Problem	Answer
1	Expressed as a decimal, what is 15 percent of 15?	2.25
2	Evaluate as a decimal: 2.6 times 4.12	10.712
3	What is the average of the values of x that satisfy: 2 x-squared minus 4 x + 21 equals 0?	1
4	If 321 days ago was a Sunday, then what is the day before two days after tomorrow?	Monday
5	What is the sine of negative pi over 6?	-1/2
6	If Katie tricycles at one foot per second for one **minute**, and at ten feet per second for thirty seconds, what is her average speed in feet per second?	4 [fps]
7	If James wins at game night, he is so excited that he has a three-fourths probability of giving his class extra credit the next day. Otherwise he has only a one-fifth chance of giving extra credit. The probability that James wins at game night tonight is one-third; what is the probability he gives extra credit tomorrow?	23/60
8	If the log base a of b is c and the log base a of c is d, express the log base b of c without logarithms?	d/c
9	What is the distance between the foci of the hyperbola the quantity x minus three squared over 16 minus the quantity y minus six squared over 9?	10
10	The 10 and all face cards are worth 10, an ace is worth 11 and other cards have their face value. If two cards are dealt from a standard 52-card deck, what is the probability of getting a total of 21?	32/663

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COLLEGE KNOWLEDGE BOWL ROUND #2 - SET 2

#	Problem	Answer
1	In what quadrant of the Cartesian plane does the point 17 comma negative 4 lie?	Fourth
2	Evaluate: 3 to the fifth power.	243
3	The log base 10 of an integer, x, is four point seven seven eight to three decimal places. How many digits are there in x?	5
4	Evaluate: Eighty-three squared minus seventy-seven squared.	960
5	If x is between 0 and 2 pi, how many values of x satisfy, sine squared of x equals one-half?	4
6	My local library has 1432 books in the science fiction section. If 838 involve space travel, 682 involve time travel, 419 involve dimensional travel, what is the most number of books that involve none of these types of travel?	594 [books]
7	I have a bet with my friends. From a standard deck I will draw one card. If it's red or a King I will get $21, otherwise I will pay $18. How many dollars should I expect to make? Assume a negative number signifies a loss.	3 [$]
8	Find the coefficient of the x to the fourth power in the product of the quantity x cubed plus 2 x squared plus 3 x plus 4 times the quantity 4 x cubed plus 3 x squared plus 2 x plus 1.	20
9	Find the determinant of a matrix whose first row is one, five, seven, the second row is two, five, nine, and the third row is six, four, six.	50
10	On a road trip, a group of friends pool their money to pay for a hotel room. If there had been one more friend, the price per person would have been two dollars less. If there had been two fewer friends, the price per person would have been six dollars more. What was the total price of the hotel room, in dollars?	[$] 144

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COLLEGE KNOWLEDGE BOWL ROUND #3 - SET 3

#	Problem	Answer
1	What is the slope of a line perpendicular to the line 8 x plus 2 y equals 35?	[m=] 1/4
2	What are the coordinates, in the form x comma y, of the point of intersection of the lines 3 x minus 5 y equals 2, and 2 x minus 3 y equals 1?	(-1,-1)
3	What is the sine squared of 30 degrees plus the sine squared of 60 degrees?	1
4	If 5 beetles are equivalent to 3 caterpillars and 15 dragonflies are equivalent to 2 beetles, how many caterpillars would be equivalent to 100 dragonflies?	8 [caterpillars]
5	The sine of a given angle in the first quadrant is 0 point 6. As a decimal, what is the cosine of that angle?	0.8
6	Two sides of a triangle are 6 and 4. Find the product of all possible integer values for the third side.	181440
7	During the first six tests of the year I got 97, 80, 95, 83, 89, and 88. Assuming all tests are weighted equally and I really want to average at least a 90 in my class, what is the minimum score I need on my seventh test to reach my goal?	98
8	Find the area of a polygon whose vertices are at the points 6 comma 0 and 0 comma negative 3 and negative 1 comma 0 and 0 comma 5.	28 [units squared]
9	What is the limit as x goes to infinity of the quantity 5 x squared plus 6 x minus 3 over the quantity 2 x squared minus 3 x plus 6?	5/2
10	Dave and Donna wanted to meet for coffee, but forgot a specific time, and just said sometime between 3 and 4. If they both arrive sometime during that frame and wait for 4 minutes, what is the probability they will be there at the same time?	29/225

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COLLEGE KNOWLEDGE BOWL ROUND #4 - SET 4

#	Problem	Answer
1	What is the area, in square centimeters, of a circle with a circumference measuring 26 pi?	169pi [cm^2]
2	A polygon with an area of 18 square centimeters and a perimeter of 12 centimeters is similar to a polygon with a perimeter of 30 centimeters. What is the other polygon's area, in square centimeters?	225/2 [cm^2]
3	How many ordered pairs of positive integers, x comma y, are there such that 2 x plus 3 y equals 50?	8
4	What is the product of the solutions to the equation: 3 x squared plus 23 x minus 8 equals 1?	-3
5	What is the harmonic mean of one-fourth and one-sixth?	1/5
6	Find the mean of the number 8, 12, 5, 5, 3, 4, 7, and 12.	7
7	What is the remainder when the quantity 2 x to the fourth plus 2 x cubed minus 3 x plus 7 is divided by the quantity x minus 2?	49
8	When x is added to the numerator and denominator of 6 over 11, the result is 2 over 3. What is x?	4
9	The coordinates of a point in polar coordinates is 5 comma pi over six radians. What are the rectangular coordinates of this point?	5 root3 over 2 comma 5 over 2
10	What is the product of 1/3 times 2/4 times 3/5 etc. up to 2008/2010?	1/2019045

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COLLEGE KNOWLEDGE BOWL ROUND #5 - SET 5

#	Problem	Answer
1	What is the sum of the integers from negative 5 to 8 inclusive?	21
2	What is the equation, in slope-intercept form, y equals m x plus b, of the line through the points three comma five and five comma negative one?	Y=-3x+14
3	In a triangle with sides 8, 15, and 17; what is the sine of the angle opposite the side of length 8?	8 / 17
4	Find the least common multiple of 231 and 280.	9240
5	If f of x equals 3 x plus 10 and g of x equals x squared minus 2, then what is f of g of 5?	79
6	Mia and Harold are walking to class. Since Mia walks faster than Harold, Harold gets a 10 second head start. The distance to the classroom is 168 feet. If they arrive there at the same time 34 seconds after Harold starts, what is Mia's walking speed in feet per second?	7 [ft per sec]
7	What is the sum of the first 12 positive perfect squares?	650
8	How many terms are there in the arithmetic sequence: 7, 20, 33 and so on until 228?	18
9	What is log base 3 of 4 times the log base 2 of 25 times log base 5 of 27?	12
10	In a triangle with sides measuring 3, 5, and 6 cm, how many centimeters long is the median to the 6 cm side?	2 times the square root of 2 [cm] or $2\sqrt{2}$

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COLLEGE KNOWLEDGE BOWL ROUND #6 - SET 6

#	Problem	Answer
1	What is the surface area of a sphere of radius 7?	196 pi [un^2]
2	What is the log base 3 of the quantity 729 squared?	12
3	What is the sum of the digits of 11 raised to the fourth power?	16
4	In a new country, they currently only have 7 cent and 17 cent coins. What is the largest number of cents for which they can't make exact change?	95 [cents]
5	If the log base 10 of 2 is zero point three zero one, then what is the log base 10 of 50 as a decimal?	1.699
6	How many numbers between 10 and 100 are multiples of 3 and/or 8?	37
7	In how many ways can you arrange the letters in the word "Statistics" spelled S-T-A-T-I-S-T-I-C-S?	50,400
8	A sphere has a radius of 5 inches, a cube has an edge length of 7 inches, and a cylinder has a base radius of 3 inches and height 9 inches. What is the ratio of the smallest of the three volumes to the largest of the three volumes?	243/500 or 243 to 500
9	What is the equation of the directrix of the parabola with equation: 4 y equals x squared minus 2 x plus 5?	y = 0
10	The following equation is a perfect cube of a linear polynomial: x cubed plus 3 k times x squared minus 9 k times x minus 27. What is k plus 4?	1

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COLLEGE KNOWLEDGE BOWL ROUND - EXTRA

#	Problem	Answer
1	Find the sum of all two digit prime numbers whose positive difference between the digits is exactly 2.	273
2	What is the slant height of a cone whose base radius is 9 and whose volume is one-thousand eighty pi?	41
3	A triangle is circumscribed about a circle of radius 7. If the sides of the triangle are 5, 6, and 7, what is the triangle's area?	63 [un^2]
4	Sheila is trying out a new grocery store. In this store, 40% of the apples have a worm. If Sheila randomly chooses 3 apples out of a large bucket, what is the probability no more than one of them is bad? Round to the nearest percent.	65 [%]

Math is Cool" Championships – 1010-11

11th & 12th Grade – October 20, 2010

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

Final Score:

KEY

First Score

(out of 20)

INDIVIDUAL MULTIPLE CHOICE - 15 minutes – 20% of team score

This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. ***Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet.*** *No talking during the test.*

DO NOT WRITE IN SHADED REGIONS

Answer		-1, 0 or 2	-1, 0 or 2
1	B		
2	D		
3	A		
4	B		
5	C		
6	C		
7	B		
8	A		
9	D		
10	E (49/25)		

"Math is Cool" Championships – 1010-11

11th & 12th Grade – October 20, 2010

Final Score:
KEY

School Name_____Team #_____

Proctor Name_____Room #_____

First Score

(out of 10)

STUDENT NAME_____

Team Contest – Score Sheet

TEAM TEST - 15 minutes – 30% of team score

*When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as **1 or 0**. Record all answers on the colored answer sheet.*

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	30		
2	-5/12		
3	$207_{[10]}$		
4	-289		
5	12		
6	8		
7	(2,5)		
8	330		
9	84		
10	$\frac{3a-a^3}{2}$		

"Math is Cool" Championships – 1010-11

11th & 12th Grade – October 20, 2010

Final Score:
KEY

First Score

School Name_____Team #_____

Proctor Name_____Room #_____

STUDENT NAME_____

PRESSURE ROUND - 10 minutes – 15% of team score

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

Pressure Round Answers

	Answer
1	80640
2	1/13
3	87
4	16/3 [ft]
5	6