"Math is Cool" Masters-2001-02

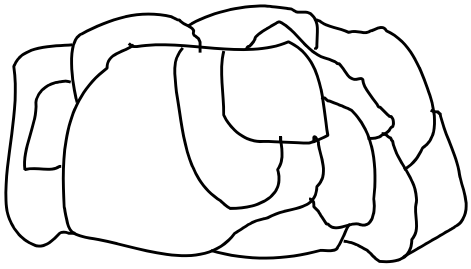
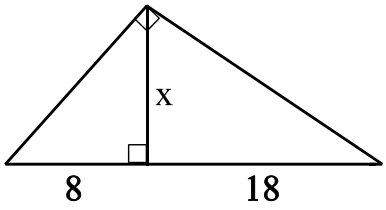
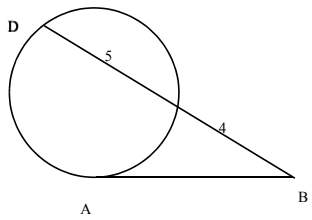
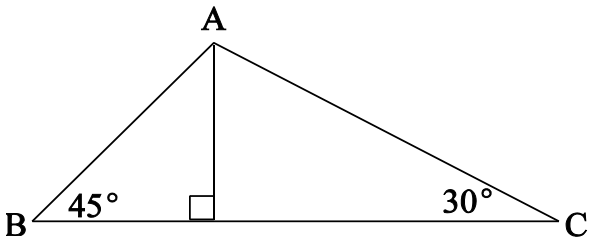
Sponsored by: Washington Trust

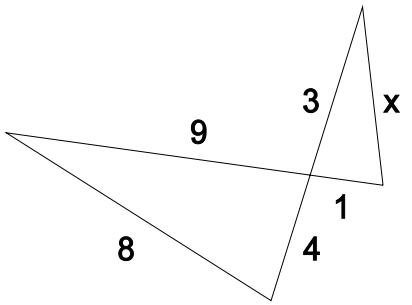
11^{th} & 12^{th} Grade - May 11, 2002

Individual Contest

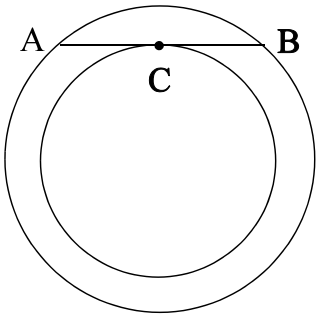
Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

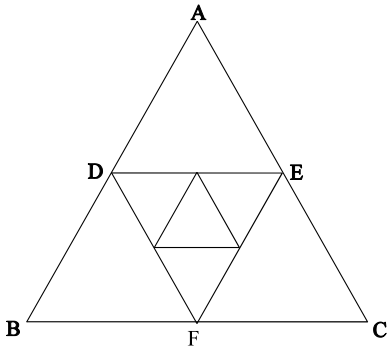
1	What is the volume of a cone, in cm^3, with height 40 cm and radius 30 cm?
2	Find the area of the triangle formed by the line x + y = 20 and the x and y axes.
3	Find the sum: 1+4+7+10+....+181.
4	What are the last three digits of 5^{2002}?
5	The sides of a right triangle form the diameters of semicircles. What is the ratio of the sum of the areas of the semicircles on the legs to the area of the semicircle on the hypotenuse?
6	A triangular number can be written as the sum of the first n whole numbers, where n is a positive integer. What is the second smallest triangular number that is also a square number?
7	What is the sum of the coordinates of the intersection of the altitudes of the triangle formed by the lines y=2, x=3 and y= -x+10?
8	Evaluate: $e^{3\ln 2}$
9	A bathtub with dimensions of 8ft, 2ft, and 30 inches drains at 5 ft^3/min. (1 foot = 12 inches) A faucet fills the bathtub at 3 ft^3/min. Assume the bathtub is completely full. How long, in minutes, will it take a full bathtub to become 1/4 full if the drain is left open and the faucet is running?
10	Give a Pythagorean triple that gives the same numerical value for the area and perimeter.
11	A "Best of the Hamburger 5" CD spins at 60 revolutions per second. If the song "Would You Like Fries With That?" lasts 4 minutes and 32 seconds and the song "Ketchup as Thick as Blood" lasts 6 minutes and 1 second, how many more revolutions does the CD make during "Ketchup as Thick as Blood" than during "Would You Like Fries With That?"
12	Which is greatest: 2^{35}, 5^{15}, or 6^{14}?

13	The country Malmania is divided into regions as shown. If each region is given a color, what is the minimum number of colors used so that no adjacent regions have the same color? 
14	Lily has twice as many flowers as Katie. Katie has 6 more flowers than James. James has 2 less than Silas. Lily has 3 times as many as Silas. How many do they have all together?
15	On an 8x12 grid of squares, a line is drawn from one corner to the opposite corner. How many squares does it pass through?
16	Find x: 
17	The inscribed circle of a regular hexagon has a radius of $\sqrt{3}$. Find the area of the hexagon.
18	Solve for x: $x^3 - 3x^2 - x + 3 = 0$
19	Find the volume of the solid given by the inequality $16x^2 + 4y^2 + 9z^2 - 144 \# 0$
20	A ball is shot from one corner of a 6 foot by 10 foot pool table at a 45° angle. How many times will it bounce off the sides before landing in a corner pocket?
21	Find the length of AB,where AB is tangent to the circle. 
22	Solve for x in the interval: (0, 2π) $\sin 2x + \cos x = 0$
23	Find the area of ^{a}ABC, given AB = 20. 

24	Determine the vertical asymptote(s) of the function $f(x) = \dfrac{x^2 - 5x + 6}{x^2 - 7x + 12}$?
25	Find the distance between the center of one face of a cube and one of the opposite vertices, given that the side length equals 10.
26	ABCD is a regular tetrahedron. A plane cuts through the midpoints of AB, AC, CD, and BD. Given that AC = 4, find the area of the intersection of the plane and the tetrahedron.
27	Find the volume of a regular pyramid with a square base of area 100 units^2 and lateral faces each of area 65 units^2.
28	What is the value of x? 
29	A beetle sits on each square of a 9x9 checkerboard. Each beetle can crawl diagonally to a neighboring square, leaving some squares empty and others with multiple beetles. What is the smallest possible number of empty squares after each beetle has moved exactly once?

Challenge Questions

30	Find the area of the "ring" (region between two concentric circles) where AB = 10, C is the midpoint of AB, and AB is tangent to the inner circle. 
31	A 13 by 13 by 13 cube consists of 13^3 unit cubes. What is the maximum number of cubes that can be seen from a single viewpoint?
32	Evaluate: $\dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \dfrac{1}{4!} + \ldots$
33	Solve for x: $\dfrac{1}{\log_{2x} 10} + \dfrac{2}{\log_{5x} 10} = 3$

34	Let the area of ^{a}ABC = 30, and let D, E, and F be the midpoints of AB, AC, and BC, respectively. ^{a}DEF is called the medial triangle of ^{a}ABC. If you continue drawing medial triangles in this manner, what is the sum of all the areas of the triangles? 
35	If x, y, and z are chosen randomly on the interval (0,1) what is the probability that the mean of x, y, and z is less than $\frac{1}{4}$?
36	How many paths are there from (0,0) to (7,7) (by moving in 1 unit increments up or to the right), that do not go above the line y = x?
37	Find the four digit number abcd such that the product of 4 and abcd equals dcba.
38	How many integer solutions are there to $x^2 + y^2 + z^2 = x^2y^2$?
39	Write $\sqrt{29+12\sqrt{5}}$ in the form $a+b\sqrt{c}$ where a, b *and* c are integers.
40	Consider 64 objects, each with a different weight. What is the minimum number of pair wise weight comparisons needed to determine the heaviest and 2^{nd} heaviest objects?

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Team Contest

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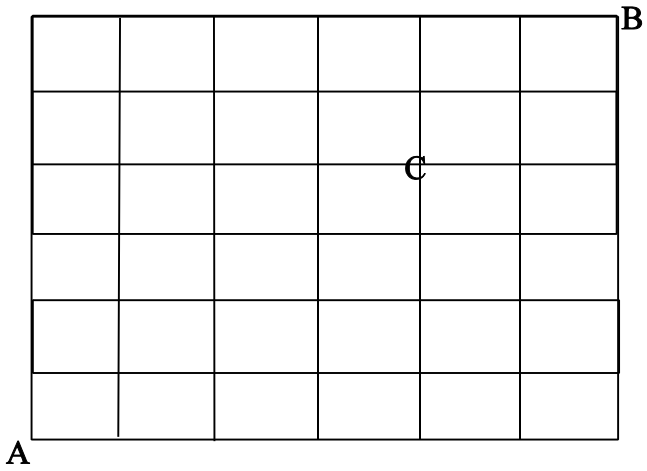
1	Max is in a bike race. He biked the first half of the race plus $\frac{1}{2}$ a mile at 30 mph. He biked $\frac{1}{2}$ the remaining distance plus half a mile at 20 mph. He then biked the remaining 15 miles at 10 mph. How many minutes did he spend biking?
2	Round Lake is in the shape of a circle, with a radius of 2000 feet. A swimmer starts in the middle of the lake and swims at a speed of 25 feet per minute due north. A runner starts due south of the swimmer and runs around the edge of the lake. How fast, in feet per minute, must the runner travel in order to arrive at the north shore of the lake at exactly the same time as the swimmer?
3	 How many ways can A travel to B, given the restriction that A can only go up or right, and cannot go through c?
4	If tan(x)=4/3, what is cos(2x)?

5	If $ab - ac - bc = -10$, $a^2 = 36$ and $b^2 + c^2 = 19$, what is $(a + b - c)^2$?
6	What is the sum of the coefficients of the expansion of $(a + b)^{12}$?
7	How many numbers less than 1,000 are the product of two square numbers greater than 1?
8	Solve for θ over $[0, 2\pi)$. $2 = 1 + \sin\theta + \sin^2\theta + \sin^3\theta + \ldots$
9	An ant and a fly both start at one corner of a $4 \times 6 \times 8$ rectangular prism. The fly can fly straight to the opposite corner, but the ant must crawl along the faces of the box. What is the difference between the distance the ant travels and the distance the fly travels assuming the ant travels the shortest distance possible?
10	$\sum_{n=1}^{100} \frac{1 + ni}{i}$ Write answer in $a + bi$ form.

"Math is Cool" Masters - 2001-02

11th & 12th Grade - May 11, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

Full Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

	Answer	1 or 0	1 or 0
21			
22			
23			
24			
25			
26			
27			
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39			
40			

"Math is Cool" Masters - 2001-02

11th & 12th Grade - May 11, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

Individual Multiple Choice Contest-Score Sheet

1st Score

Out of 18

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer		
1			
2			
3			
4			
5			
6			
7			
8			
9			

"Math is Cool" Masters - 2001-02

11th & 12th Grade - May 11, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

1st Score

Out of 10

Team Contest-Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

"Math is Cool" Masters - 2001-02

11th & 12th Grade - May 11, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Final Score:

"Math is Cool" Masters -- 2001-02

11^{th} & 12^{th} grade - May 11, 2002

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Full Name:_____

1^{st} Score

Out of 40

Individual Contest - Score Sheet
DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	12,000π [cm^3]		
2	200 [$units^2$]		
3	5551		
4	625		
5	1:1 or 1		
6	36		
7	5		
8	8		
9	15 [minutes]		
10	6,8,10 or 5,12,13 (Only one answer needed)		
11	5340[rev]		
12	6^{14}		
13	4		
14	50[flowers]		
15	16		
16	12		
17	$6\sqrt{3}$		
18	-1,1,3		
19	96π [$units^3$]		
20	6 [times]		

	Answer	1 or 0	1 or 0
21	6		
22	π/2, 7π/6, 3π/2, 11π/6		
23	$100+100\sqrt{3}$		
24	x = 4		
25	$5\sqrt{6}$		
26	4		
27	400 [$units^3$]		
28	$\sqrt{29}/2$		
29	9[squares]		
30	25π [$units^2$]		
31	469 [cubes]		
32	e-1		
33	$\sqrt[3]{20}$		
34	40		
35	9/128		
36	429		
37	2178		
38	1		
39	$3+2\sqrt{5}$		
40	67		

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School Name_____Team #_____

Proctor Name_____Room #_____

Key

Individual Multiple Choice Contest-Score Sheet

1st Score

Out of 18

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	A		
2	B		
3	E		
4	D		
5	D		
6	A		
7	C		
8	E		
9	E		

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School Name_____Team #_____

Proctor Name_____Room #_____

Key

1st Score

Out of 10

Team Contest-Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	202 (minutes)		
2	25π (ft/min)		
3	504 (ways)		
4	-7/25		
5	35		
6	4096		
7	19 (numbers)		
8	π/6, 5π/6		
9	$2\sqrt{41}-2\sqrt{29}$		
10	5050 - 100i		

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Proctor Name_____Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1.	111		
2.	10 [people]		
3.	1/3		
4.	$16\sqrt{3}$		
5.	6		