Sponsored by: Pemco Foundation October 20, 2010 High School Individual Contest

Tear this sheet off and fill out top of answer sheet on following page prior to the start of the test.

GENERAL INSTRUCTIONS applying to all tests:

- Good sportsmanship is expected throughout the competition by <u>all</u> involved.
 Bad sportsmanship may result in disqualification.
- Calculators or any other aids may not be used on any portion of this contest.
- Unless stated otherwise:
 For problems dealing with money, a decimal answer should be given.
 Express all rational, non-integer answers as reduced common fractions.
- All radicals must be simplified and all denominators must be rationalized.
- Units are not necessary unless it is a problem that deals with time and in that
- case, a.m. or p.m. is needed. However, if you choose to use units, they must be correct.
 - Leave all answers in terms of π where applicable.
 - Do not round any answers unless stated otherwise.
 - Record all answers on the colored cover sheets in the answer column only.
 - Make sure all answer sheets have all the information at the top of the sheet filled out.
 - Tests will be scored as a 0 if answers are not recorded on the answer sheets.
 - Blank answer sheets and answer sheets with no name will also be scored as a 0.

INDIVIDUAL TEST - 35 minutes

When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. The raw score will be 2 points for correct answers to problems 1-30 and 3 points for 31-40. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.

Sponsored by: Pemco Foundation October 20, 2010 High School Individual Contest

	Questions 1-30: 2 points each
1	Find the sum of 12789, 32598 and 11090.
2	Find the product of $\frac{24}{35}$ and $\frac{56}{48}$.
3	Find the distance between the points (-2, 5) and (10, -11).
4	Evaluate: $3x^2 + 5c - 2d^3$ when x = 4, c = -3 and d = 2.
5	Solve for x: 5x + 2(3x - 5) + 1 = 11 - 3x
6	What is the slope of the line 5x-2y = 11
7	What is the volume of a right circular cylinder with a radius 7 and a height of 11?
8	What are the values of x that satisfy: $ x + 3 = 19$
9	What is the mean of the following set of data: {2, 15, 17, 32, 15, 7, 3}
10	Find the sum of $\frac{x}{3}$ and $\frac{x-3}{2}$
11	What is the total surface area of a rectangular box with side lengths of 4 feet, 7 feet and 11 feet?
12	There are 40 teams competing at the Math is Cool contest today. How many ways can 1 st and 2 nd place be awarded?
13	Harvey's house of horrors is selling calculators that give incorrect answers. The calculators were selling for \$35.00 each. Since they were such big sellers they raised the price by 20%. What is the new selling price?
14	Factor $x^2 - 3x - 18$ with integer coefficients.
15	A bag has 5 red marbles, 7 green marbles and 3 purple marbles. In one draw, what is the probability the marble is not red?
16	What is the sum of the roots of the polynomial: $6x^3 - 25x^2 + 23x - 6$?
17	What is the probability of obtaining exactly 6 heads in 10 tosses of a fair coin?
18	How many ways are there to arrange the letters in the word 'LEVERAGE'?

19	What is the remainder when $2x^3 - 11x^2 + 5x + 6$ is divided by $x - 1$?
20	What is the smaller angle, in degrees, formed by the hour and minute hand of an analog clock at 4:16?
21	What is the perimeter of a rectangle that has an area of 28 and a diagonal length of $\sqrt{65}$?
22	How many lines can be made from 7 distinct points in space, no three on the same line?
23	What is the (1,3) entry of the matrix $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}^{10}$
24	Matt, Stacey, and Bertha are taking turns rolling a normal die to determine who gets the last cookie from the cookie jar. Matt goes first and wins only if he rolls a 6, else Stacey takes a turn. She wins only if she rolls a 1 or 4, else Bertha rolls. Bertha wins only if she rolls a 2, 3 or 5, otherwise the die goes back to Matt and the process is repeated until someone wins. What is the probability that Bertha wins?
25	Two concentric circles of different radii form an annulus. If the annulus has an area of 144π cm ² , what is the length, in cm, of the chord of the larger circle that is tangent to the smaller circle?
26	A line is drawn from along the lateral side of a right circular cylinder of base circumference of 1 cm and height 10 cm such that the line winds around the side 5 times. What is the least length of the line drawn, in cm?
27	What is the sum of the coefficients in the expansion of $(2x + 5y)^3$?
28	A ball is dropped off a 30 m high building. If the ball bounces up 1/3 of its drop height, what is the total distance, in meters, traveled by the ball when it has come to rest?
29	What is the remainder when 7! is divided by 11?
30	What is the probability that when a random committee of 6 people is formed from 4 guys and 6 girls that it contains 3 girls and 3 guys?

	Challenge Questions: 3 pts each
31	How many circular pieces of radius 1 inch can be cut from a 5.5 inch by 6 inch rectangular piece of cardboard?
32	Four spheres of $\sqrt{6}$ radius are arranged on a floor such that the fourth sphere is resting on the surfaces of the three other spheres which are lying on the floor tangent to each other. What is the height, in inches, of the top of this pile, measured from the floor?
33	What is the hundreds digit of 5 ¹⁶⁸ ?
34	What is the remainder when the polynomial $p(x) = x^{20} + 6x^{12} - 4x^9 + 3x^3 + 2x - 5$ is divided by $x^2 - 1$?
35	Ten identical books are to be distributed amongst three librarians. The librarians can receive any number of books, but each must receive at least one. How many ways are there to distribute the ten books?
36	What is the area of the convex polygon with vertices at: (-2,3),(3,-3),(-1,4),(0,-4),(-2,2),(2,2)?
37	Evaluate: $\sum_{x=1}^{\infty} \frac{1}{x^2 + 9x + 18}.$
38	In the math wing of a high school there are 100 lockers numbered 1 through 100. One day some students play a prank on these closed lockers. The first student begins by opening lockers with numbers that are multiples of the first prime number, the second student toggles (opens if closed and closes if opened) all the lockers that are multiple of the second prime number and so on until all prime numbers under 100 are accounted for. At the end of their prank, how many lockers are closed?
39	The local Math is Cool office has an espresso machine that is finicky with the first two cups of coffee that it makes in the morning. The first cup of coffee has a $\frac{7}{8}$ chance of being bitter. If the first cup is bitter then the
	second cup of coffee has a $\frac{7}{9}$ chance of being bitter, else the second cup of
	coffee has a $\frac{4}{9}$ probability. Given that the second cup of coffee is bitter,
	then what is the probability that the first cup was not bitter?
40	The 3-digit number abc, where a, b and c are digits 0-9 is such that 81a + 9b + c = 395. What is the number abc?

Sponsored by: Pemco Foundation 9th & 10th Grade - October 20, 2010 Individual Multiple Choice Contest

1	Evaluate: $2 + 3 \times 5 - 8 \div 4$.				
	A) $\frac{17}{4}$	B) 15	C) $\frac{-15}{4}$	D) -28	E) Answer not given.
2	What is the arithmetic mean of 3, 5, 9, 12, and 21?				
	A) 3	B) 5	C) 9	D) 10	E) 18
3	How many d	iagonals can be	e drawn in a pei	ntagon?	
	A) 5	B) 6	C) 7	D) 10	E) Answer not given.
4	What is the cm?	area, in squar	e centimeters,	of an equilate	ral triangle with sides measuring 12
	A) 36√3	B) 24√3	C) 144	D) 12√3	E) Answer not given.
5	How many p	ositive factors	does the num	ber 1152 have?	
	A) 18	B) 21	C) 24	D) 27	E) Answer not given.
6	Stacey has a 5-ounce cup, a 3-ounce cup, and a 1 quart jug of water. What is the least				
	number of pours Stacey needs to perform in order to obtain exactly 4 ounces of water in				
	a cup?				
	A) 4	B) 5	C) 6	D) 7	E) 8
7	How many so	quares of any s	size are in a 5×	5 array of uni [.]	t squares?
	A) 25	B) 55	<i>C</i>) 70	D) 85	
		E) Answer n	ot given.		
8	What is the	thousands dig	it of 11 ¹¹ ?		
	A) 0	B) 2	<i>C</i>) 3	D) 5	E) Answer not given.
9	Three stand	lard six-sided	dice are rolled	. What is the p	probability that a sum of 12 is
	obtained?	-			
	A) $\frac{1}{126}$	B) $\frac{2}{21}$	C) $\frac{4}{21}$	D) $\frac{25}{216}$	E) Answer not given.
10	A biplane is	pointed north	at an airspeed	of 80 mph. If	the plane encounters a wind
	headed 60 degrees west of south at 40 mph, how fast will the plane move in mph?				
	A) 40	B) 60	C) 80	D) 20√3	E) Answer not given.

Sponsored by: Pemco Foundation 9th & 10th Grade - October 20, 2010

Team Contest

1	In how many ways can the letters in the word "WOOHOO" be arranged?			
2	What is the sum of the distinct prime factors of the number 720?			
3	Express the base-six number 543_6 as a base-ten number.			
4	What is the sum of the first 34 terms of an arithmetic sequence with a first term of 41 and a common difference of -3?			
5	When five lines are drawn in a plane, what is the smallest number of regions that can be created if there must be at least one 90° angle and at least two 40° angles?			
6	How many positive integer factors of 3600 are also integer multiples of 72?			
7	What are the coordinates, in the form (x, y) , of the center of the hyperbola with equation $4x^2 - 2y^2 - 16x + 20y = 0$?			
8	What is the sum of all positive integer value(s) of n less than 100 such that $\sqrt{n + \sqrt{n + \sqrt{n + \cdots}}}$ will evaluate to an integer?			
9	In the following figure, how many rectangles can be drawn that do not contain the shaded square?			
10	If $\log_2 3 = a$, express the value of $\log_{12} 72$ in terms of a.			

Sponsored by: Pemco Foundation 9th & 10th Grade - October 20, 2010 Pressure Round Contest

1	How many non-congruent scalene triangles have a perimeter of 20 m and sides each measuring an integer number of meters?
2	Your trusted friend is dealt two cards from a standard 52-card deck. He looks at them and tells you that they are not of the same suit. What is the probability that they are a pair?
3	If $257 \equiv j \mod 34$, what is the largest possible two-digit number that j could be?
4	I had 2 oz. of gold this morning. Every day I spend half of the gold that I have and my father gives me 5 more ounces of gold at night. Next morning, I start spending again. If this continues indefinitely, how much gold would I eventually have in the morning before spending any?
5	What is the product of the real value(s) of g that satisfy the equation $3^{2g} - 36 \cdot 3^{g} + 243 = 0$?

Sponsored by: Pemco Foundation 9th & 10th Grade - October 20, 2010

Mental Math Contest

PERSON 1			
1.1	Solve for x if four x plus thirteen equals fifty-seven.	11	
1.2	What is the perimeter of a rectangle with length four and width seven halves?	15	
1.3	What is the sum of the next two prime numbers after fifty-three?	120	
1.4	What is eighty-six times one hundred fourteen?	9804	
PERSO	DN 2		
2.1	What is the probability of getting all heads on seven flips of a fair coin?	1/128	
2.2	What is twenty-six squared?	676	
2.3	What is the area of a square inscribed in a circle of radius 3?	18	
2.4	What is the sum of the arithmetic sequence, forty, thirty-six, thirty-two,	216	
	and so on down to eight?		
PERSC	DN 3		
3.1	Is it possible to add two prime numbers and have the sum also be a prime number - ves or no?	Yes	
3.2	What is the area of a rectangle with length four and one diagonal equal to five?	12	
3.3	What is the sum of the numbers in the geometric series: one, two, four, and so on up to thirty-two?	63	
3.4	In how many ways can three different math books, a physics book, a history book and a french book be arranged on a shelf so that the math books are kept together?	144	
PERSC	DN 4		
4.1	What is the slope of the line connecting the point four comma two and the point ten comma four?	1/3	
4.2	How many ways can I interchange the letters in the word comma, C-O-M-M- A?	60	
4.3	An equilateral triangle has a side length of 2, what is its area?	$\sqrt{3}$	
4.4	Twin primes are two prime numbers separated by two such as seventy-one	101, 103	
	and seventy-three. What is the next set of twin primes larger than seventy- one and seventy-three?		

"Math is Cool" Championships - 1010-11 Sponsored by: Pemco Foundation

9th & 10th Grade - October 20, 2010

COLLEGE KNOWLEDGE BOWL ROUND #1 - SET 1

#	Problem	Anguan
#		
1	Expressed as a decimal, what is 15 percent of 15?	2.25
2	Evaluate as a decimal: 2.6 times 4.12	10.712
3	What values of y satisfy two y squared minus seven y minus seventy-two equals zero?	8, -9/2 Both are needed
4	If 321 days ago was a Sunday, then what is the day before two days after tomorrow?	Monday
5	What are the coordinates, in the form x comma y, of the y- intercept of the parabola with equation y equals 4 x squared plus 12 x plus 9?	(0,9)
6	If Katie tricycles at one foot per second for one minute , and at ten feet per second for thirty seconds, what is her average speed in feet per second?	4 [fps]
7	If James wins at game night, he is so excited that he has a three-fourths probability of giving his class extra credit the next day. Otherwise he has only a one-fifth chance of giving extra credit. The probability that James wins at game night tonight is one-third; what is the probability he gives extra credit tomorrow?	23/60
8	If the log base a of b is c and the log base a of c is d, express the log base b of c without logarithms?	d/c
9	Jane is walking home from school. Her path requires her to walk 5 blocks west and 8 blocks north. If she only walks west and north, how many different paths could she take?	1287 [paths]
10	The 10 and all face cards are worth 10, an ace is worth 11 and other cards have their face value. If two cards are dealt from a standard 52-card deck, what is the probability of getting a total of 21?	32/663

COLLEGE KNOWLEDGE BOWL ROUND #2 - SET 2

#	Problem	Answer
1	In what quadrant of the Cartesian plane does the point 17	Fourth
1	comma negative 4 lie?	
2	Evaluate: 3 to the fifth power.	243
6		
3	Express 483.7 in scientific notation rounded to two significant	4.8 times 10 to
5	figures.	the second
		power (squared)
4	Evaluate: Eighty-three squared minus seventy-seven squared.	960
-	Find the equation of the line perpendicular to 3 x plus y equals	v - 1/3v ±5
5	5 which ages through the point 6 comma 7 in slope-intercept	y = 1/5x 15
	form?	third x plus 51
	My local library has 1432 books in the science fiction section	594 [books]
6	If 838 involve space travel, 682 involve time travel, 419 involve	
	dimensional travel, what is the most number of books that	
	involve none of these types of travel?	
7	I have a bet with my friends. From a standard deck I will draw	3 [\$]
/	one card. If it's red or a King I will get \$21, otherwise I will	
	pay \$18. How many dollars should I expect to make? Assume a	
	negative number signifies a loss.	
Q	Find the coefficient of the x to the fourth power in the	20
0	product of the quantity x cubed plus 2 x squared plus 3 x plus	
	4 times the quantity 4 x cubed plus 3 x squared plus 2 x plus 1.	
Q	Find all roots of the equation x-cubed plus 5 x squared minus	5, -4, -6 (in any
/	26 x minus 120.	order)
10	On a road trip, a group of friends pool their money to pay for a	[\$] 144
10	hotel room. If there had been one more friend, the price per	
	person would have been two dollars less. It there had been	
	two tewer triends, the price per person would have been six	
	dollars more. What was the total price of the hotel room, in	
	dollars?	

COLLEGE KNOWLEDGE BOWL ROUND #3 - SET 3

#	Problem	Answer
1	What is the slope of a line perpendicular to the line 8 x plus 2 y equals 35?	[m=] 1/4
2	What are the coordinates, in the form x comma y, of the point of intersection of the lines 3 x minus 5 y equals 2, and 2 x minus 3 y equals 1?	(-1,-1)
3	What is the area, in square centimeters, of an equilateral triangle with sides measuring 12 cm?	$36\sqrt{3}$ [cm ²]
4	If 5 beetles are equivalent to 3 caterpillars and 15 dragonflies are equivalent to 2 beetles, how many caterpillars would be equivalent to 100 dragonflies?	8 [caterpillars]
5	Evaluate the quantity 5 plus 6i times 4 plus 4 i. Answer in c plus d i form.	-4 + 44i
6	Two sides of a triangle are 6 and 4. Find the product of all possible integer values for the third side.	181440
7	During the first six tests of the year I got 97, 80, 95, 83, 89, and 88. Assuming all tests are weighted equally and I really want to average at least a 90 in my class, what is the minimum score I need on my seventh test to reach my goal?	98
8	Find the area of a polygon whose vertices are at the points 6 comma 0 and 0 comma negative 3 and negative 1 comma 0 and 0 comma 5.	28 [units squared]
9	F of x equals four plus the quantity x minus three squared for values of x greater than 3. What is f inverse of 104?	13
10	Dave and Donna wanted to meet for coffee, but forgot a specific time, and just said sometime between 3 and 4. If they both arrive sometime during that frame and wait for 4 minutes, what is the probability they will be there at the same time?	29/225

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COLLEGE KNOWLEDGE BOWL ROUND #4 - SET 4

#	Problem	Answer
1	What is the area, in square centimeters, of a circle with a circumference measuring 26 pi?	169pi [cm²]
2	A polygon with an area of 18 square centimeters and a perimeter of 12 centimeters is similar to a polygon with a perimeter of 30 centimeters. What is the other polygon's area, in square centimeters?	225/2 [cm ²]
3	How many inches are in the space diagonal of a right rectangular prism with sides of 10, 13 and 5 inches?	7 times the square root of 6 [inches]
4	What is the product of the solutions to the equation: $3 \times$ squared plus 23 x minus 8 equals 1?	-3
5	What is the area, in square centimeters, of a square inscribed in a circle with an area of 80 pi cm ² ?	160 [cm ²]
6	Find the mean of the number 8, 12, 5, 5, 3, 4, 7, and 12.	7
7	What is the remainder when the quantity 2×10 the fourth plus 2×10^{10} cubed minus 3×10^{10} f is divided by the quantity $\times 10^{10}$ minus 2?	49
8	When x is added to the numerator and denominator of 6 over 11, the result is 2 over 3. What is x?	4
9	What is the coefficient of the x to the fifth term in the expansion of the quantity x plus 1 to the eighth power.	56
10	What is the product of 1/3 times 2/4 times 3/5 etc. up to 2008/2010?	1/2019045

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COLLEGE KNOWLEDGE BOWL ROUND #5 - SET 5

#	Problem	Answer
1	What is the sum of the integers from negative 5 to 8 inclusive?	21
2	What is the equation, in slope-intercept form, y equals m x plus b, of the line through the points three comma five and five comma negative one?	Y=-3x+14
3	Express the base 3 number two one zero one two in base 10.	194 _[10]
4	Find the least common multiple of 231 and 280.	9240
5	In degrees, the three angles of a triangle are 3x, 3x plus 9, and 8x plus 3. In degrees, what is the measure of the largest angle?	99 [degrees]
6	Mia and Harold are walking to class. Since Mia walks faster than Harold, Harold gets a 10 second head start. The distance to the classroom is 168 feet. If they arrive there at the same time 34 seconds after Harold starts, what is Mia's walking speed in feet per second?	7 [ft per sec]
7	What is the sum of the first 12 positive perfect squares?	650
8	How many terms are there in the arithmetic sequence: 7, 20, 33 and so on until 228?	18
9	What is the area of the figure described by the equation: x squared minus 4 x plus y squared plus 6 y equals 10?	23 pi
10	In a triangle with sides measuring 3, 5, and 6 cm, how many centimeters long is the median to the 6 cm side?	2 times the square root of 2 [cm] or 2√2

COLLEGE KNOWLEDGE BOWL ROUND #6 - SET 6

#	Problem	Answer
1	What is the surface area of a sphere of radius 7?	196 pi [un ²]
2	What is the log base 3 of the quantity 729 squared?	12
3	Find the prime factorization of the product of 116 and 432.	2 ⁶ times 3 ³ times 29 [or] 2 to the sixth times 3 cubed times 29.
4	In a new country, they currently only have 7 cent and 17 cent coins. What is the largest number of cents for which they can't make exact change?	95 [cents]
5	I roll two dice two times. What is the probability that the first time the sum is 7 and the second time the sum is 12?	1/216
6	How many numbers between 10 and 100 are multiples of 3 and/or 8?	37
7	In how many ways can you arrange the letters in the word "Statistics" spelled S-T-A-T-I-S-T-I-C-S?	50,400
8	A sphere has a radius of 5 inches, a cube has an edge length of 7 inches, and a cylinder has a base radius of 3 inches and height 9 inches. What is the ratio of the smallest of the three volumes to the largest of the three volumes?	243/500 or 243 to 500
9	A right triangle is inscribed in a circle. If the two legs of the triangle are 16 and 30, what is the circumference of the circle?	34 pi [un]
10	The following equation is a perfect cube of a linear polynomial: x cubed plus 3 k times x squared minus 9 k times x minus 27. What is k plus 4?	1

COLLEGE KNOWLEDGE BOWL ROUND - EXTRA

#	Problem	Answer
1	Find the sum of all two digit prime numbers whose positive difference between the digits is exactly 2.	273
2	What is the slant height of a cone whose base radius is 9 and whose volume is one-thousand eighty pi?	41
3	A triangle is circumscribed about a circle of radius 7. If the sides of the triangle are 5, 6, and 7, what is the triangle's area?	63 [un ²]
4	Sheila is trying out a new grocery store. In this store, 40% of the apples have a worm. If Sheila randomly chooses 3 apples out of a large bucket, what is the probability no more than one of them is bad? Round to the nearest percent.	65 [%]

Math is Cool" Champions 9 th & 10 th Grade - Octob	pionships - 1010-11 October 20, 2010	Final Score: KEY
School Name Proctor Name	Team # Room #	First Score
STUDENT NAME		(out of 20)

INDIVIDUAL MULTIPLE CHOICE - 15 minutes - 20% of team score

This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet. No talking during the test.

DO NOT	WRITE	IN	SHADED	REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	В		
2	D		
3	A		
4	A		
5	С		
6	С		
7	В		
8	A		
9	D		
10	E [40√3]		

9th & 10th Grade - October 20, 2010

School Name_____ Proctor Name_____

_____Team #_____ Room # KFY

Final Score:

First Score

(out of 10)

STUDENT NAME

Team Contest - Score Sheet

TEAM TEST - 15 minutes - 30% of team score

When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as 1 or 0. Record all answers on the colored answer sheet.

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	30		
2	10		
3	207[10]		
4	-289		
5	12		
6	6		
7	(2,5)		
8	330		
9	84		
10	$\frac{3+2a}{2+a}$		

"Math is Cool" Char 9 th & 10 th Grade -	npionships - 1010-11 October 20, 2010	Final Score:
School Name	Team #	First Score
Proctor Name	Room #	
STUDENT NAME		

PRESSURE ROUND - 10 minutes - 15% of team score

When it is time to begin, you will be handed a packet of questions. There is a copy of the questions for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the questions (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining questions; its maximum value is two points. This process will continue until all the questions are answered and each consecutive question's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified question number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected.

Pressure Round Answers

Answer		
1	4	
2	1/13	
3	87	
4	10 [oz]	
5	6	