Mental Math

9/11	Answer	Solution
1	10 [primes]	2, 3, 5, 7, 11, 13, 17, 19, 23, 29
2	96	120 – 24 = 96.
3	5/12	There are 6 ways of getting a 7. If these are subtracted and then divided by two we get: (36-6)/2=15. 15/36 = 5/12.
4	169	The formula is just n^2. 13^2 = 169.
5	720	All letters are different so the permutations are 6!=720.
6	36π [sq in]	The Volume is: $V = \frac{4}{3}\pi r^3, so\ r = 3.$ The formula is $SA = 4\pi r^2 = 4\pi(3^2) = 36\pi$
7	20 [palindromes]	2n2 and 3n3 where n is any digit. 2(10)=20
8	10 [integers]	Conceptually, there is a range of two times 20 = 40, divided by 4 = 10. OR $-20 < 4x - 2 < 20,$ $-18 < 4x < 22,$ $-4.5 < x < 5.5$

Individual Test

9/11	Answer	Solution
1	10 [cm]	The other leg is 8 cm as $\frac{6\cdot 8}{2} = 24$. The hypotenuse is $\sqrt{6^2 + 8^2} = 10\ cm$.
2	[y=] -3, 5 [either order]	Factor the quadratic: $0 = 4y^2 - 8y - 60 = 4(y-5)(y+3)$ y = -3, 5.
3	42	The sum of the middle two numbers will be the same as the first and the last. For example: a, a+d, a+2d, a+3d.
4	[x=] 3	Simplify: $8x - 4 - x + 3 = 3x + 11,\ 4x = 12, x = 3$.
5	A C B [in that order]	Put everything to the 5th power. $A = (2^4)^5 = 16^5, B = (5^2)^5 = 25^5, C = 17^5$ Now compare the bases: 16<17<25, so A C B.
6	$\sqrt{3}$ [inches]	The altitude forms a 30-60-90 degree triangle. The side opposite the 30 degree angle is ½ the hypotenuse or 1 and the other leg is $\sqrt{3}$.
7	36 [pencils]	5.00 – 3(0.20) = 4.40. Now divide by .12, 440/12 = 36+.
8	72 [sq cm]	Since a square is a rhombus, the area is one-half the product of the diagonals. A square is also a rectangle so its diagonals are equal. The area is then 12(12)/2 = 72 square centimeters.
9	0	$(4-5)^3 + 2 - 1 = (-1)^3 + 1 = 0$
10	144 [degrees]	The sum of the exterior angles in a convex polygon is 360, so in a regular decagon, each one measures 360/10=36 degrees. The interior angle is then 180-36=144 degrees.
11	22	$g(3) = 3(3^2) - 2(3) + 1 = 27 - 6 + 1 = 22.$
12	1/9	$3^4 \cdot 9^{-3} = 3^4(3^{-6}) = 3^{-2} = 1/9$
13	264 [cubic inches]	Since each area is the product of two of the sides, the product of the areas will be the square of the volume. $V = \sqrt{33 * 44 * 48} = 11 * 2 * 3 * 4 = 264.$
14	Average [income]	A few very large incomes will pull the average up quite a bit while having little or no effect on the median.
15	1/2	The line can be written: $y = -2x + 5/2$. The perpendicular slope is the negative reciprocal of this, which equals 1/2.

9/11	Answer	Solution
16	1/13	Multiplying, there are two equations: 3a-2b=1 and 2a+3b=0. Solving for a and b: $a=\frac{3}{13}, b=-\frac{2}{13}; a+b=\frac{1}{13}$.
17	186	$a+b=31, a-b=6, (a+b)(a-b)=a^2-b^2=186.$
18	8 [pairs]	Clearly x=100 and y=1 work, since 20 and 13 are relatively prime, for every decrease of 13 in x, we can increase y by 20. Therefore x=9, 22, 35, 48, 61, 74, 87 and 100 for a total of 8 possible values.
19	11	The slope is $\frac{-1-8}{5-2}=-3$. Then $8=f(2)=-3(2)+b, b=14$. Finally, $-3+14=11$
20	$m+16$	Each of the four new numbers is 4 more than one of the previous ones. Therefore the sum is the old sum plus 4x4. $m+16$
21	5/9	The numbers 1, 4 and 6 each have one part and the numbers 2, 3 and 5 each have two parts for 9 parts total. The probability of an odd number is then (1+2+2)/9=5/9
22	76	There are 9C3 = 9(8)(7)/6=84 ways of choosing 3 points. There are 8 sets of three in the grid that are collinear. The total is then 84 – 8 = 76 possible triangles.
23	7 [mm]	Two of these altitudes are legs of the triangle. Since the hypotenuse is the longest side, choose $2\sqrt{6}$ and 5 for the legs. The area can be calculated in two ways: $\frac{5\left(2\sqrt{6}\right)}{2}=\frac{\frac{10\sqrt{6}}{7}h}{2}, h=7mm$
24	[x=] -2	Equate slopes: $\frac{2x+1-2}{x-1}=\frac{12-2}{7-1}, \quad x=-2$
25	28	Multiplying by 10=2x5 adds a zero to the end. There are plenty of factors of 2, just need to count the factors of 5. Every 5th number adds a 5, every 25th adds 2 factors, etc. 123/5 = 24 and 24/5 = 4 for a total of 24+4=28.

9/11	Answer	Solution
26	92	Make a systematic list listing the quarters and dimes – the rest can be made up in nickels.. Q – D 7 – 2,1,0 (3 ways) 6 – 4,3,2,1,0 (5 ways) 5 – 7,6,5,4,3,2,1,0 (8) 4 – 9,8,7,6,5,4,3,2,1,0 (10) 3 – 12,…0 (13) 2 – 14,…,0 (15) 1 – 17, …, 0 (18) 0 – 19, …, 0 (20) 20+18+15+13+10+8+5+3=92
27	$400/\pi$ [sq meters]	Largest area with smallest perimeter is a circle. $C = 40 = 2\pi r, r = 20/\pi$ $A = \pi r^2 = \pi\left(\frac{20}{\pi}\right)^2 = 400/\pi$
28	4096	$\log_2 \log_4 \log_8 x = 0, \quad \log_4 \log_8 x = 1, \log_8 x = 4, \qquad x = 8^4 = 4096.$
29	1/2	Normally would need to factor 63,946; but in this case, 63946/2 is odd which means all the rest of the factors are odd. Since there is only 1 factor of 2, each odd factor has an even counterpart, i.e. multiply by 2. Therefore, ½ of the factors are even.
30	11	The numbers 1, 2, 3, …10 all have multiples in the numbers 12, 13, …, 20. This just leaves 11 in the numerator that distinguishes these two lcm's.
31	1 [point]	Initially ignore the +2 at the end. The graph is negative for x<-3 and crosses the x-axis there, it only touches the x-axis at 2 and 3. Adding 2 pushes the two touches above the x-axis and there is only one crossing.
32	4	Just like the divisibility rule for 9 in base 10; the sum of the digits must be divisible by 5. To prove, write the number as: $4(5+1)^6 + 2(5+1)^5 + 3(5+1)^4 + 2(5+1)^3 + x(5+1)^2 + 2(5+1) + 3(1)$. When expanded all the terms have a power of 5 except the sum of the digits.

9/11	Answer	Solution
33	$\frac{169}{24}cm$	The center of the circle will be R cm from each of the vertices and lie on the perpendicular bisector of each of the sides. Drop the perpendicular to the short side, which divides it into two equal lengths of 5. That altitude will have length 12 by Pythagoras. The line from a base vertex to the center of the circle forms another right triangle with lengths: 5, R and 12-R which can be solved for R. $$5^2 + (12 - R)^2 = R^2, R = \frac{169}{24}$$
34	$\frac{37}{180}$	Use partial fractions to represent: $$\frac{1}{x^2 + 9x + 18} = \frac{1}{3}\left(\frac{1}{x+3} - \frac{1}{x+6}\right)$$ Thus the series telescopes to yield a sum of: $$\frac{1}{3}\left(\frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right) = \frac{37}{180}.$$
35	7	15! is divisible by 9 so that the sum of the digits must be divisible by 9 and y=7 for this to be true.
36	33	$$\frac{4n^2 - 13n + 17}{n - 4} = (4n + 3) + \frac{29}{n - 4}$$ n-4 must divide into 29, so the largest would be n=33.
37	-2	1) Translate the point and line up 1: $y = x, (-6,5)$ 2) Reflect point by switching coord: $(5, -6)$ 3) Translate back down 1: $(5, -7)$ Sum: $5 - 7 = -2$
38	16	The sum of coefficients can be found by setting x=1 and evaluating the polynomial, so set x=1 from the start. $$\Big(f\big(g(-1-1)\big)\Big)^2 = \big(f(1)\big)^2 = 4^2 = 16$$
39	$\frac{\sqrt{3}+3}{6}$	Let the side length = 1. Each triangular face of the octahedron has area: $A = \frac{s^2\sqrt{3}}{4} = \frac{\sqrt{3}}{4}$. The square base has area 1. The ratio is then: $$\frac{1 + 4\left(\frac{\sqrt{3}}{4}\right)}{8\frac{\sqrt{3}}{4}} = \frac{1 + \sqrt{3}}{2\sqrt{3}} = \frac{\sqrt{3}+3}{6}$$

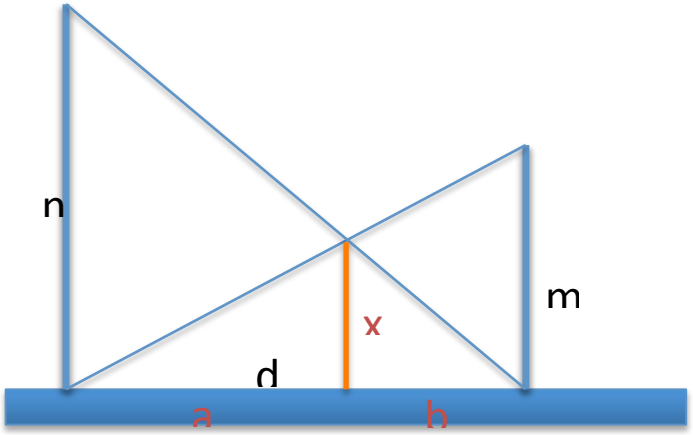
9/11	Answer	Solution
40	1	$3S = 1 + \frac{3}{3} + \frac{5}{9} + \frac{7}{27} + \cdots$ $3S\text{-}S=2S = 1 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \cdots$ $2S = 1 + 2\left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots\right) = 1 + 2\left(\frac{1}{2}\right) = 2$

Individual Multiple Choice

9	11	Answer	Solution
1	1	C	3^n cycles between 3-9-7-1 and 7^n cycles between 7-9-3-1. So product ends in 1x5x1x1=5
2	2	D	$2^x + 2^x + 2^x + 2^x = 4 \cdot 2^x = 2^2 2^x = 2^{x+2} = \left(8^{\frac{1}{3}}\right)^{x+2} = 8^{\frac{x+2}{3}}$.
3	3	A	Form a systematic list using the fact that two sides must add up to more than the third. The longest side must be 4 and two possibilities for the others 4,2 and 3,3.
4	4	B	The remainder when a positive integer is divided by 3 is 0, 1 or 2. If three numbers have the same remainder then their sum is divisible by 3. If 3 numbers all have different remainder, their sum is divisible by 3. One can choose two numbers with one remainder and two others with a different remainder as the largest set for a total 4 elements.
5	50	A	The two higher scores are two parts of the total while the lowest is only one part. Compute: 2(82)+2(90)=344 and I need 5(80)=400 to average 80. Therefore, the last score must be 400-344=56.
6	6	B	The small cubes with two faces painted will be on the interior edges = 5 on each. There are 12 such edges for a total of 60 cubes.
7	50	A	$1@4 = \frac{\binom{4}{1} - \binom{8}{4}}{1^{-4}} = 4 - \frac{8(7)(6)(5)}{4(3)(2)}$ $= 4 - 70 = -66$
8	8	C	Imagine triangles with a base of 15 and one other side of 8 that swings in an arc to make the triangles. (A) and (B) are acute while (D) is obtuse. (C) obtains the maximum height, and thus area, by being a right triangle.
9	9	B	Since 3 and 7 are roots, $P(x) = (x-3)(x-7)R(x), so\ P(5) = (5-3)(5-7)R(x) = -4R(x)$. Therefore, $P(5)$ must be a multiple of 4, so -16.
10	10	D	At each roll, 5/6 of the dice are expected to remain, so after the first roll (5/6)60 is the expected amount, then 5/6 of that, etc. $e_n = 60\left(\frac{5}{6}\right)^n$
50	5	C	Add one to both sides. $118 + 1 = 119 = 7(17) = m + n + mn + 1 = (m+1)(n+1)$. Therefore m,n = 6, 16 and $m + n = 22$
50	7	D	$(\sin\alpha + \cos\alpha)^2 - \sin(2\alpha) = \sin^2\alpha + 2\sin\alpha\cos\alpha + \cos^2\alpha - 2\sin\alpha\cos\alpha = 1$

9	11	Answer	Solution
50	10	B	e^{-x} tends to 0 no matter what polynomial it is multiplied by as x goes to infinity. The horizontal asymptote is then y=0. There is no vertical asymptote – instead, x=3 represents a hole in the graph. 1 Asymptote.

Team Test

9	11	Answer	Solution
1	1	2 : 3	One could calculate the area and then divide but it is much easier to connect the midpoints of the sides of the triangle to see 4 triangles each congruent to 1/6 of the hexagon. 4:6 = 2:3.
2	20	2	p and q are clearly odd since they don't have 2 as a factor. The sum is then even and has a factor of 2.
3	3	66 [sq cm]	Heron's Formula. $s = \frac{11+13+20}{2} = 22$ $A = \sqrt{22(22-11)(22-13)(22-20)} = \sqrt{22(11)(9)(2)} = 11(3)(2) = 66$
4	4	1/8	We can factor 8 out of the numerator and 64 out of the denominator leaving identical factors. The result is then 8/64=1/8.
5	5	18 [numbers]	The triangular numbers are given by the formula: $T_n = \frac{n(n+1)}{2}$, so either n or n+1 must be divisible by 11. This gives n=10,11, 21, 22, … 98, 99 or 18 total numbers.
6	20	$\frac{mn}{m+n}$ [feet]	 Using similar triangles: $\frac{x}{n} = \frac{a}{a+b}, \frac{x}{m} = \frac{b}{a+b} = 1 - \frac{x}{n}$. Solve for x: $x\left(\frac{1}{m} + \frac{1}{n}\right) = 1, x = \frac{mn}{m+n}$

9	11	Answer	Solution
7	7	15 [boys]	One could set up two equations and two unknowns. $$g+b=35, 87g+80b=35(84)$$ It is easier to set up a proportion, 84 is 4/7 of the way from 80 to 87. The boys to girls ratio is then 3:4 and the number of boys is (3/7)35=15.
8	8	647 [cents]	Formula: $ab-a-b=37\cdot 19-37-19$
9	9	72	The four consonants must be in spots 1-3-5-7 which can be done in 4! ways. The three vowels can be arranged in 3 ways since there are 2 I's. In total there are 24 times 3 = 72 ways.
10	20	$11110011_{[2]}$	Convert base 8 to base 2 taking 1 digit to 3 digits; back to base 16 using 4 digits to make 1. $271 \rightarrow 010111001 \rightarrow$ B9 B9 + 3A = F3 F3 $\rightarrow$ 11110011
20	6	4π	$\frac{1}{2}\sin\left(\frac{x}{2}\right)=sin\left(\frac{x}{4}\right)\cos\left(\frac{x}{4}\right)$ using the double angle formula. Factors as $\left(\cos\left(\frac{x}{4}\right)-1\right)\left(\sin\left(\frac{x}{4}\right)+1\right)=0.$ $$\cos\left(\frac{x}{4}\right)=1, x=-8\pi, 0, 8\pi$$ $$\sin\left(\frac{x}{4}\right)=-1, x=-2\pi, 6\pi$$
20	2	100	You can add multiples of one row to another without changing the determinant. If one subtracts twice the first row from the second and adds it to the third we get: $$\begin{vmatrix}1 & -2 & 3 & 2\\ 0 & 5 & -9 & -3\\ 0 & 0 & 4 & 2\\ 0 & 0 & 0 & 5\end{vmatrix}.$$ For an upper (or lower) triangular matrix, the determinant is just the product of the main diagonal. 1(5)(4)(5)=100.
20	10	$\frac{7}{12}$	Let $u=x^2-6, du=2xdx$ $$\int_{\sqrt{7}}^{3}\frac{7x}{4(x^2-6)^2}dx=\frac{7}{8}\int_{\sqrt{7}}^{3}\frac{2x}{(x^2-6)^2}dx$$ $$=\frac{7}{8}\int_{1}^{3}\frac{1}{u^2}du=\frac{7}{8}\left[-\frac{1}{u}\,|_1^3\right]=\frac{7}{8}\left(\frac{2}{3}\right)=\frac{7}{12}$$

Pressure Round

9	11	Answer	Solution
1	9	16	$e^{\left(\frac{2}{\log_4 e}\right)} = e^{2\log_e 4} = \left(e^{\log_e 4}\right)^2 = 4^2 = 16$
2	2	2304	Need the coefficient of 'x' in $(3x+4)^4$ which will be $(3x)(4)(4)(4) = 192x$ which can occur 4 ways. 3(4)(192)=2304
3	9	32 8/11 [min]	The minute hand is at 0 degrees and travels at 6 degrees per minute. The hour hand is at 90 degree and travels at ½ degree per minute. We need the minute hand to pass the hour hand by 90 degrees. $0 + 6m = 90 + \frac{m}{2} + 90.$ $\frac{11}{2}m = 180, \quad m = \frac{360}{11} = 32\frac{8}{11} minutes$
4	9	252	$(r+s)^3 = r^3 + s^3 + 3rs(r+s), 216 = r^3 + s^3 + 3(-2)(6)$ $r^3 + s^3 = 216 + 36 = 252.$
5	5	$3\sqrt{13}$ [units]	Unfolding the tetrahedron, one obtains a two-dimensional figure. AX on the base measures 3, AC measures 12 and is at an angle of 60 degrees with the base. The upper base CY parallel to the lower base measures 6. The segment XY forms two similar triangles – one twice the size of the other. Use this fact and the law of cosines to find its measure. OR, assign coordinates to X and Y. If A is the origin: $X = \left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right), Y = \left(9, -3\sqrt{3}\right)$ and use the distance formula.
9	1	30	There are three subsets {1,4,7}, {2,5,8} and {3,6,9} where each has the same remainder when divided by 3 and there are 27 (3^3) others where one of the elements is 1, 4, or 7; another is 2, 5, 8 and the third is 3, 6 or 9.
9	3	-1	Since $\cos A = \frac{3}{\sqrt{10}}, \sin A = \frac{1}{\sqrt{10}}$. $\log \sin A + \log \cos A + \log \tan A = \log (\sin A \cos A \tan A) = \log \sin^2 A = \log \frac{1}{10} = \log 10^{-1} = -1$

College Bowl Round 1

9	11	Answer	Solution
1	1	7225	All you have to do is 8(9) followed by 5(5)=7225.
2	50	20	An icosahedron has 20 equilateral triangle faces.
3	50	8	$1001 = 7^1 \cdot 11^1 \cdot 13^1$. This makes the number of positive factors at 2x2x2 = 8.
4	4	7/13	The slope of the line is -13/7, the perpendicular line has the negative reciprocal for a slope. 7/13.
5	2	86,400 [seconds]	$60\times60\times24 = 86{,}400$
6	6	0	Since the discriminant $b^2 - 4ac = 16 - 72 < 0$ the solutions are complex.
7	7	2	Mod is simply the remainder when 30 is divided by 7 – which equals 2.
8	8	2002	$14C5 = \frac{14(13)(12)(11)(10)}{5(4)(3)(2)} = 2002$
9	50	18	The average of a six-sided is 3.5, 8 sides is 4.5 and 12 sides 6.5. Total is 18.
10	10	48[mph]	The average speed is the harmonic mean of the two speeds. $AS = \frac{2(40)(60)}{40+60} = 48$ One can also choose a distance, say 120 miles and compute total distance divided by total time. $\frac{2(120)}{3+2} = 48$.
50	3	7	$\log_4 64 - \log_2 \frac{1}{16} = 3 - (-4) = 7$
50	5	50	$11_2 + 22_3 + 33_4 + 44_5 = 3 + 8 + 15 + 24 = 50$
50	9	4	Sine is negative means that y is negative but the tangent being negative means x is positive – 4th quadrant.

College Bowl Round 2

9	11	Answer	Solution
1	1	4891	$(70-3)(70+3) = 70^2 - 3^2 = 4900 - 9 = 4891$
2	2	135 [degrees]	The exterior angle will be 360/8=45 making the interior angle 180-45 = 135 degrees.
3	50	744	Clearly the GCD of 24 and 124 is 4 which means the LCM is 6 x 124 = 744.
4	4	$-\frac{12}{7}$	$\frac{-10-2}{8-1} = -\frac{12}{7}$
5	50	63,360 [inches]	$12 \times 5280 = 63{,}360$
6	6	-4	This factors as $(x+4)(x-3)$ and the smaller root is -4.
7	7	2	Two divisibility rules. 2345 / 11 $5-4+3-2=2$ $23+45=68$ which has a remainder of 2.
8	8	35	$7C3 = \frac{7(6)(5)}{3(2)} = 35$
9	50	240	There are essentially 5 units to be arranged which can be done in 5!=120 ways. There are two ways of ordering the parents for a total of 120x2=240.
10	10	154 [elevators]	From the data, each elephant destroys 1 elevator in an hour. In 2 hours then 77 elephants can destroy 154 elevators.
50	3	2	$\log_5 100 - 2\log_5 2 = \log_5\left(\frac{100}{2^2}\right) = \log_5 25 = 2$
50	5	$121_{[4]}$ OR one-two-one	$25 = 1(16) + 2(4) + 1(1) = 121_4$
50	9	$\frac{24}{25}$	$\sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}, \sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$

College Bowl Round 3

9	11	Answer	Solution
1	1	25π	The side length of the square is the diameter of the circle, that makes the radius 5 and the area of the circle: 25pi.
2	2	8	1, 1, 2, 3, 5, 8
3	50	2048	Just multiply it. Easy to remember: $2^{10} = 1024$
4	4	616	The product of the GCF and LCM will be the product of the numbers. $22(28) = 616$
5	50	90	9 choices for the 1st digit which must also be the third; there are 10 choices for the 2nd digit. 9(10)=90.
6	6	21	The number of ways two vertices can be connected is: 7C2 = 21.
7	7	348	97 + 89 + 83 + 79=348
8	8	4π	The hypotenuse is 13, which makes the semiperimeter is s=(5+12+13)/2=15. The area of the triangle (which we know to be 30) is also the s times the radius of the circle. The radius must be 2 and the area 4pi.
9	50	20	6 faces, 9 edges, 5 vertices (2 tetrahedrons glued together)
10	10	871π/4 [sq ft]	The crocodile will be able to graze over 3/4 of a circle of radius 17. There will still be a 2 foot length that will be another 1/4 circle. $A = \frac{3}{4}\pi 17^2 + \frac{1}{4}\pi 2^2 = \frac{871\pi}{4}$
50	3	even	Sine is an odd function but it doesn't matter since the cosine is done last and it is even.
50	5	5π/4 [rad]	$225\frac{\pi}{180} = \frac{5\pi}{4}\ radians$
50	9	−18	$f\big(g(0)\big) = f(-4) = -18$ $g\big(f(0)\big) = g(2) = 0$ The difference is -18.

College Bowl Round 4

9	11	Answer	Solution
1	1	222 [cubic units]	The volume is: $V = \frac{1}{3}Ah = \frac{1}{3}(37)(18) = 37(6) = 222$
2	2	381	$15\% = \frac{3}{20}.\quad \frac{3}{20}(2540) = 3(127) = 381$
3	50	784	$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2 = \left(\frac{7(8)}{2}\right)^2 = 28^2 = 784$
4	4	120	Triangle numbers $T_n = \frac{n(n+1)}{2} = \frac{15(16)}{2} = 120$
5	50	362,880	Just multiply it out.
6	6	7	Since 13 ends in 3, the unit place of the powers cycles from 3 – 9 – 7 – 1 so the 7th power will end in 7.
7	7	9	Perfect numbers are equal to the sum of their proper factors: 6, 28, 496. The 12th prime is 37. 37-28=9.
8	50	6	Two sides must add to more than the third side of the triangle. The range goes from 14 to 20 inclusive, the difference is 6.
9	9	35	Three factors means they have to be squares of prime numbers, namely 25 and 49. The geometric mean will be 5(7)=35.
10	10	12 [hours]	Standard work problem: $\frac{1}{4} + \frac{1}{x} = \frac{1}{3},\quad x = 12\ hours$
50	3	2	Since $6^{\frac{3}{2}} = \sqrt{6^3} = \sqrt{216} < 15$, the $\log_6 20 > 1.5$ and 2 is the closer integer.
50	5	14	$16x^2 + 9y^2 - 64x - 54y + 1 = 0$ $16(x^2 - 4x + 4) + 9(y^2 - 6y + 9) = -1 + 64 + 81$ $\frac{(x-2)^2}{9} + \frac{(y-3)^2}{16} = 1$ The two axes have lengths 2(3)+2(4)=14.
50	8	12/5	$f(x) = 5x + 8 = 20,\ \ 5x = 12,\ \ x = \frac{12}{5}$

College Bowl Round 5

9	11	Answer	Solution
1	1	11	$3+4(5-3)=3+4(2)=3+8=11$
2	2	$\sqrt{227}$ The square root of 227.	$\sqrt{5^2+9^2+11^2}=\sqrt{25+81+121}=\sqrt{227}$
3	50	20/3	On the y-axis, x=0; so that 3y=20, y=20/3. 0+20/3 = 20/3.
4	4	56	In this case, it is probably easier to just to add them: 1+2+4+7+14+28=56, OR note that 28 is a perfect number so the total will be 2(28)=56; OR factor it $28=2^2\cdot 7$ so the total will be (1+2+4)(1+7)=56.
5	50	120 [square units]	This is double the 5-12-13 Pythagorean triple so that it is a right triangle with area: $\frac{1}{2}(10)(24)=120$.
6	6	1/5525	There are 4 kings, then 3, then 2. The probability is: $\frac{4}{52}\cdot\frac{3}{51}\cdot\frac{2}{50}=\frac{1}{13}\cdot\frac{1}{17}\cdot\frac{1}{25}=\frac{1}{5525}$
7	7	2	This is Euler's formula, the answer is 2 regardless of the convex polyhedral. $V=12, E=30, F=20.\ 12-30+20=2$
8	8	-45	The product of the roots in a cubic equation is $-\frac{a_0}{a_3}=-\frac{225}{5}=-45$
9	50	27/2	The sum of an infinite geometric series is $S=\frac{a}{1-r}=\frac{9}{1-\frac{1}{3}}=\frac{27}{2}$
10	10	[x=] 2	$4^{2x-1}=8^x,\quad 2^{4x-2}=2^{3x},\ 4x-2=3x,\qquad x=2$
50	3	2	2+2=4 and 4 is $100_2, 11_3, 10_4, 4_5, etc$. Therefore it is a 2-digit number in only 2 bases.
50	5	450 [deg]	$\frac{5\pi}{2}\left(\frac{360}{2\pi}\right)=450°$
50	9	0	$\lim_{x\to\infty} xe^{-x}=\lim_{x\to\infty}\frac{x}{e^x}=\lim_{x\to\infty}\frac{1}{e^x}=0$ Using L'Hopital.

College Bowl Round 6

9	11	Answer	Solution
1	1	1	$\frac{5}{7+3}+\frac{4}{8}=\frac{1}{2}+\frac{1}{2}=1$
2	2	9	Since both 21 and 51 are multiples of 3, their product will be a multiple of 9 and thus have a digital root of 9.
3	50	-8	The sum of the roots is –b/a = -32/4 = -8.
4	4	2	$\frac{1}{\frac{1}{3}+\frac{1}{6}}=\frac{1}{\frac{1}{2}}=2$
5	50	120 pi [sq units]	The lateral area is the circumference of the base times the height, the top and bottom are found by pi radius-squared. $SA=2\pi(5)(7)+2(\pi)5^2=(70+50)\pi=120\pi$
6	6	0.4116	There are 4 days that it may or may not rain. $\Pr(Rain=3)=4(.7)^3(.3)=0.4116$
7	7	350 *pi* [cubic units]	The height of the full cone would be 12 using similar triangles. The volume of the frustum will be 7/8 (i.e. $1-\left(\frac{1}{2}\right)^3$) of the full volume. $V_f=\left(\frac{7}{8}\right)\left(\frac{1}{3}\right)\pi 10^2(12)=350\pi$
8	8	-3	By the remainder theorem, if x-1 divides polynomial P(x), then P(1)=0 which only happens if k=-3.
9	50	3725	There are 25=100/4 houses on the block. For an Arithmetic series, the sum is the average of the first and last number times the number of values. $S=n\frac{f+l}{2}=25\frac{101+197}{2}=25(149)=3725$
10	10	16	The cube root of 8 squared is 4 and the square root of 16 equals 4.
50	3	11	Representing the number $77_b=7b+7=84,\ b=11$.
50	5	2 pi over 7 $\left[\frac{2\pi}{7}\right]$	Keep adding 2π until it is positive. $-\frac{26\pi}{7}+2\pi<0, -\frac{26\pi}{7}+4\pi=\frac{2\pi}{7}>0$

9	11	Answer	Solution
50	9	Two-thirds [2/3]	$\lim_{x\to\infty}\frac{2x^2+2x-7}{3x^2+8x+10}=\lim_{x\to\infty}\frac{2+\frac{2}{x}-\frac{7}{x^2}}{3+\frac{8}{x}+\frac{10}{x^2}}=\frac{2}{3}$

College Bowl Extra Questions

9	11	Answer	Solution
1	1	3.875	(3+1+4+1+5+9+2+6)/8 = 31/8 = 3.875
2	2	20	There are two right triangles. The first is 2x2 with area 2 and the second 6x6 with area 18 for a total of 20.
3	3	$6\sqrt{2}$	The geometric mean is: $G = \sqrt{ab} = \sqrt{8(9)} = \sqrt{72} = 6\sqrt{2}$
4	4	38	$19 - (-19) = 38$
5	5	$6\sqrt{10}$	Use Heron's formula: s=(6+7+11)/2 = 12 $A = \sqrt{(12)(6)(5)(1)} = 6\sqrt{10}$
6	6	2	$(1+i)(1-i) = 1 - i^2 = 1 - (-1) = 2$