"Math is Cool" Masters-2003

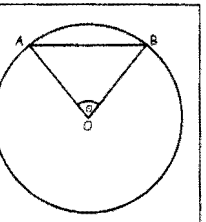
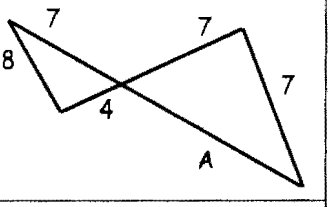
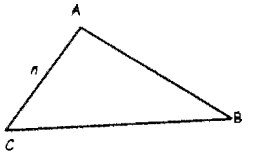
Sponsored by: Lukins & Annis PS

11th and 12th Grade - May 10, 2003

Individual Contest

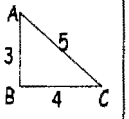
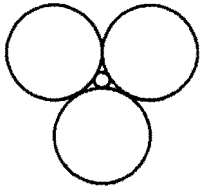
Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

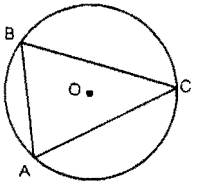
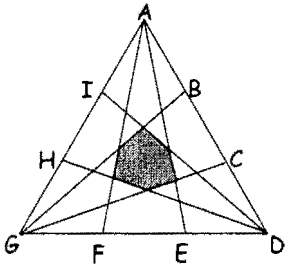
1	Evaluate: cos 45°
2	What degree measure between 0 and 360 degrees is equal to $\frac{\pi}{4}$ radians?
3	If $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, for what angle is $\sin\theta = \frac{1}{2}$?
4	What is the value of log 100?
5	Solve for x: log x + log 2x = log 18
6	If $\log_x 9 = 2$, what is x?
7	What is the maximum number of times a circle and parabola can intersect?
8	While hiking Sarah and Dillon notice that they can gain 30 feet of elevation every minute at 4000 feet and 16 feet per minute at 7000 feet. Suppose that these data are used to determine a linear model predicting their rate of elevation gain as a function of altitude. According to this model, how many feet per minute would they be able to gain at 10,000 feet?
9	A wheel on a car with radius $22/\pi$ inches is rotating at 120 revolutions per minute. How fast is the car traveling in miles per hour?
10	Solve for all real values of x: $7(x-5)^3 + 12 = 68$
11	How many solutions on the interval from 0 to 2π does tan2x - cotx = 0 have?
12	Evaluate (tan 75°)/(1 - (tan 60°)(tan 75°)) + (sin 60°)(cos 75°) / (cos 135°)
13	Simplify $5\sqrt{75} + 8\sqrt{27} - 11\sqrt{48}$ to the form $a\sqrt{b}$ where b does not have a factor that is a perfect square. Find a + b.

14	Find the measure of angle θ, in degrees, in the circle shown, if the length of the line segment AB is 1, O is the center of the circle, and the length of the line segments AO and BO is $\frac{\sqrt{2}}{2}$. 
15	Solve for A: Picture not drawn to scale. 
16	Solve for n, where: $AB = \sin 37.5°$ $CB = \cos 37.5°$ $\angle ABC = 75°$ 
17	$\log_4 3^{2^{\log_9 7^6}}$ can be simplified to the form $a\log_b c$, where c is a prime number. Find the sum of a, b & c.
18	Two points (a, b) and (c, d) on the curve defined by $f(x) = x^2$ are five units away from (6,1). Find the sum of a, b, c & d.
19	What is the area of the ellipse defined by $\frac{(x+3)^2}{16} + \frac{(y-4)^2}{25} = 1$?
20	Evaluate: $\log_{10} 2 + 2\log_{100}\sqrt{2} + 3\log_{1000}\sqrt[4]{2} + 4\log_{10000}\sqrt[8]{2}\ldots$
21	Solve for x: $\frac{5^{x^2}}{5^{5x}} = \frac{1}{625}$.

22	In triangle ABC, $AB = 5$, $BC = 6$ and $AC = 7$. Let D be on $\overline{BC}$ such that $\overline{AD}$ is the bisector of $\angle A$. What is the length of $\overline{DC}$?
23	What is the smallest positive integer greater than 1 that is a perfect square, cube and fourth power?
24	Take a 3-digit number. Subtract from it the same number with the digits reversed (ex. 543-345). If the result of such an operation is 99, what is the largest possible original number?
25	Solve for all real values of x: $\sqrt{x+10} - \frac{6}{\sqrt{x+10}} = 5$
26	Given a 10th degree polynomial with 10 negative coefficients and 1 positive coefficient, what is the maximum number of positive roots it can have?
27	Consider $f(x) = 2x^2 - 4x + c$. What value of c maximizes the product of the roots of f(x), given that at least one root is real?
28	What is the sum of the squares of the roots of $4x^2 + 3x + 2 = 0$?
29	The number 15! in base 10 is written in base 12. How many zeros does it end with?

Challenge questions

30	Simplify: $\sqrt[4]{8\sqrt[4]{8\sqrt[4]{8\ldots}}}$
31	A point is chosen at random from the interior of triangle ABC. What is the probability that it is closer to vertex C than vertices A and B? 
32	Consider points A, B, C, and D in that order on a line ℓ, where AC:BD = 3:7, and BC:AD = 1:4. Determine AB:CD.
33	A circle with radius 6 is surrounded by 3 larger congruent circles, so that every circle touches all 3 other circles. What is the radius of the larger circles? 

34	How many positive integer factors does $3^{12} - 2^{12}$ have?
35	Triangle ABC is inscribed inside circle O. If $\angle A=60°$, $\angle B=45°$, $\angle C=75°$ and the radius of circle O is 1, what is the area of $\triangle ABC$? 
36	What is the product of the four complex fourth roots of 81?
37	Evaluate the sum $\frac{1}{3+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+1}$.
38	Two identical carafes, labeled A and B, are filled 2/3 full, carafe A with red sand and carafe B with green sand. Carafe B is poured into carafe A until it is full. Carafe A is then poured into B until B is filled. Finally, B is poured back into A until A is full. Assume that the sand is mixed thoroughly after every transfer. What is the ratio of red sand to green sand in carafe A after the final transfer?
39	Given that the arithmetic mean of x and y is 10 and their geometric mean is 7, what is $x^2 + y^2$?
40	The equilateral triangle ADG below has side length $\sqrt{\frac{80\sqrt{3}}{3}}$. Also, $\overline{AB} = \overline{BC} = \overline{CD} = \overline{DE} = \overline{EF} = \overline{FG} = \overline{GH} = \overline{HI} = \overline{IA}$. Find the area of the shaded region in the figure: 

Math is Cool" Championships-2003

Sponsored by: Kimberly- Clark and Lukins & Annis PS

9th, 10th , 11th & 12th Grade - May 10, 2003

Individual Multiple Choice Contest

1	What is the remainder when $6x^3 + 10x^2 + x + 8$ is divided by $2x^2 + 1$? A) $-2x + 3$ B) 0 C) $-3x + 5$ D) $4x + 6$ E) Answer not given
2	What is the equation of a parabola passing thraugh the points (1, 2), (-3, 14) and (3, 20). A) $y = 5x + 3$ B) $y = 2x^2 + x - 1$ C) $y = -x^2 + 7x - 4$ D) $y = x^2 + 3x - 2$ E) Answer not given
3	How many natural numbers are 4 digits long when expressed in base 8, but 5 digits when expressed in base 6? A) 2800 B) 421 C) 2600 D) 821 E) Answer not given
4	Factor completely: $4a^2 - 9b^2 - c^2 - 6bc$ A)(2a-3b-c)(2a+3b-c) B) (2a-3b-c)(2a-3b+c) C) $(2a-3b-c)^2$ D) $(2a+3b+c)^2$ E) Answer not given
5	What shape is the graph of $4x^2 - 3x + 2y^2 - 8 = 0$? A) straight line B) parabola C) hyperbola D) ellipse E) Answer not given
6	What is the equation of the directrix of the parabola $x^2 + 8y = 0$? A) $y = 2$ B) $y = -2$ C) $x = 2$ D) $x = -2$ E) Answer not given
7	Solve for x in the following equation for x in terms of a, b, c and d: $a - cx + bx^2 = d$ A) $\frac{c \pm \sqrt{c^2 - 4b}}{2b}$ B) $\frac{c \pm \sqrt{c^2 - 4b(a-d)}}{2b}$ C) $\frac{c \pm \sqrt{4b(a-d)}}{2b}$ D) $\frac{c \pm \sqrt{c^2 - 4b(a-d)}}{2}$ E) Answer not given
8	What is the largest number of points in which three triangles (which do not share any portions of sides) can intersect? A) 12 B) 13 C) 14 D) 15 E) Answer not given
9	What is the first time after 3:00 when the hour and minute hands of a 12-hour analog clock are 120° apart? A)3:38:10 B) $3:38:10\frac{10}{11}$ C) $3:38:10\frac{14}{15}$ D) 3:38:11 E) Answer not given

"Math is Cool" Masters-2003

Sponsored by: Lukins & Annis PS

11th & 12th Grade - May 10, 2003

Team Contest

Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the colored cover sheet.

1	Consider the 4x4x4 array of lattice points whose coordinates are either 0, 1, 2, or 3. How many right rectangular solids have eight of these points as vertices and have edges of integer length?
2	Three real numbers are randomly chosen between 0 and 10. What is the probability that their sum is less than 10?
3	Consider a process in which a three-digit number (e.g. 246) is concatenated with itself reversed (e.g. 642) to produce a 6-digit number (e.g. 246642). What is the largest natural number which is guaranteed to be a factor of possible 6-digit numbers produced by this process?
4	Express $0.08\overline{3}_9$ as a fraction (base 10) in lowest terms.
5	What is the tens digit of $6^{315} + 3^{11}$?
6	What is the largest integer by which all even, perfect squares must be divisible?
7	What is the length of the semi-major axis of the ellipse defined by the equation $4x^2 + y^2 + 32x - 4y + 52 = 0$?
8	If c chickens can lay e eggs in d days; how many days will it take r fewer chickens to lay the same number?
9	Evaluate: $\sin\frac{7\pi}{12}$
10	Eho is shooting free throws. He makes his first shot and misses his second shot. For each shot after the second, the probability that he makes that particular shot is equal to the ratio of the number of successful shots to the number of attempted shots. What is the probability that Eho will have made exactly 50 shots after 100 attempts?

"Math is Cool" Masters-2003

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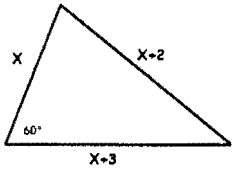
11th & 12th Grade - May 10, 2003

Pressure Round

1	Using the equation for a plane passing through the points (3,0,0) (0,-4,0) and (0,0,5) in the form $Ax + By + Cz = D$, where the greatest common divisor of A, B, C, and D is 1 and D is positive, find $A + B + C + D$.
2	If each side of a certain equilateral triangle decreases by 2 inches, the area decreases by $5\sqrt{3}/2$ square inches. What was the original area, in square inches, of the triangle?
3	Evaluate: $2003^3 - 2002^3$.
4	Solve for all possible values of a: $\log_3 a^2 = \log_3 a^5 - 6$
5	Solve for x: Figure not drawn to scale. 

"Math is Cool" Masters-2003

Sponsored by: Kimberly- Clark and Lukins & Annis PS

9th , 10th , 11th & 12th Grade - May 10, 2003

Mental Math Contest

Express all answers in terms of radicals and π, where applicable, unless otherwise instructed.

Person #1		
1	How many prime numbers less than 30 are divisible by 3?	1
2	Evaluate: $\sqrt{8^2 + 15^2}$ ***Read as: Evaluate the square root of the quantity eight squared plus fifteen squared***	17
3	What is the geometric mean of 12 and 108?	36
4	Solve for x: $2^x = 1024$ ***Read as: solve for x: two to the power x equals 1024.***	[x =] 10
Person #2		
1	What is the measure, in degrees, of one interior angle of a regular pentagon?	108[°]
2	What is the sum of the reciprocals of the first three primes?	31/30
3	What is the surface area of a sphere with radius ½?	π
4	Solve for x: $2x^2 - 3x + 1 = 0$. ***Read as: Solve for x: two times x squared minus 3 times x plus 1 equals zero.***	x=1 or x=1/2 Both answers needed
Person #3		
1	What is the slope of a line with x-intercept (12, 0) and y-intercept (0, 8)?	-2/3
2	What is the greatest common factor of 196 and 280?	28
3	What is the maximum possible volume of a rectangular prism with a surface area of 24 square centimeters?	8 [cm^3]
4	If the sum of three consecutive integers is 1560, what is the largest number?	521
Person #4		
1	How many diagonals can be drawn in a regular octagon?	20
2	If a + b = 90, and a and b are both primes, what is the largest possible value of a?	83
3	Write as a repeating decimal: 6/11	.545454... or $.\overline{54}$
4	A cylinder filled with maple syrup has a radius of 2 and a height of 27. If it is emptied into 3 congruent spheres, and each are completely filled, what is the radii of the spheres?	3

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9th ,10th ,11th , & 12th Grade - May 10, 2003

College Knowledge Bowl Questions #1		
1	If Jason flipped 6 fair coins, what is the probability of his getting at least 2 heads?	57/64
2	How many natural numbers less than 1000 are multiples of both 6 and 8?	41
3	How many points with integer coordinates are exactly 6 units away from (0,0)?	4
4	What are the coordinates of the point on the segment AB that is two-thirds of the way from A to B, where A has coordinates (2,4) and B has coordinates (-1,1)?	(0,2)
5	At what point does the line 3x + 5y = 8 intersect the line -2x + 7y = 5?	(1,1)
6	7, a, b, c, 11 is an arithmetic sequence. What is b?	9
7	For what values of x is the following inequality true? $\frac{x+3}{x+3} \le 0$ ***Read as: For what values of x is the following inequality true? The quantity x + 3 divided by the quantity x + 3 is less than or equal to 0.***	None or empty set or no value exists or null set
	Number 8 is an extra question. Only use it if needed.	
8	An urn contains 8 orange marbles and 12 black marbles. If two marbles are drawn without replacement, what is the probability they are the same color?	47/95

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9th ,10th ,11th , & 12th Grade - May 10, 2003

College Knowledge Bowl Questions #2		
1	How many vertical asymptotes does $f(x) = \frac{4}{x^2+1}$ have? ***Read as: How many vertical asymptotes does f of x equals 4 divided by the quantity x squared plus 1 have?***	0 or none
2	A string long enough to fit exactly around a square with diagonal $6\sqrt{2}$ also fits exactly around a circle. What is the radius of the circle?	$12/\pi$
3	The Mathketeers hike 3 miles north of Eigen lake, then 8 miles east, then 7 miles south, then 10 miles west in time for dinner. How far, in miles, are they from Eigen Lake?	$2\sqrt{5}$ [mi]
4	Evaluate: 396 times 404	159984
5	How many ways can the Caltech Ultimate frisbee team choose a starting lineup of 7 people from a group of 10?	120
6	Solve the system of equations: x + 5y + 2z = 0 2x + 8y + 3z = 1 -x - y + 2z = 0	x = 3, y = -1, z = 1
7	Find the y-intercepts: $x^2 - 8x + y^2 + 4y - 5 = 0$	(0,1) and (0, -5)
	Number 8 is an extra question. Only use it if needed.	
8	Eric has a die weighted so that it is twice as likely to roll an even number as it is to roll an odd number. Suppose he rolls the die twice. What is the probability that the numbers he gets sum to 7?	4/27

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9^{th} ,10^{th} ,11^{th} , & 12^{th} Grade - May 10, 2003

College Knowledge Bowl Questions #3		
1	What is the least common multiple of 48, 36, and 45?	720
2	What is the sum of the first 12 odd numbers?	144
3	Express 172 in base 2.	10101100
4	Simplify: $i^{19} + i^{82} + i^{7} + i^{29}$	$-1-i$
5	At which points do $y = x^2$ and $y = 3x - 2$ intersect?	(1,1) and (2,4)
6	What is the area of a regular hexagon with side length 2?	$6\sqrt{3}$
7	For what value of c does $2x^2 + 8x + c = 0$ have exactly one real solution?	c=8
Number 8 is an extra question. Only use it if needed.		
8	How many multiples of 13 are between 3000 and 4000?	77

Math is Cool Masters -- 2003

11th and 12th Grade - May 10, 2003

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	$\sqrt{2}/2$		
2	45[°]		
3	$\pi/6$		
4	2		
5	3		
6	x=3		
7	4		
8	2 [feet/min]		
9	5 [mph]		
10	7		
11	6		
12	-1		
13	8		
14	90[°]		
15	1/2		
16	$\sqrt{3}/2$		
17	56		
18	8		
19	20π		
20	$2\log_{10}2$ or log4 or $\log_{10}4$		

	Answer	1 or 0	1 or 0
21	x=1 or x=4 Both answers needed		
22	7/2		
23	4096		
24	998		
25	26		
26	2		
27	2		
28	-7/16		
29	5		
30	2		
31	1/4		
32	1:5 or 1/5		
33	$12\sqrt{3}+18$		
34	32		
35	$\frac{3+\sqrt{3}}{4}$ or $\frac{1}{4}\sqrt{12+6\sqrt{3}}$ (Only need one answer)		
36	-81		
37	1		
38	14:13 or 14/13		
39	302		
40	2		

Math is Cool Masters -- 2003

11th and 12th Grade - May 10, 2003

School Name_____Team #_____

Proctor Name_____Room #_____

Key

1st Score

Out of 18

Individual Multiple Choice Contest-Score Sheet

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer	-1, 0 or 2	-1, 0 or 2
1	A		
2	B		
3	A		
4	E		
5	D		
6	A		
7	B		
8	E		
9	B		

4. $(2a-3b-c)(2a+3b+c)$ 8. 18

Math is Cool Masters -- 2003

11th and 12th Grade - May 10, 2003

School Name_____Team #_____

Proctor Name_____Room #_____

Key

1st Score

Out of 10

Team Contest-Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	216		
2	1/6		
3	11		
4	67/648		
5	2		
6	4		
7	4		
8	$\frac{cd}{c-r}$		
9	$\frac{\sqrt{6}+\sqrt{2}}{4}$		
10	1/99		

Math is Cool Masters -- 2003

11th and 12th Grade - May 10, 2003

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1	77		
2	$\frac{49\sqrt{3}}{16}$ [square inches]		
3	12,030,019		
4	9		
5	[x =] 5		