# "Math is Cool" Masters – 2010-11
## 8th Grade – December 11, 2010
## Individual Multiple Choice Contest



Temperature of Liquid
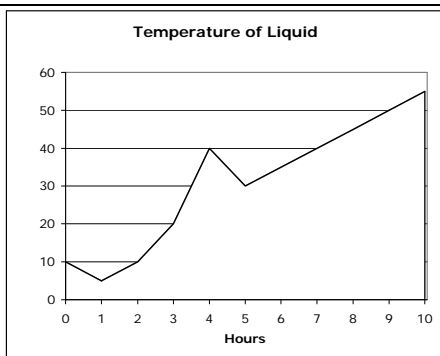
**FOR QUESTIONS 1, 2 and 3, please refer to the following chart:**

The graph at left records the temperature of a strange liquid over a ten-hour time period. After exactly one hour, the temperature of the liquid was 5°C.

**1** At which time below was the temperature increasing at the fastest rate?
A) 0.5 hours    B) 3.5 hours    C) 4.5 hours    D) 5.5 hours    E) 8.5 hours

**2** Sampson noticed a trend develop starting at hour 5 and continuing until the end of the 10th hour. If this trend had continued and Sampson had measured the temperature at the end of the 15th hour, what temperature would he have measured?
A) 55 °C    B) 75 °C    C) 80 °C    D) 85 °C    E) 100 °C

**3** What was the average temperature of the liquid in degrees C for the first four hours of the experiment?
A) 12.5 °C    B) 15 °C    C) 17.5 °C    D) 20 °C    E) Answer not given
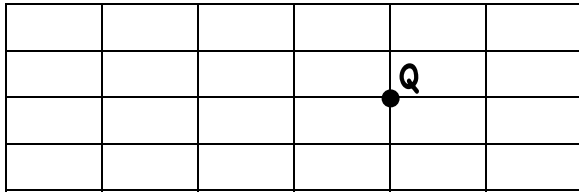
## USE THIS TABLE FOR PROBLEMS 4, 5, & 6

| Die name | P(1) | P(2) | P(3) | P(4) | P(5) | P(6) |
|---|---|---|---|---|---|---|
| Die A | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 |
| Die B | 1/3 | 0 | 1/6 | 1/6 | 0 | 1/3 |
| Die C | 0 | 0 | 1/2 | 1/2 | 0 | 0 |
| Die D | 0 | 1/3 | 1/6 | 1/6 | 1/3 | 0 |

Delilah has four cubical dice labeled A, B. C, and D. All are equally likely to land on any side. However, some have multiple sides with the same number (and lack some numbers all together). The table above shows the probability of rolling a specific number with a die, where P(x) is defined to be the probability of rolling a number x. (For example: the probability of rolling a 2 with die D is 1/3.)

**4** Delilah rolls die A and die B. What is the probability she rolls "snake eyes" (that is, both dice show the number 1)?
A) 1/18    B) 1/36    C) 1/9    D) 0    E) Answer not given

**5** Delilah rolls each die two times and adds the values of the two rolls. Which of the four dice have the same probability of rolling a sum of seven?
A) A and B    B) A,B, and C    C) B,C, and D    D) B and D    E) A, B, C, and D

**6** Delilah rolls die A and die B at the same time. What is the probability that the number showing on die A is larger than the number showing on die B?
A) 0    B) 5/12    C) 1/2    D) 5/6    E) Answer not given

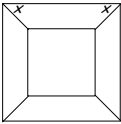**For problems 7, 8, and 9, consider the following diagram of Bennett's floor (figure not necessarily drawn to scale):**



1 ft

Bennett's floor is composed of identical 1 ft x 6 in tiles as laid out above.

| 7 | While Bennett waits for his coffee to brew, he passes time by counting the number of squares he can see on his floor. How many squares can Bennett see?<br>A) 0      B) 15      C) 23      D) 24      E) 38 |
|---|---|
| 8 | What is the maximum number of tiles a single straight line drawn on Bennett's floor could intersect?<br>A) 6      B) 7      C) 8      D) 9      E) Answer not given |
| 9 | An ant starts on the upper left corner of Bennett's floor and walks towards the lower right hand corner of the floor. Only traveling down or right, how many ways can the ant reach the lower right hand corner given that he travels only on the gridlines and passes through point Q?<br>A) 50      B) 90      C) 100      D) 102      E) 210 |
| 10 | In the figure below, a smaller circle and a larger circle are externally tangent to each other and to line j at the same point. Additionally, each circle is tangent to line k, as shown. What is the ratio of the area of the larger circle to the area of the smaller circle?<br><br>A) 6:1      B) 7:1<br><br>C) 8:1      D) 9:1<br><br>E) 10:1 |

# "Math is Cool" Masters – 2010-11
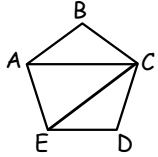
## 8th Grade – December 11, 2010
## Team Contest

| 1 | Simplify $\dfrac{2}{3+\dfrac{2}{5+\dfrac{1}{2}}}$. Your answer should be a reduced common fraction. |
|---|---|
| 2 | Eight times my number is less than 500, but 18 times my number is more than 1000.  What is the median of all possible integer values of my number? |
| 3 | A photocopy shop sells pre-paid cards to use in their self-service copy machines.  Each page copied deducts 8¢ for a copy on letter-size paper or 11¢ for a copy on legal-size paper.  A card has the value of $2.00 and is used up exactly.  Find the largest number of copies that could have been made, given that at least one copy is legal-sized. |
| 4 | When $p$ and $q$ are both prime numbers and $q-p=10$, we will call this pair of prime numbers a "ten-away prime pair". How many ten-away prime pairs exist with $q<100$? |
| 5 | The outer square shown in the figure (not to scale) has an area of 12.25 square cm. The four trapezoids are congruent, with altitude 1.5 cm and $\angle x = 45°$. Find the number of cm in the perimeter of the inner square. |
| 6 | Bob borrows $40,000 (called the "principal") at 4.5% annual simple interest.  Bob pays the lender $200 per month. During the first year of the loan, by how many dollars is the principal of the loan reduced? (NOTE: When a loan is repaid, the borrower first must pay the interest due before any money is applied toward reducing the principal.) |
| 7 | How many right triangles with integer side lengths and hypotenuse less than 100 units have an even number of sides whose lengths are even numbers? |
| 8 | In the equations below, each different letter stands for a different digit, and a given letter always stands for the same digit.  What number does $ebc$ stand for?  $$a+b+c=d \qquad b+d=e \qquad ab\overline{)ebc}^{\,dd}$$ |
| 9 | Today's date can be written as 12/11/10. This is a set of descending consecutive integers. The next such date will be February 1, 2100 (02/01/00). In how many days from today will it be February 1, 2100? Remember leap years have 366 days, as opposed to the regular 365 days. Leap years occur on years that are multiples of four, except years that are centuries that are not multiples of 400. |
| 10 | A clock time can be expressed either as "$M$ minutes after $C$ o'clock" or as "$N$ minutes before $D$ o'clock", where $M,N,C,D$ are all integers, $0 \le M,N \le 60$, and $0 < C,D < 13$. (For example, 7:15 can be expressed as 15 minutes after 7 o'clock or as 45 minutes before 8 o'clock.) For how many times in the hour between 2 PM and 3 PM is the sum of the digits of $M$ and the digits of $C$ equal to the sum of the digits of $N$ and the digits of $D$? |

# "Math is Cool" Masters – 2010-11

## 8th Grade – December 11, 2010
## Pressure Round Contest

| 1 | Polygon ABCDE is a regular pentagon.  What is the degree measure of angle ACE?  |
|---|---|
| 2 | I am thinking of a 3-digit counting number with all its digits different. Exactly one of its digits is in both the sum of 7798 and 1255 and the difference between 7798 and 1255.  Exactly one of its digits is in both the square of 87 and the square root of 5476.  One of its digits is in the majority of the first 7 factorials (ie, 1!, 2!, …, 7!) when they are multiplied out.  What is the largest number I could be thinking of? |
| 3 | The vertices of a rectangle have the following $(x,y)$ coordinates: $(7,-4)$, $(-5,12)$, $(7,12)$, and $(-5,-4)$.  Give the coordinates, in $(x,y)$ form, of the point of intersection of the diagonals of this rectangle. |
| 4 | The average of 14 different integers is 5. The largest of the 14 integers is 20, and their range is 25. At least $n$ of the 14 integers MUST be negative. What is $n$? |
| 5 | Using exactly 2010 unit cubes, I build a solid rectangular prism (box-shape) that is at least two units on each edge. How many different prisms could I have built?  (To be different, the set of edge lengths must be different. Two congruent prisms that are oriented differently are not different.) |

# "Math is Cool" Masters – 2010-11

## 8th Grade – December 11, 2010
## Mental Math Contest

| | | |
|---|---|---|
| **PERSON 1** | | |
| 1.1 | What is the volume in cubic feet of a cone with a radius of two feet and height of three feet? | $4\pi$ [ft$^3$] |
| 1.2 | Evaluate: thirteen times twelve. | 156 |
| 1.3 | What is the probability of drawing either a red ace or a ten from a standard fifty-two card deck? | 3/26 |
| 1.4 | What is the area in square inches of an isosceles right triangle with hypotenuse of length six inches? | 18 [in$^2$] |
| **PERSON 2** | | |
| 2.1 | What is the mean of the following set of numbers: three, three, four, five, five, six, three, six, ten? | 5 |
| 2.2 | What is the least common multiple of 4, 6, and 8? | 24 |
| 2.3 | For what radius will the numerical value of a circle's area and its circumference be equal, disregarding units? | 2 [units] |
| 2.4 | What is the largest area in square feet that can be encompassed by four yards of rope? | 36/$\pi$ [ft$^2$] |
| **PERSON 3** | | |
| 3.1 | What is the positive difference between three to the third and two to the fifth? | 5 |
| 3.2 | Five friends all shake hands with each other once. How many total handshakes occur? | 10 [shakes] |
| 3.3 | What is the probability of obtaining only two heads on three flips of a fair coin? | 3/8 |
| 3.4 | A wheel of radius seven inches is rolling at twenty revolutions per minute along the ground. How far, in feet, does the wheel travel in half an hour? | $700\pi$ [ft] |
| **PERSON 4** | | |
| 4.1 | What is the complement of twenty-five degrees? | 65 [deg] |
| 4.2 | What is eleven to the third power? | 1331 |
| 4.3 | What is the value of x if three x minus eleven equals five? | 16/3 |
| 4.4 | Two concentric circles have radii thirteen feet and five feet. What is the area in square feet of the space outside the circle of radius five feet and inside the circle of radius thirteen feet? | $144\pi$ [ft$^2$] |

# "Math is Cool" Masters – 1010-11

## 8th Grade – December 11, 2010

## COLLEGE KNOWLEDGE BOWL ROUND #1 – SET 1

| # | Problem | Answer |
|---|---------|--------|
| 1 | What is the least common multiple of ten, eleven, and twelve? | 660 |
| 2 | What is the area in square inches of an equilateral triangle with sides of length five inches? | $\frac{25\sqrt{3}}{4}$ [square inches] |
| 3 | What are the odds in favor of obtaining a five in one roll of a fair cubical die? Express your answer as a ratio in the form a to b. | 1 to 5 |
| 4 | If three times my favorite number is equal to seven minus twice my favorite number, then what is my favorite number? | 7/5 |
| 5 | Evaluate: Seven hundred thirty-five times six hundred seventy-five. | 496125 [four hundred ninety-six thousand one hundred twenty-five] |
| 6 | What is the point of intersection of the lines: x equals four and y equals two x minus three? Express your answer as a coordinate pair. | (4,5) [four comma five] |
| 7 | What is the volume in cubic meters of a cone with diameter eight meters and height of six meters? | $32\pi$ [cubic meters] |
| 8 | Let y equal seven minus x plus x squared. What is the value of y when x equals eleven? | 117 |
| 9 | What is the probability of obtaining two heads on four flips of a fair coin? | 3/8 |
| 10 | How many positive integers under one thousand have an odd number of positive integer factors? | 31 [integers] |

# "Math is Cool" Masters – 1010-11

## 8th Grade – December 11, 2010

## COLLEGE KNOWLEDGE BOWL ROUND #2 - SET 2

| # | Problem | Answer |
|---|---------|--------|
| 1 | What is the probability of obtaining a sum of eight or nine on a roll of two dice? Express your answer as a fraction. | 1/4 |
| 2 | What is the perimeter in meters of a rhombus with sides of length one-half meter? | 2 [meters] |
| 3 | What is the y-intercept of y minus four equals seven x plus one? Express your answer as a coordinate pair. | (0,5) |
| 4 | Seven is fifty percent of twenty-five percent of one hundred seventy-five percent of what number? | 32 |
| 5 | What is the quotient of the largest factor and the fourth largest factor of seven hundred twenty? | 4 |
| 6 | Convert the following number from base two to base four: one-zero-one-zero-one. | 111 [base 4] |
| 7 | What is the area in square centimeters of a trapezoid inscribed in a semicircle of radius two centimeters, if three sides of the trapezoid are congruent to each other and the fourth side is the diameter of the semicircle? | $3\sqrt{3}$ [cm$^2$] |
| 8 | How many positive integers less than one hundred have at least one odd positive integer as one of their factors? | 99 [integers] |
| 9 | Evaluate: Seventy-five squared minus twenty-five squared | 5000 |
| 10 | A square is inscribed in a circle of radius two centimeters. What is the probability of selecting a point within the circle that also is within the square? | $\dfrac{2}{\pi}$ |

## COLLEGE KNOWLEDGE BOWL ROUND #3 - SET 3

| # | Problem | Answer |
|---|---------|--------|
| 1 | What is the length in feet of the diagonal of a rectangle with sides of length nine feet and forty feet? | 41 [ft] |
| 2 | What is the greatest common factor of thirty-six, forty-two, and fifty-four? | 6 |
| 3 | If eight spiders are equivalent to five beetles and fifteen dragonflies are equivalent to two beetles, how many spiders are equivalent to seventy-five dragonflies? | 16 [arachnids] |
| 4 | What is the probability of obtaining an even number when randomly selecting a positive integer less than one hundred? Express your answer as a fraction. | 49/99 |
| 5 | If f of x is defined as x squared plus x minus two, then what is f of five? | 28 |
| 6 | What is the sum of the seven smallest positive perfect squares? | 140 |
| 7 | What is the positive difference between the product and the sum of eighteen and twenty-four? | 390 |
| 8 | A wheel of diameter sixteen inches rolls at a speed of fourteen revolutions a minute for fifteen minutes. How far in inches does the wheel travel? | $3360\pi$ [inches] |
| 9 | A circle centered at three comma four contains the point seven comma seven. What is the area of this circle in square units? | $25\pi$ [un$^2$] |
| 10 | Gerry has 60 feet of fencing and he wants to enclose the largest possible rectangular area, while using his house as one side of the rectangle. What is the greatest area in square feet that he can enclose? | 450 [ft$^2$] |

# "Math is Cool" Masters – 1010-11

## 8th Grade – December 11, 2010

## COLLEGE KNOWLEDGE BOWL ROUND #4 – SET 4

| # | Problem | Answer |
|---|---------|--------|
| 1 | If x squared plus y squared equals one, then what is the largest possible value of x? | 1 |
| 2 | What is the area in square units of the region defined by the axes and the lines: y equals eight and x equals seven? | 56 [un$^2$] |
| 3 | I have a bag of twenty red balls and ten green balls. What is the probability that in two draws without replacement, you get two green balls from this bag? Express your answer as a fraction. | 3/29 |
| 4 | What is the sum of the factors of twenty-eight? | 56 |
| 5 | What is the area in square units of a circle with diameter six y units? | 9y$^2$π or 9π y$^2$ [un$^2$] |
| 6 | What is the area in square yards of a one hundred thirty-five degree sector of a circle of radius twelve yards? | 54π [yd$^2$] |
| 7 | Stacey and Bertha each write down a positive one-digit integer on a piece of paper. If they reveal their numbers and multiply them together, what is the probability that their product is even? | 56/81 |
| 8 | What time is just as long after 5:58 PM as 1:32 PM is after 9:16 AM? | 10:14 PM |
| 9 | How many prime factors does six thousand six have? | 5 [primes] |
| 10 | If x plus y equals one and x squared plus y squared equals seventeen-ninths, then what is the value of x squared times y squared? | 16/81 |

# "Math is Cool" Masters – 1010-11

## 8th Grade – December 11, 2010

## COLLEGE KNOWLEDGE BOWL ROUND #5 - SET 5

| # | Problem | Answer |
|---|---------|--------|
| 1 | If a bacterium doubles its population every ten minutes, then how many minutes would three bacteria take to reach a population of at least forty-eight bacteria? | 40 [min] |
| 2 | What is the distance in units from the point three comma seven to the origin? | $\sqrt{58}$ [units] |
| 3 | How many factors does one hundred ninety-eight have? | 12 [factors] |
| 4 | What is the surface area in square decimeters of a cube with edge length of square root three decimeters? | 18 [dm$^2$] |
| 5 | If zero and one are the zeroth and first terms, respectively, in the Fibonacci sequence, then what is the sixth number in this sequence? | 8 |
| 6 | What is the probability of drawing a spade or a king from a standard fifty-two card deck? | 4/13 |
| 7 | What is the digit in the hundreds place of the product of the three smallest two digit prime numbers? | 4 |
| 8 | What is the sum of the coefficients of the terms in the expansion of the quantity three x plus one to the third power? | 64 |
| 9 | A string is wrapped tautly around two externally tangent right cylinders of radius three centimeters. How many centimeters are in the shortest possible length of the string? | $6\pi + 12$ [cm] |
| 10 | What is the reciprocal of the sum of reciprocals of three, four, and five? | 60/47 |

# "Math is Cool" Masters – 1010-11

## 8th Grade – December 11, 2010

## COLLEGE KNOWLEDGE BOWL ROUND #6 – SET 6

| # | Problem | Answer |
|---|---------|--------|
| 1 | What is the sum of the first 10 counting numbers? | 55 |
| 2 | What is the sum of the first three digits to the right of the decimal in the decimal expansion of four ninths? | 12 |
| 3 | What is the area in square feet of a right triangle with hypotenuse of thirteen feet and integer side lengths? | 30 [ft$^2$] |
| 4 | What is the number of distinct dollar amounts of change I can make with one quarter, two dimes, and three nickels? Include zero cents as one amount. | 13 [amounts] |
| 5 | If I flip a fair coin and roll a die, what is the probability of obtaining a head on the coin and two on the die? | 1/12 |
| 6 | What is the units digit of seven factorial? | 0 |
| 7 | If the first three triangular numbers are one, three, and six, then what is the sixth triangular number? | 21 |
| 8 | What is the volume in cubic meters of a cube with a space diagonal of length square root of six meters? | $2\sqrt{2}$ [m$^3$] |
| 9 | What is the product of the mean, median, and mode of the following set of numbers: two, seven, two, eleven, nine, fifteen, thirty-one? | 198 |
| 10 | How many integer coordinate pairs represent points that are within three units of the origin? | 29 [coordinate pairs] |

## COLLEGE KNOWLEDGE BOWL ROUND - EXTRA

| # | Problem | Answer |
|---|---------|--------|
| 1 | Let R be the region bounded by the x-axis, the graph of y equal the absolute value of x, and the two vertical lines x equals negative four and x equals nine. What is the area of R, in square units? | $\frac{97}{2}$ [square units] |
| 2 | What is the number of ways can you rearrange the letters in the word MIDDLE, spelled M-I-D-D-L-E? | 360 [ways] |
| 3 | What is the volume in cubic centimeters of a solid cylinder of radius four centimeters and height of three centimeters, with a cone of the same dimensions cut out of it? | $32\pi$ [cubic centimeters] |

Extra

# "Math is Cool" Masters – 2010-11
## 8th Grade – December 11, 2010

<div style="border:1px solid">KEY</div>

School Name_____Team #_____

Proctor Name_____Room #_____

**STUDENT NAME_____**

**INDIVIDUAL MULTIPLE CHOICE - 15 minutes – 20% of team score**

*This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response.  It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. **Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet**. No talking during the test.*

## DO NOT WRITE IN SHADED REGIONS

| | Answer | -1, 0 or 2 | -1, 0 or 2 |
|---|---|---|---|
| 1 | B | | |
| 2 | C | | |
| 3 | B | | |
| 4 | A | | |
| 5 | D | | |
| 6 | B | | |
| 7 | C | | |
| 8 | D | | |
| 9 | B | | |
| 10 | D | | |
| | | | |

# "Math is Cool" Masters – 2010-11

## 8th Grade – December 11, 2010

School Name_____Team #_____

Proctor Name_____Room #_____

## STUDENT NAME_____

First Score

(out of 10)

### Team Contest – Score Sheet

**TEAM TEST - 15 minutes – 30% of team score**

*When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as* **1 or 0**. *Record all answers on the colored answer sheet.*

### DO NOT WRITE IN SHADED REGIONS

| | Answer | 1 or 0 | 1 or 0 |
|---|---|---|---|
| 1 | $\frac{22}{37}$ | | |
| 2 | 59 | | |
| 3 | 22 [copies] | | |
| 4 | 10 [pairs] | | |
| 5 | 2 [cm] | | |
| 6 | [$] 600 | | |
| 7 | 0 [triangles] | | |
| 8 | 924 | | |
| 9 | 32559 [days] | | |
| 10 | 6 [times] | | |
| | | | |

# "Math is Cool" Masters – 2010-11
## 8th Grade – December 11, 2010

School Name_____Team #_____

Proctor Name_____Room #_____

## STUDENT NAME_____

**PRESSURE ROUND - 10 minutes – 15% of team score**

*When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.*

## Pressure Round Answers

| | Answer |
|---|---|
| 1 | 36 [°] |
| 2 | 752 |
| 3 | (1,4) |
| 4 | 2 |
| 5 | 6 [prisms] |