"Math is Cool" Masters-1999-00

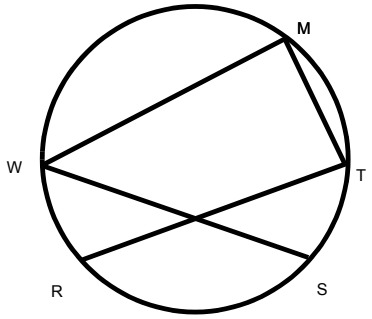
11^{th} -12^{th} Grade -April 29, 2000

Individual Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.
Record all answers on the green cover sheet.

1. Simplify: $\dfrac{y+\dfrac{2y}{y+2}}{1+\dfrac{4}{y^2-4}}$

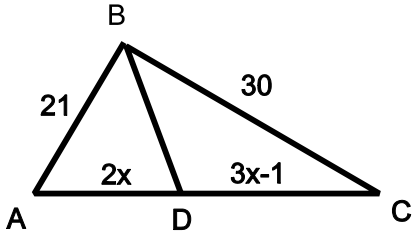
2. In the circle, minor arc RS = 40E,
then $m\text{p}W + m\text{p}T =$



3. A, B, C and D are four different weights. When they are placed on a balance scale, the following observations are made:
A and B exactly balance C and D
A and C together out weigh B and D together
C is lighter than D
Arrange the weights in order from heaviest to lightest.

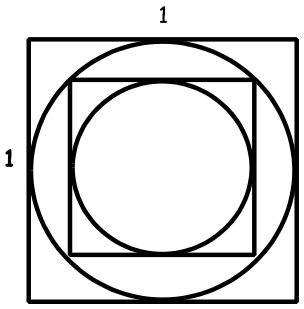
4. If a>b>0 then
a) $\dfrac{1}{a}=\dfrac{1}{b}$ b) $\dfrac{1}{b}>\dfrac{1}{a}$ c) $\dfrac{1}{b}<\dfrac{1}{a}$ d) $\dfrac{1}{b}<0$ e) $\dfrac{1}{a}<0$

5. In $\hat{I}$ ABCD, BD bisects pB. Then X=



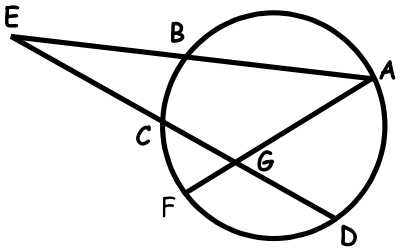
6. A line through (2,3) has x-intercept twice its y-intercept. Its equation is:

7. All are circles or squares. What is the area of the smaller circle?



8. If m+1 varies inversely as 2n-1 and if n=13 when m=3, find n when m=24.

9. BE=5, AB=7, CE=4 and AG=6.
Then GF=

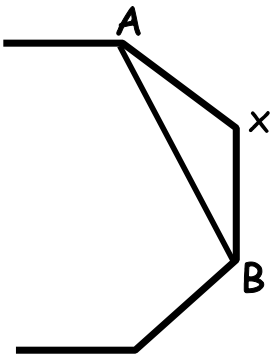


10. In $\hat{I}$ ABC, if a=2b and mρB=30E, then mρA=

11. If logX = 3log3 + $(\frac{1}{2})$log5 - 2log7 then x=

12. If 5x-7y=3 and -2x+4y=0, then x+y=

13. Ax and Bx are two adjacent sides of a regular polygon. If the measure of angle ABx equals 1/3 the measure of angle AxB, how many sides does the regular polygon have?



14. One root of x^2-kx+18=0 is twice the other. Then k=

15. A boy ran up a hill at $1\frac{1}{2}$ mph and came down the hill at $4\frac{1}{2}$ mph. The trip took him 6 hours. How far, in miles, is it to the top of the hill?

16. In a city, 85% of the taxi cabs are green and 15% are blue. A taxi cab has an accident, and a witness who is accurate 80% of the time identifies the cab as blue. What is the probability that it was a blue rather than green cab involved in the accident?

17. A number that is 11 less than twice the square root of 49 is:

18. If (6/5)x=10 then 3x=

19. N is a 3-digit number whose units digit is 3. What is the probability that N is divisible by 3?

20. Three kinds of nuts are mixed in a can. How many nuts must you take to be sure you have at least n nuts of one kind?

21. Suppose a, b and c are integers between 0 and 9. Then the number represented by the infinite repeating decimal 0.abcabcabcabc... is a rational number. Express it as a fraction.

22. The yearly changes in population census of a town for four consecutive years are, respectively, 25% increase, 25% increase, 25% decrease, 25% decrease. To the nearest percent, what is the net change over the four years?

23. Simplify: $$\cfrac{1}{1+\cfrac{2}{1+\cfrac{2}{1+\cfrac{2}{1+....}}}}$$

24. How many diagonals does a regular 2000-sided polygon have?

25. The ordered pair (2,-3) is the common solution of the system 2x+3y=A and 5x-2y=B. What is A+B?

26. A cyclist rides 30 km at an average speed of 9 km/hr. At what rate must she cover the next 10 km in order to bring her overall average speed up to 10 km/hr?

27. Consider the linear function f(x)=ax+b. If the x-intercept is (-32,0) and the y-intercept is (0,24), determine a+b.

28. Gregg can mow 600 square yards of grass in 1 $\frac{1}{2}$ hours. At this rate, how many minutes would it take him to mow 600 square feet?

29. Find the max of : x^2+5x+7 on [-3,5].

30. You have 5 liters of 10% HCI and 3 liters of 30% HCI. You want 1 liter of 16% HCI. How much 30% solution should be used?

31. What is the sum of the roots of the equation $2x^3+x^2-25x+12=0$.

32. Angle θ is in standard position and its terminal side contains the point $(-4, \sqrt{33})$. Then sinθ=

33. Solve over the domain 0E<x<360E: 2cos(x+45E) = 1.

34. The value of $25^{\frac{1}{2}-\log_5\sqrt{2}}$.

35. $\binom{13}{6}+\binom{13}{7}=\binom{14}{x}$. What is x?

36. X is chosen randomly from the interval 0 to 1. What is the probability that 3sin8x>0?

37. Evaluate $(\log_3 5)(\log_3 7)(\log_5 7)$

38. Simplify: $\|5+12i|-|40+30i\|$

39. If $\sec^2\theta-\tan^2\theta=1$, what is θ?
a) 30E b) 45E c) 60E d) 90E e) Any angle for which tanθ is defined.

40. What is the ratio of the area of a hexagon inscribed in a circle of radius 1 to the area of a triangle circumscribed about a circle of radius 1.

"Math is Cool" Masters-1999-00

11^{th} -12^{th} Grade -April 29, 2000

Individual Multiple Choice Contest

1. If $i^2=-1$ and $x=1+i$, then $\frac{1}{x^2}$ is equal to:

   a) $\frac{-1}{2}i$ b) $\frac{1}{2}i$ c) 2i d) -2i e) none of these

---

2. What is the shortest distance between the lines y=x and y=x+2?

   a) $\sqrt{2}$ b) $\sqrt{3}$ c) 2 d) 3 e) none of these

---

3. Town X and Town Y are separated by 400 miles. A car going from X to Y travels at 50 mph. How fast must the car travel from Y to X to obtain an average speed of 100 mph?

   a) 100 mph b) 150 mph c) 300 mph d) 450 mph e) not possible

---

4. If log2+log15.6=log7.8-logx, then x=

   a) 1 b) 1/4 c) 31.2 d) 7.8 e) 5.8

---

5. Let q be a binary operator, defined for positive integers, such that $aqb=a^2+b^2$. Which of the following are true?

   I. q is commutative

   II. q is associative

   III. q is distributive over standard addition

   a) I only b) II only c) I and II only d) II and III only e) I, II and III

---

6. x is a non-zero real number. $x^{-\frac{1}{3}}=$

   a) $\frac{-x}{3}$ b) $\sqrt[3]{x}$ c) $\sqrt[3]{-x}$ d) $\sqrt[3]{\frac{1}{x}}$ e) $\frac{1}{x^3}$

---

7. Beth's basket contains 1998 red balls, 1999 blue balls and 2000 green balls. Every minute she removes two balls at random from her basket. If the balls are the same color, she discards them. Otherwise, she puts the balls back in her basket. After a certain amount of time passes, she is left with only one ball in her basket. What color is it?

   a) red b) blue c) green d) either blue or green e) not enough information given

---

8. Rewrite $8^{\log_{64}5}$ exactly without using the log symbol.

   a) $\sqrt[3]{5}$ b) $\sqrt{5}$ c) 5 d) 25 e) 125

---

9. The arithmetic mean of the four solutions of $x^4-28x^3+265x^2-1006x+1560=0$ is

   a) 7 b)-7 c) 28 d) -28 e) $5\sqrt{3}$

"Math is Cool" Masters-1999-00

11^{th} -12^{th} Grade -April 29, 2000

Team Contest

Express all answers as reduced fractions unless stated otherwise.
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1. Solve for x:. x^2-1999x+2000=0

2. A work crew of 3 people requires 3 weeks and 2 days to do a certain job. How long would it take a work crew of 4 people to do the same job if each person on both crews works at the same rate as each of the others? Note: each week contains 6 work days.

3. For what integers value of x is the following inequality satisfied?
$$\frac{x^2+10x+24}{x^2+6x+8} \le 0$$

4. 3 coins are picked at random from an unlimited supply of pennies, nickels and dimes. Given that the total value of coins chosen exceeds 15¢, what is the probability at least 2 dimes were chosen?

5. How many ways are there to divide 10 identical pennies among Robert, Bill, Eric, Greg and Tim if each person must receive at least one penny and Robert must receive an even number of pennies.

6. Diophantus spent 1/6th of his life in childhood, 1/12th in youth and 1/7th more as a bachelor. Five years after his marriage was born a son who dies 4 years before his father, at ½ his father's final age. How old was Diopheutus when he died?

7. Find all solutions of the following equation
$[\cos x \sec x-(\sin x+\cos x)^2]^2+\cos^2 2x=1$ on the interval $[0,\pi)$.

8. Define $f(x)=\log_x 81$ for $x>1$
Simplify: $\dfrac{1}{f(\log 125)}-\dfrac{1}{f(\log 5)}$

9. Jina has an urn filled with 4 green balls and 7 red balls. She draws one randomly and places it in a second urn that originally contained 3 green balls and 3 red balls. If Jina now draws a new ball from the second urn, what is the probability the ball is green?

10. Simplify $\left(i-\sqrt{3}\right)^9$ where $i^2=-1$

"Math is Cool" Masters-1999-00

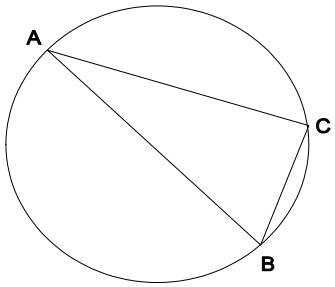
11^{th} -12^{th} Grade -April 29, 2000

Pressure Round Contest

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.

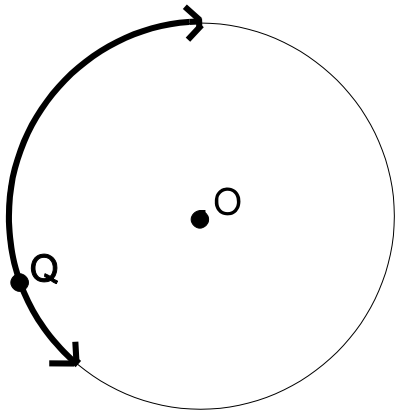
1. Given a three dimensional tic-tac-toe type game consisting of four levels of 4-by-4 squares, how many ways could you get 4 in a row?

2. In the diagram, AB is the diameter of the circle. If AB=4 and the area of triangle ABC is 4, calculate the perimeter of the triangle ABC.



3. Any number of the form 1000a + 100b + 10a + b where a and b are integers is always divisible by what prime?

4. Simultaneously, two particles start at point Q, on circle O, and move in opposite directions around the circle. If the speed of one particle is 4 times that of the other, then the particles will next meet at point P on the circle. If pPOQ is acute, what is *m*pPOQ, in degrees?



5. A sequence of numbers t_1, t_2, t_3, . . . is defined by:

$t_1=7$ and $t_{n+1}=\sqrt{\left|(t_n)^2-16\right|}$. What is the value of t_{80}?

"Math is Cool" Masters-1999-00

9^{th} -12^{th} Grade -April 29, 2000

Mental Math Contest

Express all answers as reduced fractions in terms of radicals and π, where applicable, unless otherwise instructed.

Person #1

1. What is the fourth term in the geometric progression that begins: 1, 2, . . .
2. What is the area of a rectangle with diagonal of 5 and a long side of 4?
3. What is the sum of the first 5 primes?
4. 2 coins are flipped simultaneously. What is the probability of 1 head and 1 tail appearing?

Person #2

1. Find the x-value where x^2+x+1 reaches a minimum?
2. Two runners are running around a 1 mile track. One runner runs a 6-minute mile and the other runs an 8-minute mile. They start at the same place. How many minutes go by until the slower runner is lapped by the faster runner?
3. Robert's prize fish weighs 45 pounds plus 1/6 of its weight. How many pounds does the fish weigh?
4. Factor completely: $X^4+3x^3+2x^2$

Person #3

1. How many times larger than 12^{72} is 12^{144}?
2. What is the units digit of 13^{44}?
3. In what base does 26x4=116?
4. Six people are in a circle and begin counting in turn. (i.e., the first person says one, the second two, ... and they continue around). Which person (1, 2, 3, 4, 5 or 6) will be the one to say 2000?

Person #4

1. What is the slant height of a right circular cone with volume 30π and radius 3?
2. Factor completely: $x^2+30x+225$
3. How many distinct prime factors does 2000 have?
4. Bill has 6 red socks and 4 blue socks. Bill first draws a red sock. How many more socks must Bill draw to guarantee he has a pair of red socks?

"Math is Cool" Masters-1999-00

9^{th} -12^{th} Grade -April 29, 2000

College Knowledge Bowl Questions #1		
1	What is the median of the mode, median and mean of the following list of test scores? 60, 70, 85, 60, 95	70
2	Factor completely: $x^3 + 9x^2 + 26x + 24$	(x+2)(x+3)(x+4)
3	Simplify $i^{2000} + i^{-2000} + i^{1999} - i^{-1999}$ where $i^2=-1$	2-2i or 2(1-i)
4	The population of Holleyville increased 25% two years ago and then decreased 25% last year. The population is now 4500 people. What was the population before the two changes?	4800
5	If a, b, c, d, e are positive integers and ab is even, bc is even, cd is odd, de is even. Of a, b, c, d, e, which must be even?	b and e
6	What is $\sin\frac{19\pi}{6}$?	-1/2
7	What value(s) of k will make the following to be factored as the square of a linear polynomial? $4x^2+kx+49$	±28
Number ***8*** is an extra question. Only use it if needed.		
8	How many ways can you rearrange the letters in the word "BASSOON"?	1260

"Math is Cool" Masters-1999-00

9^{th} -12^{th} Grade -April 29, 2000

College Knowledge Bowl Questions #2		
1	What is the sum of the positive factors of 48?	124
2	Where do the following two functions intersect? $f(x)=x^2+2$ $g(x)=-2x^2+8$	$\left(\sqrt{2},4\right)$and $\left(-\sqrt{2},4\right)$
3	$f(x)=x+3$, $g(x)=x^2-9$, $h(x)=3-2x$ What is $h(f(g(x^2)))$?	$15-2x^4$
4	What is the vertex of the parabola $f(x)=2x^2-3x+1$?	$\left(\frac{3}{4},\frac{-1}{8}\right)$
5	Three water pipes are used to fill a swimming pool. The first pipe alone can fill the pool in 8 hours, the second in 12 hours, and the third in 24 hours. If all three pipes are opened at the same time, how many hours will it take to fill the pool?	4 (hours)
6	Express $0.31\overline{7}$ as a simple fraction	143/450
7	Ryan, Mark and Nadav are running around a track. Ryan runs 12 laps per hour, Mark runs 6 laps per hour and Nadav runs 3 laps per 2 hours. If they all start together, how many minutes will it take them to all return to the beginning at the same time?	40 minutes
Number ***8*** is an extra question. Only use it if needed.		
8	The slope of line L is $m=-\sqrt{3}$. What is the positive inclination of line L in degrees?	120E

"Math is Cool" Masters-1999-00

9^{th} -12^{th} Grade -April 29, 2000

College Knowledge Bowl Questions #3		
1	A certain cube has a side length of 2. If we triple its volume to form a new cube, what is its new side length?	$2\sqrt[3]{3}$
2	Simplify: $\frac{8!}{2\cdot 4\cdot 6\cdot 8\cdot 10\cdot 12\cdot 14\cdot 16}$	1/256
3	What is the remainder when 6! is divided by 7?	6
4	What is the area of a square circumscribed about a circle with circumference 1?	$\frac{1}{\pi^2}$
5	Bill and Robert are playing a game. In an Urn there are 6 marbles, one black and five white ones. Bill draws a marble and then passes the Urn to Robert who draws a marble and passes the Urn to Bill. After all the marbles are drawn the person with the black marble wins. What is the probability that Bill wins?	1/2
6	Two fair six-sided dice are rolled. What is the probability a sum of 7 is rolled given that at least one 3 is rolled?	2/11
7	Which is larger 3^{180} or 2^{300} ?	2^{300}
Number ***8*** is an extra question. Only use it if needed.		
8	Bill has a right, circular cylinder of volume 10. If Bill doubles the radius of the cylinder, what is the new volume of the cylinder?	40

"Math is Cool" Masters -- 1999-00

11th & 12th grade - April 29, 2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Full Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet
DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	$(y^2 + 2y-8)/y$	also factored	
2	160E		
3	A>D>C>B or ADCD		
4	b		
5	7		
6	x+2y=8		
7	$\frac{\pi}{8}$		
8	5/2		
9	4		
10	90E		
11	$\frac{27\sqrt{5}}{49}$		
12	3		
13	5		
14	±9		
15	6 3/4 miles		
16	12/29 . .41		
17	3		
18	25		
19	1/3		
20	3n-2		

	Answer	1 or 0	1 or 0
21	$\frac{abc}{999}$		
22	-12%		
23	$\frac{1}{2}$		
24	1,997,000		
25	11		
26	15		
27	24 3/4		
28	10		
29	57		
30	3/10 L or .3 L		
31	-1/2		
32	$\frac{\sqrt{33}}{7}$		
33	15E,225E		
34	5/2		
35	7		
36	$1-\frac{\pi}{8}$		
37	$(\log_3 7)^2$ or $(\log 7)^2/(\log 3)^2$		
38	37		
39	e		
40	1/12 or 1:12		

"Math is Cool" Masters -- 1999-00

11th & 12th grade - April 29, 2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Individual Multiple Choice Contest-Score Sheet

1st Score

Out of 18

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points. Answers need to be recorded letters.

DO NOT WRITE IN SHADED REGIONS

Answer		-1, 0 or 2	-1, 0 or 2
1	A		
2	A		
3	E		
4	B		
5	A		
6	D		
7	B		
8	B		
9	A		

"Math is Cool" Masters -- 1999-00

11th & 12th grade - April 29, 2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

1st Score

Out of 10

Team Contest-Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer	1 or 0	1 or 0
1	$\left(1999 \pm \sqrt{398001}\right)/2$		
2	2 weeks 3 days or 15 days		
3	-6, -5, -3		
4	7/16		
5	40		
6	84		
7	$\left\{x \middle\| 0 \le x < \pi, x \ne \frac{\pi}{2}\right\}$		
8	1/4		
9	37/77		
10	-512i		

"Math is Cool" Masters -- 1999-00

11^{th} & 12^{th} grade - April 29, 2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Mental Math - Score Sheet

1	8
2	12
3	28
4	1/2
1	-1/2
2	24(minutes)
3	54(pounds)
4	$x^2(x+1)(x+2)$
1	12^{72}
2	1
3	9
4	2
1	$\sqrt{109}$
2	$(x+15)^2$
3	2
4	5

"Math is Cool" Masters -- 1999-00

11th & 12th grade - April 29, 2000

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1	76 ways (24 on diagonals)		
2	$4\sqrt{2}+4$		
3	101		
4	72E		
5	1		