"Math is Cool" Championships-1999-00

February 18, 2000

Individual Contest, High School 11-12

Express all answers as reduced fractions unless stated otherwise.
Leave answers in terms of π where applicable.
Do not round any answers unless stated otherwise.

1. What is the reciprocal of the reciprocal of the reciprocal of 3/2?

2. What is the measure in degrees of an interior angle in a regular, 36 sided polygon?

3. Given an isosceles, right triangle with two sides of length $\sqrt{2}$, what is the length of the other side?

4. How many odd, composite numbers are there between 2 and 50?

5. If $a_1=3$ and $a_{n+1}=2a_n-1$, what is a_{10}?

6. Factor completely: $8x^4-64xy^3$

7. Evaluate: $4^{\log_4 8}$

8. What is the amplitude of the following function:
$f(x)=4\sin(3x-1)$

9. Given log(a)=.3, log(b)=.4 and log(c)=.5, what is $\log\frac{a^2b}{c}$?

10. What is the sum of the following geometric series?
$1+\frac{3}{4}+\frac{9}{16}+\frac{27}{64}+\ldots..$

11. If 8 men can assemble 16 machines in 12 days, how many days will be required for 15 men to assemble 50 machines?

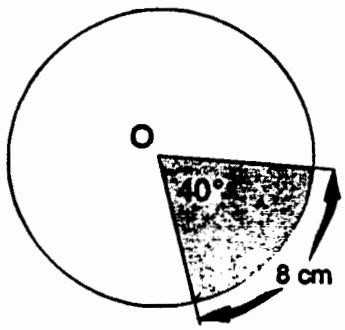
12. In a geometric sequence of 4 terms, all of which are real numbers, the product of the first three terms is 1 and the product of the last three terms is 27/8. Find the last term.

13. Ten chips, numbered 1 through 10, are mixed in a bowl. Two chips are drawn successively, and without replacement from the bowl. What is the probability the sum of the chips is 10?

14. If $4^x - 4^{x-1} = 24$ then $x =$

15. Determine all values of x and y such that xy, x/y and x-y are all equal and $y \neq 0$.

16. O is the center of the circle shown. Find the area of the sector shaded in.



17. Solve for y:
$y-k+(k-2)(k+5)=(k+5)(k-3)+2k+5$

18. An isosceles triangle has integral sides and perimeter of 8. What is its area?

19. A point is in the interior of a circle of radius 2. What is the probability it is further than 1 unit from the center?

20. The first three terms of a geometric progression are $\sqrt{2}$, $\sqrt[3]{2}$, $\sqrt[6]{2}$. What is the fourth term?

21. x equals 7 more than x divided by $\frac{1}{2}$. What is x?

22. The surface area of a cube is numerically equal to its volume. What is the length of the diagonal through the middle of the cube?

23. What is the probability of rolling a sum of 8 on two, fair, 10-sided dice?

24. What is the surface area of a sphere with diameter 1?

25. Robert is twice as old as Bill. In 7 years, Robert will be three fourths of twice Bill's age. How old is Bill now?

26. Stan drives from point A to point B at 40 mph. He returns from point B to point A at 30 mph. He then repeats this trip, this time traveling 10 mph one way and 20 mph the other way. What was his average speed overall?

27. Find all real solutions to: $x^6+7x^3+12=0$

28. How many positive integers less than 500 are divisible by either 3 or 4?

29. What is the area of a triangle with sides 7, 8 and 9?

30. Spencer and Nadav are playing tennis. If the probability that Spencer wins any given game is 4/5, what is the probability that Spencer wins at least 1 game if 3 games are played?

31. If $\sin\theta=3/10$ and $\sec\theta<0$, what is $\cos 2\theta$?

32. Simplify: $\frac{\log 2}{\ln 8}\cdot\frac{\ln 16}{\log 32}$

33. Evaluate: $\sum_{i=1}^{\infty}\frac{1}{5^i}$

34. f(x) is an odd function and a polynomial. The remainder when f(x) is divided by x+3 is 7. What is f(3)?

35. How many ways are there to divide 4 men and 2 women into two committees of 3 if there must be at least 1 woman per committee?

36. Solve for x:
$4\cos^2 x=2\sin x+\sqrt{2}\,(2\sin x+1)+4$ on the interval $[0, 2\pi)$

37. Find 4 distinct, positive integers, such that one subset of three sums to 25, another to 30, another to 32 and another to 36.

38. At McDonalds you can order McNuggetts in boxes containing 6, 9 or 20 pieces. By ordering two boxes of 6 you can get 12. But you cannot order 13, since no combination of 6, 9 and 20 adds up to 13. What is the greatest number of McNuggets that you cannot order?

39. Sam has 2 ostrich eggs. Sam lives in a building with 36 floors. He is told that if he drops an ostrich egg from a certain floor or higher, the egg will break. If he drops an egg from any lower floor, the egg will not break. His job is to determine the lowest floor at which the egg will break. What is the minimum number of drops required that will guarantee that Sam will know the distinct floor as soon as the second egg breaks?

40. A committee of 7 is to be chosen from a group consisting of 10 women and 8 men. On the committee there are at least 2 men, but there must be more women than men. In how many ways may the committee be formed?

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Individual Multiple Choice Contest, High School 11-12

1. $(2^{-1}+3^{-1})=$

a) 5^{-2} b) 5^{-1} c) 5/6 d) 1/6 e) 3

---

2. If a, b and c are prime numbers with a+b=c and $1 < a < b < c < 40$, then the number of possible sets {a,b,c} is

a) 0 b) 1 c) 4 d) 5 e) 11

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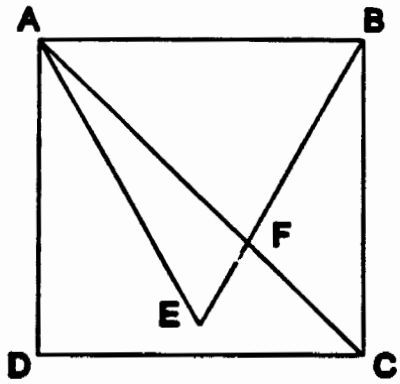
3. If ABCD is a square and ABE is an equilateral triangle, then the measure of ∠BFC, in degrees is

a) 90 b) 105 c) 75 d) 120 e) 100



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4. If the radius of a circle is increased by 1, then the ratio of the new circumference to the new diameter is:

a) π b) π+2 c) $\frac{2\pi+1}{2}$ d) $\frac{2\pi-1}{2}$ e) π-2

---

5. If $A=3^{x}+3^{-x}$ and $B=3^{x}-3^{-x}$, then $A^2-B^2=$

a) $2(3^{2x})$ b) $2(3^{-2x})$ c) 0 d) 4 e) 12

---

6. Three fair 6-sided dice are thrown. What is the probability that the sum of the numbers showing is six?

a) 1/36 b) 1/108 c) 1/216 d) 5/36 e) 10/216

---

7. Suppose that 10% of a population is left-handed. Further, 20% of the lefties are geniuses while 1% of the righties are geniuses. Assume that everyone is either a leftie or a rightie and no one is both. If a random genius is selected from the population, the probability the selectee is also a leftie is closest to:

a) 40% b) 50% c) 60% d) 70% e) 80%

---

8. Which step in the following proof is false?

Step 1. $3>2$ and $\log(\frac{1}{2})=\log(\frac{1}{2})$

Step 2. Since $3>2$, then $3\log(\frac{1}{2})>2\log(\frac{1}{2})$

Step 3. If $3\log(\frac{1}{2})>2\log(\frac{1}{2})$ then $\log(\frac{1}{2})^3>\log(\frac{1}{2})^2$

Step 4. If $\log(\frac{1}{2})^3>\log(\frac{1}{2})^2$, then taking antilogs given $(\frac{1}{2})^3>(\frac{1}{2})^2$ or $1/8>1/4$

a) Step 1 b) Step 2 c) Step 3 d) Step 4

e) the proof is valid

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9. If $g(x)=1-3x$ and $f(g(x))=9x^2-6x+5$, then $f(1)=$

a) -2 b) 0 c) 5 d) 8 e) 53

"Math is Cool" Championships-1999-00

February 18, 2000

Team Contest, High School 11-12

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1. The integers greater than 1 are arranged as follows (four in each row, in five columns):

a	b	c	d	e
2	3	4	5	
	9	8	7	6
10	11	12	13	
	17	16	15	14

If the pattern is continued, 2000 will occur in which column?

2. In the expansion of $(a+b)^n$ there are $n+1$ dissimilar terms. Find the number of dissimilar terms in the expansion of $(a+b+c)^{10}$.

3. How many minutes after 2 pm will the hands of a clock first be at right angles to each other?

4. What is the sum of all two-digit whole numbers that divide 109 with a remainder of 4?

5. Solve for x: $\frac{1}{2^{1999}} - \frac{1}{2^{2000}} = 2^x$

6. Emilie has 36 pairs of shoes. 25 of them are black, 18 of them have a buckle and 15 of them are dress shoes. Also, Emilie mentions 7 of the black shoes have a buckle and 6 dress shoes have a buckle. Five of the dress shoes are black and have a buckle. All of her shoes have at least one of these characteristics. How many black dress shoes do not have a buckle?

7. Evaluate: $\frac{x^{37}-1}{x^{36}+x^{35}+x^{34}+\ldots+x+1}$ where x=5.

8. If a,b,c,d,e are the solutions to the following quintic equation: $3x^5+4x^4-7x^3-9x^2+3x-4=0$. What is $a \cdot b \cdot c \cdot d \cdot e$?

9. Solve the following for x: $\log_{10}(x+3)+\log_{10}(x-2)=\log_{10}(2x)$

10. Given $f(x)=|3\sin 7x|$. What is the period of f?

"Math is Cool" Championships-1999-00

February 18, 2000

Pressure Round Contest, High School 11-12

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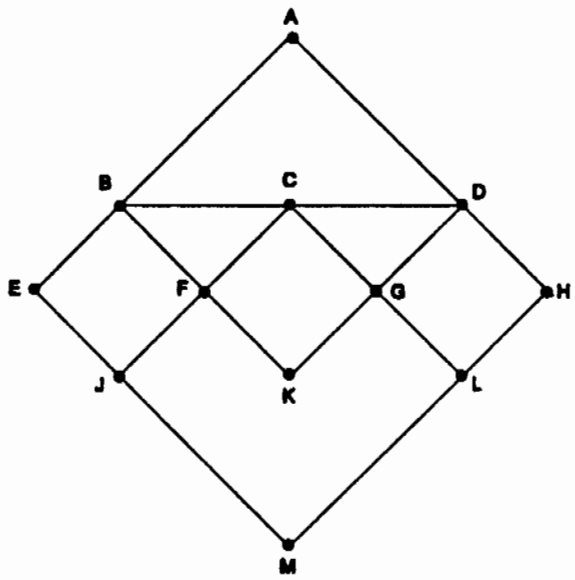
1. Spencer's company employs 99 level 2 workers and one level 1 worker (Spencer). The level 2 employees have an average salary of $10,000. When Spencer's salary is included the average doubles. How much does Spencer make?

2. If a number end in zeros, the zeros are called terminal zeros. For example 84,000 has three terminal zeros, but 80,400 has just two. Let N be the product of all natural numbers from 1 through 20:
N=1x2x3x4x . . . x20.
How many terminal zeros will N have when it is written in standard form?

3. Jude wants to trace the following figure without lifting his pencil from the paper and without using the same line between points more than once. From which point(s) can Jude start tracing if he wants to accomplish this task?



4. Beth, Tasha, Spencer and Nadav want to cross a bridge at night. It takes Beth 10 minutes, Tasha 5 minutes, Spencer 2 minutes and Nadav 1 minute to cross the bridge. Up to two people may cross the bridge at a time, and when two people cross, it takes the longer amount of time to cross the bridge. Since it is night, any person or any pair of people crossing the bridge must carry a flashlight. Among the 4 people there is only one flashlight. What is the minimum amount of time required for all 4 people to cross the bridge.

5. A bug wants to move from the origin (0,0,0) to the point (5,-6,4) in a 3D coordinate system. The bug's movement is limited to the following: each move the bug can move 1 unit in the positive x or z direction, or 2 units in the negative y direction. How many different paths can the bug take to get to its destination?

"Math is Cool" Championships-1999-00

February 18, 2000

Mental Math, High School 11-12

1. What is the volume of a sphere with surface area of 36π?
2. In degrees, what is the sum of the interior angles of a 10-sided polygon?
3. What is 51 squared?
4. Three fair six-sided dice are rolled. What is the probability of getting at least 2 even numbers?

1. What is $\frac{1}{6} \cdot 6!$
2. Find all positive real solutions to $x^2+7x-10=0$
3. What is the volume of a square pyramid with a height of π and a base diagonal of $\sqrt{6}$?
4. If Sean scores a 100 on 4 exams and a 76 on 2 exams, what is his average score?

1. Express without logs: $\left(\frac{1}{9}\right)^{\log_3 6}$
2. In a group of 10 girls, each girl shakes hands with the 9 others. How many handshakes occur?
3. Two fair 12-sided dice are rolled each with the integers 1 through 12 on their faces. Express as a reduced fraction the probability that the sum of the two faces that come up is 4.
4. Express i^{i^2} in a+bi form where a and b are real:

1. What is the arithmetic mean of the three roots of $2x^3 + 11x - 250 = 0$
2. Solve $4 \cdot 2^{3x} = \frac{1}{8}$
3. Give the sum of the two base six numerals in base six $25_6 + 44_6$
4. Which function has the larger period and what is the period?
   f defined by f(x) = 4tan(5x)
   g defined by g(x) = 9sin(11x)

"Math is Cool" Championships-1999-00

February 18, 2000

High School 11-12

College Knowledge Bowl Questions #1		
1	Let i denote $\sqrt{-1}$ Simplify: $i^{534} + i^{68} + i^{9}$	i
2	Find the smallest positive integer that has remainder 1 when divided by 5 and remainder 2 when divided by 3.	11
3	An Urn contains 10 marbles of which 2 are red and 8 are blue. If all the marbles are drawn without replacement, what is the probability that the second one drawn is red?	1/5
4	If $\log x=.7$ and $\log y=.2$, what is $\log(x^2y^2)$?	1.8
5	What is the period of the function $2\sin(3x+7)+\cos(2x+\pi)$	2π
6	What is the center of the circle given by $x^2-2x+y^2+6y=97$?	(1,-3)
7	$\sin(\pi/6)+\sin^2(\pi/6)+\cos^2(\pi/6)$	3/2 or $1\frac{1}{2}$
Number 8 is an extra question. Only use it if needed.		
8	For what value of x is the following inequality true? $x^2-4x+3>x^2+2x$	$x<\frac{1}{2}$

"Math is Cool" Championships-1999-00

February 18, 2000

High School 11-12

College Knowledge Bowl Questions #2		
1	What is the length of the longest diagonal on a box with width 7, height 3 and length 5?	$\sqrt{83}$
2	Evaluate $\lim_{x\to\infty} \frac{(x+3)(5x^2+3x+2)}{12x^4+13x^3+14x^2+15x+16}$	0
3	Find the vertical asymptotes: $f(x)=\frac{x^2+7x+6}{x^3-5x^2-14x}$	x=0, x=7 and x=-2
4	Bill, Robert and Stan have 10 pieces of candy. How many ways can they divide the candies among them if each person must receive at least one piece?	36
5	Evaluate $8^{\log_2 5}$	125
6	Let i denote $\sqrt{-1}$ Express in a+bi form the reciprocal of 2+3i.	$\frac{2}{13}-\frac{3}{13}i$
7	Express $\sin^2\left(\frac{\pi}{8}\right)$ in the form $a+b\sqrt{2}$ where a and b are rational.	$\frac{1}{2}-\frac{1}{4}\sqrt{2}$
Number 8 is an extra question. Only use it if needed.		
8	Find the vertex of the parabola: $y= 36x^2 + 12x-336$	(-1/6, -337)

"Math is Cool" Championships-1999-00

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High School 11-12

College Knowledge Bowl Questions #3		
1	Find the vertical asymptotes: $f(x)=\frac{2x^3+12x^2+22x+12}{x^2+6x+8}$	x = -4
2	What is the area of a Rhombus with diagonals 8 and 3?	12
3	From a pool of 10 people one needs to pick one president, one vice-president, and 2 general board members. How many ways is there to do this?	2520
4	Evaluate: $\log_8 16$	4/3 or $1\frac{1}{3}$
5	Let i denote $\sqrt{-1}$ What is the sum of the reciprocals of 1+i and 1-2i in the form a + bi?	$\frac{7}{10}-\frac{1}{10}i$
6	What is the area of the circle given by: $x^2-10x+y^2-6y+25=0$	9π
7	What is the period of sin(sin(4x+2))	$\pi/2$
Number **8** is an extra question. Only use it if needed.		
8	What is the remainder when $x^{10}-1$ is divided by x-2?	1023

"Math is Cool" Championships -- 1999-00

High School - Grades 11-12 - February 18, 1999

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Full Name:_____

1st Score

Out of 40

Individual Contest - Score Sheet

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	2/3		
2	170°		
3	2		
4	10		
5	1025		
6	$8x(x-2y)(x^2+2xy+4y^2)$		
7	8		
8	4		
9	.5 or 1/2		
10	4		
11	20 days		
12	9/4		
13	4/45		
14	5/2		
15	$x=-\frac{1}{2}$ $y=-1$		
16	$144/\pi$ cm^2		
17	y=2k or 2k		
18	$2\sqrt{2}$		
19	3/4		
20	1		

	Answer		
21	-7		
22	$6\sqrt{3}$		
23	7/100		
24	π		
25	7		
26	19.2 mph or 19 1/5 mph or 96/5 mph		
27	$-\sqrt[3]{4},\sqrt[3]{3}$ or $\sqrt[3]{-4},\sqrt[3]{-3}$ or $-2^{\frac{2}{3}},-3^{\frac{1}{3}}$		
28	249		
29	$12\sqrt{5}$		
30	124/125		
31	41/50		
32	4/15		
33	1/4		
34	-7		
35	6		
36	$\frac{5\pi}{4}, \frac{7\pi}{6}, \frac{7\pi}{4}, \frac{11\pi}{6}$		
37	16,11, 9, 5		
38	43		
39	8		
40	18816		

"Math is Cool" Championships -- 1999-00

High School - Grades 11-12 - February 18, 1999

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Individual Multiple Choice Contest-Score Sheet

1st Score

Out of 18

Correct responses are worth 2 points, incorrect responses are worth -1 point and no response is 0 points.

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	c		
2	d		
3	b		
4	a		
5	d		
6	e		
7	d		
8	b		
9	c		

"Math is Cool" Championships -- 1999-00

High School - Grades 11-12 - February 18, 1999

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Team Contest-Score Sheet

1st Score

Out of 10

DO NOT WRITE IN SHADED REGIONS

	Answer		
1	C		
2	66		
3	27 3/11 (or 300/11) minutes		
4	71		
5	x=-2000		
6	9		
7	4		
8	4/3		
9	3		
10	$\frac{\pi}{7}$		

"Math is Cool" Championships -- 1999-00

High School - Grades 11-12 - February 18, 1999

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Pressure Round - Score Sheet

	Answer		
1	1,010,000		
2	4		
3	J,L		
4	17 minutes		
5	27720		

"Math is Cool" Championships -- 1999-00

High School - Grades 11-12 - February 18, 1999

School Name_____Team #_____

Proctor Name_____Room #_____

Key

Mental Math - Score Sheet

1	36π
2	1440
3	2601
4	$\frac{1}{2}$
1	120
2	$\frac{-7+\sqrt{89}}{2}\left(or\ \frac{-7}{2}+\frac{\sqrt{89}}{2}\right)$
3	π
4	92
1	1/36
2	45
3	1/48
4	-i (or 0 - i)
1	0
2	-5/3 (or -1 2/3)
3	113_6
4	f $\pi/5$ (or 36°)