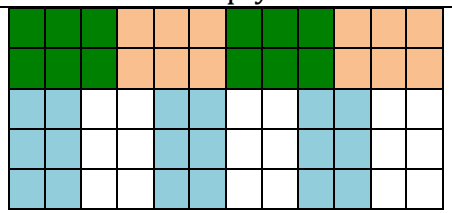
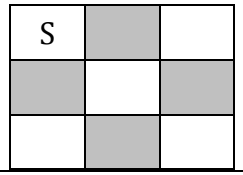
Mental Math

9/11	Answer	Solution
1	27	Probably easier to do $\left(\frac{6}{2}\right)^3 = 3^3 = 27$.
2	120	6(8)+6(12)=6(20)=120
3	81	$\sqrt{36} + \sqrt{9} = 6 + 3 = 9 = \sqrt{81}$
4	3/8	There are 8 permutations (HHH, HHT, …, TTT) and 3 of them (3C2) that have two heads.
5	120	$17^2 - 13^2 =$ $(17 - 13)(17 + 13) =$ $4(30) = 120$
6	20	Any set of three points will make a triangle. The number of ways of choosing 3 points is 6C3 = 6(5)(4)/6 = 20.
7	4	$\frac{\log(81)}{\log(3)} = \log_3 81 = 4$
8	100	Has to be a perfect square to have an odd number of factors.

Individual Test

9/11	Answer	Solution
1	1/27	$2^3 \cdot 6^{-3} = \frac{2^3}{6^3} = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$
2	79	Common difference is 7 making the next two terms: 36 and 43 totaling to 79.
3	60 [sq in]	The other leg is $\sqrt{17^2 - 8^2} = 15$. The area is (1/2)8(15) = 60 sq inches.
4	0	Simplifying: $4x + 7 = 6x + 15 - 8; 7 = 2x + 7; x = 0$
5	9 [diagonals]	A diagonal can be drawn from each of the 6 vertices to 3 others but this double counts them. 6(3)/2=9.
6	2	Solving for y (slope intercept form). $y = 2x - \frac{5}{2}$ A slope of 2.
7	25π [sq cm]	First, find the radius. $2\pi r = 10\pi, r = 5.\ A = \pi r^2 = 25\pi.$
8	ABC [in that order]	$2^3 < 3^2 < 10^1$ Raise each to the 10th power and $2^{30} < 3^{20} < 10^{10}$
9	125 [cu un]	Finding the side length: $\sqrt{s^2 + s^2} = 5\sqrt{2};\ s = 5.\ V = s^3 = 125$ cubic units.
10	90 [°]	The diagonals in <u>any</u> rhombus are perpendicular and thus the angles are 90 degrees.
11	2	$f(-2) = 2(-2)^2 + (-2) - 4 = 8 - 2 - 4 = 2$
12	[*x*=] -11, 3 [either order]	Factor the quadratic: $0 = x^2 + 8x - 33 = (x + 11)(x - 3)$ x = -11 or 3.
13	41 [donuts]	Buy three dozen for $15 (36), with the $3 leftover buy 5 more donuts. Total 36+5 = 41 donuts.
14	1	11 (mean) – 10 (median) = 1
15	49/5	$(1 + 2) \cdot 3 + \frac{4}{5} = 3 \cdot 3 + \frac{4}{5} = 9\ \frac{4}{5} = \frac{49}{5}$
16	B) -0.8 [Either is acceptable]	Clearly it is a negative correlation – larger x means smaller y. It is not perfect though as there are points off the line thus -08 is the best choice.

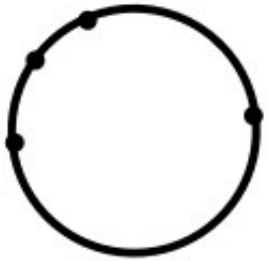
9/11	Answer	Solution
17	609π [in²]	The lateral surface area is π times the radius of the base times the slant height of the cone. Using the base area, its radius is 21 in. The volume is $V = \frac{1}{3}\pi r^2 h$. $2940\pi = \frac{1}{3}(441)h;\ \ h = 20$. The slant height is $\sqrt{20^2 + 21^2} = 29$. The lateral area is then $609\pi sq\ in$.
18	16 [subsets]	There are 4 primes. In any subset each may or may not be in the set for a total of $2^4 = 16$ subsets. The empty set is a valid subset.
19	10 [stickers]	 Uses all of the area – 10 stickers.
20	[$]1	The expected value is the sum of the dollar outcomes times its respective probability. The chance of either all heads or all tails is ¼. $10(1/4) - 2(3/4) = 4/4 = 1$
21	6720 [ways]	The multinomial value is: $\frac{8!}{3!} = 8(7)(6)(5)(4) = 6720\ ways$.
22	4	Easy to enumerate. The longest side must be 5 or less as 6 is over half the perimeter. So, 5-5-1, 5-4-2, 5-3-3 and 4-4-3 for a total of 4.
23	90 [%]	12 (80) = 8(75) + 4(x) or realize that 8 students is twice 4 students, so the 4 students must average twice as far from 80% as the first 8. x must be 90%.
24	$\sqrt{65}$	POQ is a right triangle with PO is the hypotenuse $\sqrt{9^2 - 4^2} = \sqrt{65}$. Or PQ² = 5(5+2*4)
25	225/2 [cm²]	The area ratio is the square of the perimeter ratio. $A = \left(\frac{30}{12}\right)^2 18 = \frac{25}{4}18 = \frac{225}{2}\ sq\ cm$
26	1/24	There are 24 ways of permuting the order of the gifts and only 1 way that each critter gets its own present. 1/24.
27	16	Change everything to base 2 and use the laws of exponents. $\frac{\left(2^{\frac{3}{2}} \cdot 8^{-2}\right)^{\frac{1}{3}}}{16^{-\frac{3}{2}} \cdot \sqrt{2}} = \frac{2^{\frac{1}{2}} 2^{3(-2)\left(\frac{1}{3}\right)}}{2^{4\left(-\frac{3}{2}\right)} 2^{\frac{1}{2}}} = \frac{2^{-\frac{3}{2}}}{2^{-\frac{11}{2}}} = 2^4 = 16$

9/11	Answer	Solution
28	4280 [ways]	Take the total number of combinations of 4 members out of 20 and subtract the number of combinations of all algebra or all geometry students. 20C4-12C4-8C4=4845-495-70 = 4280 ways.
29	85	When converting to base ten the individual numbers convert to: $(2^2-1)+(3^2-1)+\cdots+(6^2-1)=85$.
30	284	One could use the formula or simply evaluate directly. $$2^2+3^2+\cdots+9^2=\frac{9(10)(19)}{6}-1^2=284$$
31	4	One can get to all the shaded squares. 
32	$\left(-\infty,\frac{4}{3}\right]$	Since it is over the Reals, $$4-3r\geq 0;\frac{4}{3}\geq r. \quad \left(-\infty,\frac{4}{3}\right]$$
33	14 [nickels]	We start with the two given equations with n, d and q integers: $$n+d+q=32\ and\ 5n+10d+25q=355$$ $$or\ n+2d+q=71$$ Subtracting we get $d+4q=39$. We can't have $d=2q\ or\ 2d=q$ as the other won't be an integer. However, multiplying the 1st equation by 2 and subtracting we get: $-n+3q=7$. Now if there are twice as many nickels we get q=7 and n=14 add d=11 to get \$3.55.
34	60 [terms]	$(a+b)^3$ has 4 terms. Less clear, but $(c+d+e)^4$ has 15 terms. Basically, we must divide the exponent 4 between the three variables. The formula is 6 choose 2 = 15. In total: 4x15 = 60
35	12	By Wilson's theorem, if n is a prime number then $(n-1)!+1$ is divisible by n.
36	4	$$\theta=0,\pi,2\pi,3\pi$$ $$4\pi>10;\quad 2^{4\pi}>2^{10}=1024>1000$$ $$If\ r\geq 1, then\ \theta\geq 0.$$
37	0	The sum of the solutions is 0 since the coef. of x^2 is 0. The solutions must then be –a, 0, a. These have a product of 0=b
38	7	Enumeration: (5), (2,3), (4,1), (2,2,1), (3,1,1), (2,1,1,1), (1,1,1,1,1)

9/11	Answer	Solution
39	5/12	$\frac{1}{x^2+4x+3}=\frac{A}{x+1}+\frac{B}{x+3}, A=\frac{1}{2}, B=-\frac{1}{2}$ $so\ sum=\frac{1}{2}\left[\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{5}+\cdots\right]=\frac{1}{2}\left[\frac{5}{6}\right]=\frac{5}{12}$
40	$\left(2,\frac{\pi}{6},2\right)$ or $(2,30°,2)$	In the xy-plane the point $(\sqrt{3},1)$ is at a 30 degree angle with the x-axis and is 2 units from the origin.

Individual Multiple Choice

9	11	Answer	Solution
1	1	C	Find the first few powers: 005, 025, 125, 625, 3125 and it can be seen that for powers greater than 2, all odd powers have a 1 and even powers have a 6 in the hundreds place. (C)
2	2	C	The Remainder Theorem states that when a polynomial P(x) is divided by (x-a), the remainder is P(a). P(1) = 10 (C)
3	3	A	Follow the order of operations. $-1-(-1-1)^{-1}(-1)=-1-\left(-\frac{1}{2}\right)(-1)=-1-\frac{1}{2}=-\frac{3}{2}\ (A)$
4	50	B	Place the 3 tails. There are 4 (B) places to put the 3 consecutive heads. _ T _ T _ T _
5	5	B	$x=\sqrt{20+\sqrt{20+\cdots}}=\sqrt{20+x}$ Squaring both sides. $x^2-x-20=(x-5)(x+4)$ The positive solution is x=5 (B)
6	6	A	The ant must first make it from the centroid to the midpoint of the common edge of the two faces. It will be 1/3 of the height of the triangle. $\frac{1}{3}(2\sqrt{3})$ Doubling this to include the distance to the other centroid. $\frac{4\sqrt{3}}{3}\ (A)$
7	50	E [$x=-4$]	The equations of the axis of symmetry will be: $x=-\frac{b}{2a}=-\frac{24}{6}=-4\ (E)$

9	11	Answer	Solution
8	8	B	Start with a drawing of the two circles and the interior tangent. The radii drawn from the center to the tangent points are perpendicular. Extend the radii in the smaller circle by 12 and connect to the other center. A right triangle is made with a hypotenuse of 40 (distance between centers) a leg of 20 (8+12) other leg congruent to the internal tangent. $= \sqrt{40^2 - 20^2} = \sqrt{1600 - 400} = 20\sqrt{3}\ (B)$
9	9	D	$\log 10^{(\log_{81} 49)(\log_7 36)(\log_6 27)} = (\log_{81} 49)(\log_7 36)(\log_6 27)\ (\log 10)$ Using the change of base formulas. $= \frac{\log 49}{\log 81}\ \frac{\log 36}{\log 7}\ \frac{\log 27}{\log 6}$ Using power rule for logs. $= \frac{2\log 7}{4\log 3}\ \frac{2\log 6}{\log 7}\ \frac{3\log 3}{\log 6} = 3\ (D)$
10	50	B	$4 = 2^2$so each two digits in base 2 will be one digit in base 4. 01 is 1, 10 is 2, 11 is 3, 01 is 1. So: 1 01 11 10 01 -> 11321 base 4 (B).
50	4	A	All circles have an eccentricity of 0. (A)
50	7	E [0]	The second power has only one 1 in the (1,3) entry. After that, all higher powers are the zero matrix. 0 (E).
50	10	A	The four perpendicular bisectors meeting in a point just means that the intersection point is equidistant from the 4 vertices. So, only that it is inscribed in a circle, i.e. cyclic. (A) 

Team Test

9	11	Answer	Solution
1	1	15	60=2*2*3*5 so multiplying by 3*5=15 gives a perfect square = 900.
2	20	160 [degrees]	The sum of the measures of the exterior angles is 360 degrees. Since they are equal and there are 18 of them, each measures 20 degrees. The interior angle, supplementary to the exterior angle will be 160 degrees.
3	3	7.5 [hours]	Solve for the amount of time to make one sweater. $\frac{1}{3}+\frac{3}{5}=\frac{1}{x};\ x=\frac{15}{14};\ \ 7x=\frac{15}{2}=7.5\ hours$
4	4	$[-87,\infty)$	Complete the square: $c(b)=5b^2+40b-7=5(b+4)^2-7-80$ So the parabola has a minimum of -87. The interval is then $[-87,\infty)$.
5	5	(1/2, 2, -3)	Add the last two equations to get $3k=6, k=2$. Substitute this value into the first equations to get $4j+2l=-4$. This and the 3rd equation can be solved, the ordered triple is $\left(\frac{1}{2},2,-3\right)$.
6	20	29/100	$x^2+7x+10=(x+2)(x+5)=0.\ x=-2,-5.$ The sum of squared reciprocals is: $\frac{1}{2^2}+\frac{1}{5^2}=\frac{25+4}{100}=\frac{29}{100}$.
7	7	$464_{[8]}$	Don't convert, just carry at 8 (3*4=4 carry 1, etc.)
8	8	2	Square both sides: $8+\sqrt{60}=a+b+2\sqrt{ab}=\ a+b+\sqrt{4ab}$ This suggests two equations: $a+b=8\ and\ ab=15$ Solutions are *a*=5 and *b*=3 with *a*-*b*=2.
9	9	$\frac{\sqrt{2}+\sqrt{6}}{2}$	Set the value equal to x, square both sides and factor $\sqrt{2}$ out of the square root. This gives the equation: $x^2=1+\sqrt{2}x$. Use the quadratic formula and ignore the negative result. $x=\frac{\sqrt{2}+\sqrt{6}}{2}$
10	20	$2\pi-4$	The area of the four semicircles is equal to the area of the square plus the area of the petals as the semicircles count that area twice. The area of the petals is then $2\pi r^2-2^2=2\pi-4$

9	11	Answer	Solution
20	2	30 [degrees]	The dot product between the vectors is: $$\sqrt{3}(1) + (1)\sqrt{3} = 2\sqrt{3}$$ while each has a magnitude of 2. Therefore, $$\cos\theta = \frac{2\sqrt{3}}{2 \cdot 2} = \frac{\sqrt{3}}{2}; \quad \theta = 30°.$$
20	6	774	A multiple of a row or column can be added to another row or column without changing the determinant. Add multiples of the second column to the others to get: $$\begin{bmatrix} -13 & 3 & 2 & -6 \\ -28 & 4 & 1 & -3 \\ -41 & 6 & 9 & -9 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$ Which has the same determinant as the original and the last row and second column can be eliminated. Next add multiples of the second row to the others to get: $$\begin{bmatrix} 43 & 0 & 0 \\ ? & 1 & -3 \\ ? & 0 & 18 \end{bmatrix}$$ The determinant is simply 43(18) = 774.
20	10	$\frac{3}{64}$	Consider a situation where the three points lie in the 45-degree arc. Rotate so that the left most point is at the top. Any one of 3 points could be this point. The other two points must each lie in the 1/8 of the circle clockwise from it. $$prob = 3\left(\frac{1}{8}\right)\left(\frac{1}{8}\right) = \frac{3}{64}$$ A more exact solution would involve integrating the continuous probabilities.

Pressure Round

9	11	Answer	Solution
1	9	7/72	There are 216 equally likely cases. If the first one is a 1 then there are 6 ways of getting a total of 7 on the other two dice. If the first is a 2, there are 5 ways, etc. In total, there are 6+5+4+3+2+1=21 ways. $$\frac{21}{216}=\frac{7}{72}$$
2	2	[$] 144 [.00]	If n is the number of friends and p the price then: $$np=(n+1)(p-2)=(n-2)(p+6)$$ which yields two equations: $$-2n+p=2\ and\ 6n-2p=12$$ n=8, p=18 and the price of the room is $144.
3	3	$\frac{1651}{31}$	$$d=a+g+h$$ $$=\left(\frac{3+15+75}{3}\right)+\left(\sqrt[3]{3\times15\times75}\right)+\left(\frac{3}{\frac{1}{3}+\frac{1}{15}+\frac{1}{75}}\right)$$ $$=\frac{1651}{31}$$
4	9	42	5(x+3)=60, means x+3=12 and x=9. So 3(9+5)=3(14) =42.
5	5	19	Noting the denominator of 990 means the second and third decimal places repeat, that is, 0.abcbcbcbc. One could divide to get the first three decimals. $$a\frac{bc}{99}=\frac{99a+bc}{99}=10\left(\frac{937}{990}\right)$$ $$or\ 99a+bc=937$$ The decimal expansion is: $0.9\overline{46}$ and the sum of the digits is 19.
9	1	$\frac{1}{3}$	Let x be the answer. Rewriting the expression: $$x=\frac{1}{2+3x}\ or\ (3x-1)(x+1)=0$$ Only the positive solution of 1/3 is correct.

9	11	Answer	Solution
9	4	$\frac{70\pi}{3}$ [u^2]	Completing the square to put the ellipse into standard form. $$4(x+1)^2 + 9(y+2)^2 = 100 + 4 + 36 = 140$$ $$\frac{(x+1)^2}{35} + \frac{(y+2)^2}{\frac{140}{9}} = 1$$ The area is $ab\pi$, $$\sqrt{35\left(\frac{140}{9}\right)}\,\pi = \frac{70}{3}\pi$$

College Bowl Round 1

9	11	Answer	Solution
1	1	102	8 times 11 is 88 plus 14 is 102.
2	50	3375 [cm^3]	15 times 15 times 15 = 3375 cu cm.
3	50	465	31C2 = 31(30)/2 = 31(15) = 465.
4	4	12 [values]	Exact change can be made for any multiple of 5 cents from 5 to 60 cents making 12 values.
5	2	8	In order the elements are: -5, 1, 3, 8, 14, 14, 19 making the median 8.
6	6	6 [ways]	Because the twins and triplets are identical and must sit next to each other, the observer cannot distinguish between them in the groups. Therefore the problem is reduced to arranging 3 groups. 3!=6.
7	7	720	$(2x+3)^5 = (5C0)(2x)^5 + (5C1)(2x)^4(3^1) + (5C2)(2x)^3(3^2) + \cdots$ $(5C2)2^3(3^2) = 10(8)(9) = 720.$
8	8	9/2	$512 = 2^9 = 4^{\frac{9}{2}}$
9	50	32	$3024 = 2^4 3^3 7 = 2^2(2^2 3^3 7)$ so there are 3x4x2 multiples of 4 and also $3^2(2 \cdot 3 \cdot 7)$ gives 8 more multiples. Total is 32.
10	10	Four-zero-one-zero [base 9]	$236_9 \times 16_9 =$ Multiply as usual but in base 9. For example, 6x6=36 which is 40 base 9 so put down a 0 and carry the 4.
50	3	5/3	$\log_8 32 = \left(\frac{1}{3}\right)\log_2 32 = \frac{5}{3}$
50	5	108	The first term is then $\frac{8}{\left(\frac{1}{3}\right)^2} = 8(9) = 72$ And the sum is $\frac{72}{1-\frac{1}{3}} = 72\left(\frac{3}{2}\right) = 108.$
50	9	384 [ways]	First place the four couples around a round table (3! Ways), place an empty chair in any of the 4 spots between couples and finally each member of the four couples can swap seats (2^4=16). Total (6)(4)(16) = 384.

College Bowl Round 2

9	11	Answer	Solution
1	1	5	There are 2 fives and only one of the rest.
2	2	960	$(83^2 - 77^2) = (83 - 77)(83 + 77) = 6(160) = 960$
3	50	$\frac{15}{2}$ or Fifteen halves[feet]	Use similar triangles. $\frac{5}{12} = \frac{x}{12+6}; x = \frac{15}{2}$
4	4	672	$21(2^5) = 21(32) = 672$
5	50	32	$7 = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{7}{4}\right)x$
6	6	$\frac{1}{15}$	There are 90 two-digit numbers and includes 4^2 to 9^2. $\frac{6}{90} = \frac{1}{15}$.
7	7	1	$7^5 = (6+1)^5$ When expanded, each term except the last has a 6 in it.
8	8	35	$\frac{n(n-3)}{2} = 16n$ $n^2 - 3n = 32n.$ $n = 35$
9	50	310	We are looking for the sum of the first 20 evens minus the first 10 evens. (20)(21)-(10)(11) = 420-110 = 310.
10	10	2/3	The first term is 1 and the common ration is -1/2. The sum is: $s = \frac{a}{1-r} = \frac{1}{1--\frac{1}{2}} = 2/3.$
50	3	$\frac{25}{\pi}$ [feet^2]	The shape will be a circle. The radius is $r = \frac{10}{2\pi} = \frac{5}{\pi}$. The area is then $\pi\left(\frac{5}{\pi}\right)^2 = \frac{25}{\pi}$ sq feet.
50	5	$\frac{27\sqrt{3}}{2}$ [cm^2]	The hexagon can be broken into 6 equilateral triangles of side length 3. Each one has area $\frac{s^2\sqrt{3}}{4} = \frac{9\sqrt{3}}{4}$. Hexagon area is then $\frac{27\sqrt{3}}{2}$ sq cm.

9	11	Answer	Solution
50	9	$\frac{2+\sqrt{3}}{4}$ The quantity two plus root 3, over 4.	Use the half-angle formula, $\cos^2 15 = \frac{1+\cos(30)}{2} = \frac{1+\frac{\sqrt{3}}{2}}{2} = \frac{2+\sqrt{3}}{4}$

College Bowl Round 3

9	11	Answer	Solution
1	1	1287	The product of the GCF and the LCM is the product of the numbers: 33(39)=1287.
2	2	25 [grams]	$\left(\frac{5}{8}\right)40 = 25$ grams of shredded wheat.
3	50	2 or "2 to 1"	The side length makes no difference in the ratio. The outer circle has diameter equal to the diagonal of the square $s\sqrt{2}$ and the inner circle has diameter s. The ratio of the areas is then $\sqrt{2}^2 = 2$.
4	4	3 & 1/3	4-1=3 and 5/6 – 3/6=2/6=1/3. So $3\frac{1}{3}$.
5	50	7	61, 67, 71, 73, 79, 83, 89
6	6	$-\frac{1}{3}$	The slope is (27/9)=3. Perpendicular slope is -1/3.
7	7	$\frac{1}{27}$	$81^{-\frac{3}{4}} = \frac{1}{\left(81^{\frac{1}{4}}\right)^3} = \frac{1}{3^3} = \frac{1}{27}$
8	8	4 [hours]	$9000 < 2\,(2)^{\frac{t}{4}}$ $4500 < 2^{\frac{t}{4}}$ $2^{12} = 4096 < 4500 < 8192 = 2^{13}$ $\frac{13}{4} \leq 4 = t$
9	50	[$] 20	By subtracting, Squirrel got 3/8 of the pie. $\frac{8}{3}(\$7.50) = \$20$.
10	10	$3\sqrt{3}$ [in^2]	2/3 of the height of the triangle is 2 (the centroid). With a height of 3, the base will be $2\left(\frac{3}{\sqrt{3}}\right)$. The area is then $3\sqrt{3}$.
50	3	$\frac{8}{5}$	$\frac{20^4}{10^5} = \frac{2^4 10^4}{10(10^4)} = \frac{8}{5}$
50	5	-1	f(3x)=2(3x)^2+2(3x)-1, so f(x)=2x^2+2x-1 and f(-1)=-1.
50	9	1	(-4,5) -> (-4,-5) - > (6,-5)

College Bowl Round 4

9	11	Answer	Solution
1	1	91	Brute force. 216-125=91
2	2	2304	Can just multiply it out or use the shortcut (48-25)*100 + (50-48)^2 = 2300+4 = 2304.
3	50	$\frac{3}{2}$	The sum of the roots of $ax^2 + bx + c$ is-b/a
4	4	60 [cm^2]	Draw the altitude; the side length of 8 is the hypotenuse. The height is opposite the 30-degree angle and equals 4. The area is then 4(15) = 60 sq cm.
5	50	64π [feet^2]	The diameter of the pool is 18-2=16 and has radius 8. The area is then 64π sq ft.
6	6	60060	Thirteen books total with repeats. Cancel where you can. $\frac{13!}{3!\,4!\,6!} = 60060$
7	7	36	It will be one of these forms. p^8 *or* $p_1^2 p_2^2$, *let* $p_1 = 2, p_2 = 3$. The integer is 4(9)=36.
8	50	216/5	The first term will be 1(6)(6) = 36. The sum is: $\frac{36}{1-\frac{1}{6}} = \frac{216}{5}$
9	9	9455	n(n+1)(2n+1)/6 = 30*31*61/6 = 9455
10	10	3	$j(k) = (3)\,5^k - 72 = 303$ $(3)5^k = 375,$ $5^k = 125, k = 3$
50	3	60 [liters]	Butter is the limiting ingredient. 18 liters of butter goes with 9 liters of peanut and together make up 45% of the mixture. $(9 + 18)\left(\frac{100}{45}\right) = 60$ liters.
50	5	$80\sqrt{3}$ [sq m]	Icos = 20 faces that are equilateral triangles. Each one has an area of $\frac{s^2\sqrt{3}}{4} = 4\sqrt{3}$. Total area is $80\sqrt{3}$ sq meters/
50	8	(-1, -18) or negative one comma negative eighteen.	One can find the x-coord by –b/2a and then plug it in to get the y-coord. Or complete the square: $y = 8x^2 + 16x - 10$ $= 8(x + 1)^2 - 18$ Thus (-1, -18)

College Bowl Round 5

9	11	Answer	Solution
1	1	7848	36 (218) = 7848
2	2	3600 [outfits]	Multiply the number of each type. 5(3)(2)(5)(6)(4) = 18(10)(20) = 3600 outfits.
3	50	24 [factors]	Factor 1152. $1152 = 2^7 3^2$ The number of factors is (7+1)(2+1) = 24.
4	4	262 [cm^2]	A=2[7(8) + 7(5) + 8(5)] = 2[131] = 262 sq cm.
5	50	5 [years]	$T = 2F$ $(F + 10) = \frac{3}{4}(T + 10)$ $4(F + 10) = 3(2F + 10)$ F is 5 years.
6	6	220	One can manually add them but there is a formula. S=n(n+1)(n+2)/6 = 10(11)(12)/6 = 220.
7	7	7.5 or seven point five	We know that 7.5(7.5)=56.25. 7.4(7.4) is less than 55.
8	8	60 [m]	The ball drops 30m and twice the sum of an infinite geometric sequence with first term (1/3)(30)=10 and ratio 1/3. $30 + 2\frac{10}{1 - \frac{1}{3}} = 30 + 3(10) = 60\ m.$
9	50	5 [sq un]	One can embed the triangle inside a rectangle and subtract the right triangles or use the "shoelace" method. **1** **1** 3 **3** **4** 4 20 **5** **2** 6 2 **1** **1** 5 The numbers in the 1st and 4th columns are diagonal products. The area can be found by adding each of these two columns subtracting and divide by 2. (25-15)/2 = 5 sq un.

9	11	Answer	Solution
10	10	x between negative three and three non-inclusive.	The square root must be taken on non-negative numbers. It also cannot be zero since it is in the denominator. $9 - x^2 > 0,$ $-3 < x < 3.$
50	3	$8/15$	The angle is part of a right triangle with one leg of 8 and a hypotenuse of 17. By Pythagoras the other leg is 15. It makes the tangent $8/15$.
50	5	$11 - 2i$	$(2 + i)(4 - 3i) = \big(8 - 3(-1)\big) + (4 - 6)i = 11 - 2i$
50	9	$1/4$	$\sin 15 \sin 75 = \sin 15 \cos 15 = \left(\frac{1}{2}\right) 2 \sin 15 \cos 15 = \left(\frac{1}{2}\right) \sin 30 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

College Bowl Round 6

9	11	Answer	Solution
1	1	11025	10(11) * 100 + 5(5)
2	2	360 [gallons]	72 hours = 3 days. Fill rate = 35(24) = 840. Net is 120 gallons a day for a total of 360 gallons.
3	50	224	Factor 84. $84 = 2^2(3)(7)$ The sum of factors. $S = (1+2+4)(1+3)(1+7) = (7)(4)(8) = 224.$
4	4	752	Brute force. 2048-1296 = 752.
5	50	10.712 or ten point seven one two.	(2.6)(4.12) = 10.712
6	6	68 [cents]	The savings is 3.4% and 0.034 (20) = .68
7	7	220	Average = 20; 11 terms. 20*11 = 220
8	8	3/2 [un]	The altitude to the base of length six has measure 4 making the area of the triangle 12. The area is also the semi-perimeter (5+5+6)/2=8 times the radius of the inscribed circle which is 12/8 = 3/2.
9	50	$11551_{[6]}$	Add normally but remember to carry at 6 and not at 10. $3442_6 + 4105_6 = 11551_6$
10	10	35	1200 is close to n(n+1) n is somewhere between 34 and 35, i.e. there have been 34 34's already but not 35 35's.
50	3	5	Only need to solve: $30 = 4x + 10$ $20 = 4x; x = 5.$
50	5	Line	Perpendicular bisector between the two points.
50	9	-64	$(1+i)^{12} = ((1+i)^2)^6 = (2i)^6 = 64(-1) = -64$

College Bowl Extra Questions

9	11	Answer	Solution
1	1	30 [edges]	Euler's Formula: E=F+V-2. It has 20 faces and thus 12 vertices making 30 edges.
2	2	$\frac{1}{2652}$	$\left(\frac{1}{52}\right)\left(\frac{1}{51}\right)=\frac{1}{2652}$
3	3	One, two, and four	$y=x^2-8x+3$ Open upward so must go through I and II. Vertex is at x=4, y = -13 (IV) and has a positive y-intercept so it misses III.
4	4	7/15	Either two Blue or two Red. $\frac{6}{10}\left(\frac{5}{9}\right)+\frac{4}{10}\left(\frac{3}{9}\right)=\frac{42}{10(9)}=\frac{7}{15}$
5	5	19	$a+b=5\ \rightarrow$ $25=(a+b)^2$ $=a^2+b^2+2ab$ $=a^2+b^2+2(3)$ Answer = 25-6=19.
6	6	$\frac{3}{5}i-\frac{4}{5}j$	The magnitude is $\sqrt{15^2+20^2}=25$ so we scale the original vector by 25. $\frac{3}{5}i-\frac{4}{5}j$
7	7	5/4	$\lim_{x\to 2}\frac{x^2+x-6}{x^2-4}=\lim_{x\to 2}\frac{(x-2)(x+3)}{(x-2)(x+2)}=\lim_{x\to 2}\frac{x+3}{x+2}=\frac{5}{4}$
8	8	y = 0. Must includes the "y="	As x approaches minus infinity, e^x approaches 0.