Mental Math

9/11	Answer	Solution
1	30	$1^2 + 2^2 + 3^2 + 4^2 = 1 + 4 + 9 + 16 = 30$
2	10	If the radius of the circle is just a little greater than the apothem (distance to each side) then the circle will intersect each side twice or 10 total.
3	159	47 + 53 + 59 = 159
4	1	A circle with a circumference of pi has a diameter of 1 ($C = \pi d$) which is also the side length of the square giving it an area of 1.
5	125	With a mean of 100, the six numbers must total to 600, if two of them total 100 (2*50) then the other four add to 500 giving a mean of 500 / 4 = 125.
6	7/8	This leaves out the chance of all heads or all tails each with probability 1/16. $1 - 2\left(\frac{1}{16}\right) = \frac{7}{8}$.
7	0	The five points will make a regular pentagon. Picking three of them will guarantee at least two are consecutive. Regardless of the choice of the third point, the triangle will be isosceles. The can be no scalene triangles.
8	11	The coefficient will be the sum of roots taken two at a time. 1(2) + 1(3) + 2(3) = 11.

Individual Test

9/11	Answer	Solution
1	6	$$4-\frac{2^3-4^2}{4}=4-\frac{8-16}{4}=4-(-2)=6$$
2	[x=] 5	This simplifies to: $4x+10=30, x=5$
3	24	There are only two possibilities. $7^2+25^2$is not a perfect square but $25^2-7^2=24^2$.
4	90	The new average is: $$\frac{9(85)+(1+2+3+\cdots+9)}{9}=85+\frac{45}{9}=90$$
5	-1	$$h(-1)=\frac{(-1)^2+1}{-1-1}=\frac{2}{-2}=-1$$
6	36 [degrees]	Each interior angle will be $\frac{3(180)}{5}=108$ degrees. Since the diagonal makes an isosceles triangle, the base angles are $\frac{180-108}{2}=36$ degrees. The angle between the diagonals is then $108-2(36)=36$ degrees.
7	$3x+2y=5$ *or* $y=-\frac{3}{2}x+\frac{5}{2}$	To be parallel, the coefficients of x and y remain the same, $3x+2y$ not fit it to (1,1) by substituting those values for x and y. $$3x+2y=5$$ One can change it to slope-intercept form but not necessary.
8	1/4	$$\frac{2^8+2^8+2^8+2^8}{8^4}=\frac{4(2^8)}{(2^3)^4}=\frac{2^{10}}{2^{12}}=\frac{1}{2^2}=\frac{1}{4}$$
9	37 [crayons]	A full box of crayons is $8(20)+50=210$ *cents*. If I buy four of them, it costs $8.40 leaving $1.60. Another box costs 0.50 leaving $1.10 for crayons and I can but 5 more. Total crayons is 4(8)+5=37.
10	60 [ways]	Leaving out the 2 R's for the ends, there are 5 letters that we can permute and 2 of them are I's. The total is then: $\frac{5!}{2!}=\frac{120}{2}=60$.
11	$\frac{100}{\pi}$ [sq. cm]	Since $C=2\pi r, r=\frac{10}{\pi}$. $A=\left(\frac{10}{\pi}\right)^2\pi=\frac{100}{\pi}sq\ cm$.
12	4 [cm]	If we draw all the diagonals, they form 6 equilateral triangles with side length 2. The longest diagonal is simply two of those sides and measures 4 cm.
13	[x=] 0, -5, 2 [any order]	Take the $10x$ over to the other side. $$x(x+5)(x-2)=0, \quad x=\{0,-5,2\}$$

9/11	Answer	Solution
14	70 [sq in]	The diagonals are perpendicular to each other and the area of a quadrilateral with this property is one-half the product of the diagonals. $A = \frac{1}{2}(14)(10) = 70\ sq\ in.$
15	800	Factor as the difference between two squares. $54^2 - 46^2 = (54-46)(54+46) = 8(100) = 800.$
16	302.5	$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$ The average is: $$\frac{\left(\frac{10(11)}{2}\right)^2}{10} = \frac{55^2}{10} = \frac{3025}{10} = 302.5$$
17	25	The product of the two missing numbers will just be $5^2 = 25$
18	$5+5i$	$(1+2i)(3-i) = (3 - i + 6i - 2i^2) = 5 + 5i; since\ i^2 = -1.$
19	23	Two cases: $k - 7 < 12; k < 19\ AND\ k - 7 > -12; k > -5$. So all the integers from -4 to 8, 23 in all. OR, one can read this as the distance from k to 7 is less than 12. There will be 11 on each side of 7 and of course 7 itself for a total of 2(11)+1=23.
20	8	There are 2 equilateral and thus isosceles triangles and 6 more that use three consecutive vertices for a total of 8.
21	3	This is a geometric series with first term, 2, and common ratio, 1/3, $$S = \frac{a}{1-r} = \frac{2}{1-\frac{1}{3}} = \frac{2}{\frac{2}{3}} = 3$$
22	$64 - 16\pi.$	The side length of a square is 4(2)=8 so the area of the square is 64. Each circle has area $\pi r^2 = 4\pi$ and there are 4 of them. The shaded area is then: $64 - 16\pi$.
23	[a =] 5/4	All you have to do is look at the (1,2) element of the sum. $4a + 3 = 8, a = \frac{5}{4}$.
24	26	Solve using two cases: $4 - 2x \geq 0\ and\ 4 - 2x < 0$. Solutions are -1 and 5. So sum of squares is 1 + 25 = 26
25	1007	Using Log Rules, the sum of logs is the log of the product. After canceling, the right hand side is just: $\log\frac{2}{2014}$. The equation simplifies to $\frac{1}{x} = \frac{2}{2014}$, and the result follows.
26	(8, -6)	Reflecting (0,4) about the line $y = -x$ gives the point (-4, 0). Reflecting that about the line $x = 2$ gives the point (8,0). Finally, reflecting about $y = -3$ leads to D=(8, -6).

9/11	Answer	Solution
27	\$113	Gary gives away \$20 more than he got so he started with the average \$93 plus \$20 equals \$113.
28	18	First subtract the first row from the third, which does not change the determinant and expand about the first column. $$\begin{vmatrix} 2 & -2 & 1 \\ 0 & 3 & -1 \\ 0 & 3 & 2 \end{vmatrix} = 2\begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix} = 2(9) = 18$$
29	$\sqrt{17}$	The first equation can be rewritten: $x = 4(2^t) - 2$. Solve the second equation for 2^t and plug into the first to get $x = 4y - 6$ so it is a straight line. When t=0, the point is (2,2) and when t=1 the point is (6,3). Use the distance formula to get a length of $\sqrt{17}$.
30	1/3	The volume of the cone is: $V = \frac{1}{3}\pi r^2 h = 18\pi$. The surface area of the cylinder is: $SA = 2\pi r^2 + 2\pi rh = 54\pi$. The ratio is then 1:3 or 1/3.
31	50	An icosahedron is made up of 20 equilateral triangle faces. Each triangle has 3 sides but are shared with another triangle. Total = $20 + \frac{3(20)}{2} = 50$.
32	$\frac{375\pi}{4}$	The area of the outside circle is: $A = \pi 20^2 = 400\pi$. If we slide the left shaded region, we almost fill a quarter of the large circle except for one quarter of the smallest circle. The shaded region is then: $$A = \frac{400\pi}{4} - \frac{25\pi}{4} = \frac{375\pi}{4}$$
33	$3\sqrt{5}$	Complete the square: $$x^2 - 6x + y^2 + 10y = 11$$ $$x^2 - 6x + 9 + y^2 + 10y + 25 = 11 + 9 + 25 = 45$$ $$r = \sqrt{45} = 3\sqrt{5}$$
34	$x = 1$	Once factored: $y = \frac{(x+1)(x+2)}{(x+1)(x-1)}$. So there is a vertical asymptote at x=1 and a "hole" at x=-1.
35	$\frac{\pi}{2}$	If we multiply both sides of the equation by r, we get: $r^2 = r\sin\theta + r\cos\theta$. Now substituting rectangular coordinates: $x^2 + y^2 = x + y$. Once the squares are completed. $$x^2 - x + \frac{1}{4} + y^2 - y + \frac{1}{4} = \frac{1}{2}$$ $$r^2 = \frac{1}{2} and\ A = \frac{\pi}{2}$$

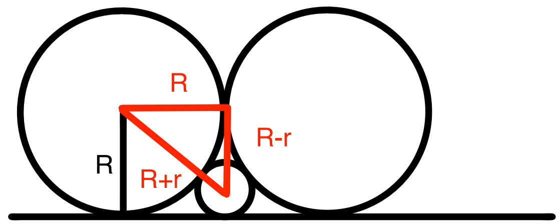
9/11	Answer	Solution
36	$\frac{4\sqrt{6}}{3}$	The triangle has vertices: (0,0), (x, 4-x) and (4-x, x). Equating the distance apart leads to the solution. OR Realize by symmetry that the altitude to the line x+y=4 must meet at (2,2) giving a height of $2\sqrt{2}$. The side length is then: $$2\sqrt{2}\ \frac{2}{\sqrt{3}} = \frac{4\sqrt{6}}{3}$$
37	99	N can be written as 111…10 – 99, where the first number is a string of 99 ones followed by a zero. This then evaluates to 111…1011, a string of 97 ones followed by 011. The sum of the digits is 97*1 + 0 + 1 + 1 = 99.
38	24	The number is divisible by 4 since it ends in 40 and has remainder 6 when divided by 9 since the sum of the digits is 114. The only number from 0 to 35 that works is 24.
39	25	Let $u = x^2 + x - 2$. The equation becomes $u = 2\sqrt{u + 15}$, which has solution u = 10 (u = -6 is not a solution). This means that x = 3 or -4.
40	64	The product of the base and height needs to be 240. The hypotenuse of the right triangle created by half the base and the height needs to be an integer. Half the base, the height and one of the other sides form a Pythagorean triple. This only works when the base is 30 or 16, with respective heights of 8 and 15.

Individual Multiple Choice

9	11	Answer	Solution
1	1	C	$$3^4 - 5! + \frac{1}{10^{-3}} = 81 - 120 + 1000 = 961$$
2	2	C	Powers of 3 end in 3, 9, 7, 1, … so we need to find the remainder dividing 447 by 4 = 3 and take the 3rd in the list. 7
3	3	D	This is 3-4-5 right triangle. The tangent is the opposite over the adjacent side = 4/3.
4	4	C	$X = 3^{44} = 81^{11}, Y = 8^{33} = 512^{11}, Z = 121^{11}$ Now compare bases to see Y>Z>X
5	50	D	The number of 5 element subsets from the 9 possible elements is 9 choose 5. $$\binom{9}{5} = \frac{9!}{5!\,4!} = \frac{9(8)(7)(6)}{(4)(3)(2)} = 9(7)(2) = 126.$$
6	6	B	In base 10, the smallest 5-digit base 3 number is 81 and the largest is 342. The smallest 3-digit base 5 number is 25 and the largest is 124. The numbers from 81 to 124 inclusive work for a total of 44.
7	50	E (60)	Since 75=5*5*3, at a minimum we need a 5 and a 9 to make perfect cubes. So m=45 and n = 15. m+n=60.
8	8	B	The product is i to the power of $1 + 2 + 3 + \cdots + 2014 = \frac{2014(2015)}{2} = (1007)(2015)$. In modulo 4, this is equivalent to $(3)(3) \equiv 9 \equiv 1$. So the expression is equal to $i^1 = i$.
9	9	A	If one term of the sequence is $\frac{x}{y}$, the next will be $\frac{x+2y}{x+y}$. In the limit, these will be arbitrarily close, so equate them to solve for x/y. $$\frac{x}{y} = \frac{x+2y}{x+y};\ \ x^2 + xy = xy + 2y^2$$ $$\frac{x^2}{y^2} = 2, \quad \frac{x}{y} = \sqrt{2}$$
10	10	B	All we need to look at is the highest degree term of the numerator and denominator. $$\frac{(3x^2+1)(2x+4)}{8x^3+6x^2+1} = \frac{6x^3+\cdots}{8x^3+\cdots} \to \frac{6}{8} = \frac{3}{4}$$
50	4	C	The remainder theorem states that the remainder when $P(x)$ is divided by $x - a$ is $P(a)$. The remainder is then $(-2)^4 - 7(-2)^3 + 4(-2)^2 + 11 = 16 - (-56) + 16 + 11 = 99$

9	11	Answer	Solution
50	7	C	Equation simplifies to $2\cos^2(3\alpha) = 3/2$, or $\cos(3\alpha) = \pm\sqrt{3}/2$. On the interval, the sum of the solutions is $(1+5+7+11+13+17)\pi/18 = 3\pi$. Alternatively, we can leave it in terms of sine.
50	10	C	One way would be to convert to complex numbers and multiply. $$(2+2i)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) = (\sqrt{3}-1)+(\sqrt{3}+1)i$$ And the sum of the coordinates is: $2\sqrt{3}$

Team Test

9	11	Answer	Solution
1	1	$\frac{1}{2014}$	The product is: $$\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\cdots\left(\frac{2013}{2014}\right)=\frac{1}{2014}$$ after canceling.
2	20	78,624,000	The number of plates is: $$26(25)(24)(10)(9)(8)(7)=78{,}624{,}000$$
3	3	$[r=]\frac{R}{4}$	 Form the right triangle as above then: $$R^2+(R-r)^2=(R+r)^2 \;\; or\; r=\frac{R}{4}$$
4	4	-1, 1, 4 (any order)	From the Rational Root theorem, the possible rational roots are: $\pm1, \pm2, \pm4$. Use synthetic division to factor as: $(x-1)(x+1)(x-4)$ *with roots*, $-1, 1,$ *and* 4.
5	5	31	Slope of line is -1/4. Solution with smallest x-coordinate is (4,31). Therefore, more solutions can be obtained by subtracting 1 from the y-coordinate and adding 4 to the x-coordinate. This leads to y-coordinate values of 31, 30, ... 1. So there are 31 lattice points.
6	20	90	Solve the equation $\frac{180(n-2)}{n}=24+38\left(\frac{360}{n}\right)$.
7	7	$-1+2i$	To get rid of the imaginary part in the denominator we multiply by the complex conjugate. $$\frac{(1+i)^2(3-i)}{2-2i}\frac{2+2i}{2+2i}=\frac{(2i)(3-i)(2+2i)}{8}=\frac{(2i)(8+4i)}{8}=-1+2i$$
8	8	420	The only possible products achieved by more than 1 pair of numbers are 12 and 6. If it were the latter, the sum of the two numbers used to obtain 6 are both odd, so Stacey would be able to tell which numbers Richard chose. So the answer is 7! / 12 = 420.

9	11	Answer	Solution
9	9	600	One can show that $x(x+1)(x+2)(x+3)+1=(x^2+3x+1)^2$ Plug in x=23 to get the desired result. Actually, for this problem, it can be reasoned to be very close to (24)(25)=600.
10	20	-24	$$\frac{n^2+n-8}{n+4}=n-3+\frac{4}{n+4}$$ Therefore $n+4$ must be $\pm1,\pm2,\pm4$ and $n=-8,-6,-5,-3,-2,0$ which sum to -24.
20	2	$[y=]\frac{5}{3}$	Use properties of logs and the fact that it is 1-1. $$\log_3(3y+1)-\log_3(y-1)=\log_3(9)$$ $$\frac{3y+1}{y-1}=9,\quad 3y+1=9y-9,\quad 6y=10,\quad y=\frac{5}{3}$$
20	6	2π	$$\sin(2\theta)=\pm\frac{\sqrt{3}}{2}$$ $$2\theta=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}, since\ 2\theta<2\pi$$ Dividing by 2 and adding gives 2π.
20	10	-196	$$a_0=N, a_n=n^2-a_{n-1}, a_{20}=14$$ $$a_1=1-N,\quad a_2=4-(1-N)=3+N,\quad a_3=9-(3+N)=6-N$$ $$a_4=16-(6-n)=10+N,$$ $$a_5=25-(10+N)=15-N$$ $$14=a_{20}=\frac{(20)(21)}{2}+N=210+N,\quad N=-196$$

Pressure Round

9	11	Answer	Solution
1	9	16	The prime factors of 60 are 2, 3 and 5. Half of the numbers have no factor of 2 and two-thirds of those have no factor of 3. Finally, four fifths of the ones remaining have no factor of 5. $$60\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{4}{5}\right)=16$$
2	2	13	The sum of the zeros is (-3 / -1) = 3 and the sum of the zeros taken two at a time is (2 / -1) = -2. If the zeros are a, b, c, and d. $$(a+b+c+d)^2=a^2+b^2+c^2+d^2+2(ab+ac+ad+bc+bd+cd)$$ $$3^2=a^2+b^2+c^2+d^2+2(-2)$$ giving an answer of 13.
3	9	13	If the roots are A and B and A<B, then A+B=p and AB = q. Since q is prime, A=1. Thus, B is prime. Overall, p and q are consecutive primes, making the only candidates 2 and 3. 9 + 4 = 13.
4	9	-2	Straightforward manipulation of the arithmetic sequence formulae yield $$\frac{a+(a+99d)}{2}100=100a+4950d=100$$ $$\frac{(a+100d)+(a+199d)}{2}100=100a+14950d=200$$ $$10000d=100$$ $$d=\frac{1}{100}\, and \log_{10} d=-2$$
5	5	2	Repeated applications of log rules shows that $N=2014^{something}$, hence N is an even number. The smallest positive prime factor of N is thus 2.
9	1	$\frac{84}{85}$	$$\sin\left(\cos^{-1}\frac{15}{17}+\tan^{-1}\frac{4}{3}\right)$$ $$=\sin\left(\cos^{-1}\frac{15}{17}\right)\cos\left(\tan^{-1}\frac{4}{3}\right)+\sin\left(\tan^{-1}\frac{4}{3}\right)\cos\left(\cos^{-1}\frac{5}{17}\right)$$ $$=\frac{8}{17}\frac{3}{5}+\frac{4}{5}\frac{15}{17}=\frac{84}{85}$$
9	4	$[f(x)=]\frac{x-2}{3}$	Use the given equation, then replace x by 1-x. $f(x)-2f(1-x)=x\ and$ $f(1-x)-2f(x)=1-x$. Now solve for $f(x)$.

College Bowl Round 1

9	11	Answer	Solution
1	1	1/6	There are six possibilities, 1+6, 2+5, …6+1. The probability is 6/36 = 1/6.
2	2	225	By hand: $1^3+2^3+3^3+4^3+5^3=225$. Or use the formula, $\left(\frac{5(6)}{2}\right)^2=15^2=225$.
3	50	17/72	$$\frac{1}{8}+\frac{1}{9}=\frac{8+9}{72}=\frac{17}{72}$$
4	4	15	Vertical angles are equal. $$7x+17=8x+2,\quad x=15.$$
5	50	501	Put everything in powers of 10. $$\frac{1000^{300}}{100^{200}}=\frac{10^{900}}{10^{400}}=10^{500}$$ A one followed by 500 zeros is 501 digits.
6	6	False	False, in fact, unless one of them is impossible, they can't be independent as knowledge that A occurs, means B cannot occur.
7	7	210	Multiply the number of ways of choosing the mean times the number of ways of choosing the women. $$\binom{5}{2}\ \binom{7}{2}=10(21)=210$$
8	8	96	The perimeter is clearly 36. If the altitude is dropped to the base of 10, it forms two 5-12-13 triangles and the area is 5(12)=60 and the sum is 96.
9	50	6	In this case, the easiest solution is 6! = 720 which has remainder of 6. It is also a special case of Fermat's Little Thrm.
10	10	36	Number the candies from 1 to 10. Pick two of the spaces between the numbers and it forms a division of the candy. There are 9 choose 2 ways of picking the spaces which equals 36.
50	3	0.9 or zero point nine	$$\log(y^2x)=2\log(y)+\log(x)$$ $$=2(-0.2)+1.3=0.9$$
50	5	-i or negative i	The powers of i are $i,-1,-i,1$. $$i^{291}+i^{263}+i^{-3}=-i-i+i=-i$$
50	9	4/15	$f(x)=4x-15x^2=x(4-15x)$ has zeros at $x=0,\frac{4}{15}$ so it's maximum is at x=2/15. $$f\left(\frac{2}{15}\right)=\frac{2}{15}(4-2)=\frac{4}{15}$$

College Bowl Round 2

9	11	Answer	Solution
1	1	9	There are 11 numbers but 23 and 29 are prime leaving 9 composite numbers.
2	2	3204	Can just multiply it out or note that the percentages multiply to .9%=1%-.1%. The answer is then 3560-356=3204.
3	50	28	The side length with then be 7 and the perimeter is 4(7)=28.
4	4	9/25	The volume is $V = \frac{1}{3}\pi r^2 h$. If the volumes are the same, h has to be $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$ of the other.
5	50	40 $[un^3]$	The volume is $V = \pi r^2 h$ so if the radius doubles the volume goes up by $2^2 = 4$ making the new volume 40.
6	6	53	Clearly 2014 is divisible by 2 giving 1007. It takes some trial and error to get a factor of 19. 1001=7*11*13 so one can rule them out. 1007/19 = 53, a prime number.
7	7	11	Classic Formula: 7*3 – 7 – 3 = 11. Once you show you can get 12=4(3), 13=7+2(3) and 14=2(7), all the rest can be made by just adding more 3 cent stamps.
8	8	8	One can divide it out but it should be commonly known as 0.428571428571… so the 9th digit is 8.
9	50	33333	Each place has 24 1's, 2's, etc. for a total of 24(15) dividing by 120 = 3. The answer is then 33333.
10	10	141/1225	There are 50 choose 3 ways of picking 3 cards. Of the two remaining queens 1 must be chosen and of the other 48 cards, choose 2. $$\frac{\binom{2}{1}\binom{48}{2}}{\binom{50}{3}} = \frac{2}{1}\frac{48(47)}{2}\frac{6}{50(49)(48)} = \frac{141}{1225}$$
50	3	π/5	The period of tangent is π so the period of $\tan(5x-1)$ $is\ \frac{\pi}{5}$
50	5	2916/π $[un^2]$	The total circumference will be 6(18)=108 and the radius will be $54/\pi$. The area will be: $$A = \pi\left(\frac{54}{\pi}\right)^2 = \frac{2916}{\pi}$$
50	9	28	Eric doesn't have to choose at least one of each. One way to solve it is to add 3 flowers to the collection bringing it to 9 and demand at least one of each (then take one of each away). This happens in 8 choose 2 ways or 28.

College Bowl Round 3

9	11	Answer	Solution
1	1	10	Three consecutive integers form an arithmetic sequence whose middle number is equal to the arithmetic mean. Thus, y = 729 / 3 = 243. The largest one is then 244 and the sum of the digits is 2+4+4=10.
2	2	105	The slope is 3.5 so if x changes by 30, f changes by 3.5(30) = 105.
3	50	50	$(x+20)+x+(210-3x)=180$ $230-x=180;\ \ x=50.$
4	4	$12.00	If each dimension is doubled, the volume will increase by 2(2)(2)=8. 8(1.50) = $12.00
5	50	1/2	Find x such that f(x)=8. $6x+5=8;\ \ x=\frac{1}{2}$.
6	6	37	The correct total is: $\frac{52(53)}{2}=1378.$ 1378 – 1341 = 37.
7	7	(18,0) or eighteen comma zero	The x-coordinate of the vertex is halfway between the two zeros. V(8,2), Z(-2,0), the other zero will be at (18,0).
8	8	[$]3000	One can set up two equations and two unknowns. OR, if all invested at 8%, you'd get $960, all at 9% $1080. You made $1050, ¾ of the way from 960 to 1080. One-fourth of the money, $3000 should be at 8%.
9	50	192 [units squared]	The diagonal of the rectangle will be 20. The dimensions are 12x16, using the Pythagorean triple 12-16-20. The area is 192.
10	10	3	$x^3-13x+12$ $=(x-1)(x+4)(x-3)$ The largest root will be 3.
50	3	84	Use Heron's Formula $s=\frac{13+14+15}{2}=21$ $A=\sqrt{21(8)(7)(6)}=21(4)=84$
50	5	No solutions	Cosine is the x-coordinate and is non-negative in the 4^{th} quadrant.
50	9	250	Two-thirds of the numbers are not divisible by 3 (and thus 9 or 12) and three-fourths are not divisible by 4 (and thus 8). $500\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)=250.$

College Bowl Round 4

9	11	Answer	Solution
1	1	2/13	$$m = \frac{13-11}{20-7} = \frac{2}{13}$$
2	2	7/8	The probability is one minus the probability of all tails. $$1 - \frac{1}{8} = \frac{7}{8}$$
3	50	12	Each quadrant has one point on the axis, ie (5,0) and two points (3,4) and (4,3). The total number of points is 4(3)=12 points.
4	4	512	The ratio of the 6th and 3rd terms is $r^{6-3} = \frac{1}{8}$, so r = ½ and the first term is $64(2)(2) = 256$. The sum is: $$\frac{256}{1-\frac{1}{2}} = 512.$$
5	50	0, 1, and 8	Consider the integers of the form: 3k, 3k+1, and 3k+2. When cubed, $(3k)^3$ is divisible by 9. $(3k+1)^3 = 27k^3 + 27k^2 + 9k + 1$ which has remainder 1 and $$(3k+2)^3 = 27k^3 + 54k^2 + 18k + 8$$ with remainder 8.
6	6	0.08	The tax is \$0.98. $$\frac{98}{1225} = \frac{2}{25} = 0.08$$
7	7	72/425	There are 13 ranks to choose from and one choosing 2 of the 4 cards from that rank. The other card comes from the remaining 48 cards. $$\frac{\binom{13}{1}\binom{4}{2}\binom{48}{1}}{\binom{52}{3}} = \frac{13(6)(48)(6)}{52(51)(50)} = \frac{24(3)}{17(25)} = \frac{72}{425}$$
8	8	-196	$$x^2 + kx + 49$$ k needs to be plus or minus 14 with a product of -196.
9	50	1035	$$6 + 15 + 24 + \cdots + 132$$ There are a total of $\frac{132-6}{9} + 1 = 15$ numbers with an average of $\frac{6+132}{2} = 69$. The total is then 15(69)=1035
10	10	377 [base 8]	The largest 4-digit base 4 number will be $4^4 - 1 = 255 = 3(64) + 7(8) + 7$ or 377 base 8.

9	11	Answer	Solution
50	3	2	$\log_6(4) + \log_6(9) = \log_6(36) = 2$
50	5	$\frac{1}{4}$	Half-angle formula $\sin^2\left(\frac{\pi}{8}\right) = \frac{\left(1 - \cos\left(\frac{\pi}{4}\right)\right)}{2} = \frac{\left(1 - \frac{\sqrt{2}}{2}\right)}{2} = \frac{1}{2} - \frac{1}{4}\sqrt{2}$ $A + B = \frac{1}{4}$
50	9	2	$y = x^3 - 3x^2 + 4$ $= (x+1)(x-2)^2$ The graph crosses at -1 and touches at 2 for a 2 times total.

College Bowl Round 5

9	11	Answer	Solution
1	1	1470	Just multiply: 10(21)(7)=1470.
2	2	144	1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144.
3	50	[$] 19.62	$18 + 18(.09) = 18 + 1.62 = \$19.62$
4	4	15129	We just need to square 123 to get 15129.
5	50	30	Since they are evenly spaced, the sum of 5th and 11th elements will be the same as the sum of the 7th and 9th . 10+20=30.
6	6	25 pi	6-8-10 is a Pythagorean triple which means that it is a right triangle and 10 is the diameter with area 25π.
7	7	9	$2592 = 2^5 3^4$ Exponent of 2 can be 0, 2, or 4. Exponent of 3 can also be 0, 2, or 4. So 3*3 = 9 possibilities.
8	8	$\sqrt{3}/_4$	Let the side length be 1, the perimeter is 6 and the area of the hexagon (6 equilateral triangles) equals $6\left(\frac{\sqrt{3}}{4}\right)$ and the ratio is $\frac{\sqrt{3}}{4}$.
9	50	1 + i	$\frac{2}{1-i}\frac{1+i}{1+i} = \frac{2(1+i)}{2} = 1 + i$
10	10	$y = 3x - 2$	The center is equidistant to these two points meaning it will be on the perpendicular bisector of the line segment between the two points. The slope between the points is: $\frac{5-3}{-1-5} = -1/3$ and the midpoint is $\left(\frac{5+(-1)}{2}, \frac{3+5}{2}\right) = (2,4)$. The line has slope 3 and equation: $y = 3x - 2$.
50	3	2 pi / 3	$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$
50	5	3 / 2	$(\sin(15) + \cos(15))^2 = \sin^2(15) + \cos^2(15) + 2\sin(15)\cos(15) = 1 + \sin(30) = 1 + \frac{1}{2} = \frac{3}{2}$
50	9	2	When written in the form: $y - k = 4p(x - h)^2$ p is the distance between the focus and vertex (and the vertex and directrix). Since we have $8x^2$, p=2.

College Bowl Round 6

9	11	Answer	Solution
1	1	90 [degrees]	The angles in a pentagon add to 3(180) = 540. Subtracting the known angles leaves 180 divided by two equals 90 degrees.
2	2	220	One can just list them and add but the formula is: $$\frac{n(n+1)(n+2)}{6} = \frac{10(11)(12)}{6} = 220$$
3	50	3	$800 = 2^{32}\ 5^2$ so the only odd factors are 1, 5, and 25; all the rest have at least one factor of 2.
4	4	2209 [sq cm]	Divide by 4 to get side length and then square it. $$\left(\frac{188}{4}\right)^2 = 47^2 = 2209$$
5	50	1015	$$1^2 + 2^2 + \cdots + 14^2 = \frac{(14)(15)(29)}{6} = 1015$$
6	6	[c =] 1/3	For one root, the discriminant must be 0. $$2^2 - 4(3)(c) = 0; c = \frac{1}{3}$$
7	7	$(3, -3)$ or 3 comma negative 3.	Use a weighted average of the points. $$\left(\frac{1}{3}(1) + \frac{2}{3}(4), \quad \frac{1}{3}(1) + \frac{2}{3}(-5)\right) = (3, -3)$$
8	8	-8	$$3 = \frac{2x-5}{x+1};$$ $$3x + 3 = 2x - 5; x = -8$$
9	50	6π [sq un]	Divide by 36. $$\frac{x^2}{(\sqrt{18})^2} + \frac{y^2}{(\sqrt{2})^2} = 1$$ The area of this ellipse is then $\sqrt{18}\,\sqrt{2}\,\pi = \sqrt{36}\,\pi = 6\pi$
10	10	3/13	Consider the first round. Trung wins with probability $\frac{5}{8} = \frac{10}{16}$. Berta can only win if Trung does not and she flips a head, probability = $\frac{3}{8}\left(\frac{1}{2}\right) = \frac{3}{16}$. Each round is the same, so Berta's odds are 3:10 giving probability 3/13.
50	3	10	Each term will look like: $x^a y^b z^c$ where $a + b + c = 3$ so it is just like the flower distribution problem. The number of ways this can be done is 5 choose 2 = 10.

9	11	Answer	Solution
50	5	16	First square it, then raise that to the 4 power. $(1+i)^8 = ((1+i)^2)^4 = (2i)^4 = 16$
50	9	π [sq units]	Multiply by r and change to rectangular coord. Complete the square and the graph is a circle of radius 1. Area = π.

College Bowl Extra Questions

9	11	Answer	Solution
1	1	50	The sum looks like: $$-1+2-3+4-\cdots+100$$ Pairing these, it is a sum of 50 1's = 50.
2	2	$8+4\sqrt{13}$ 8 plus 4 root 13. [units]	Use the distance formula to get the distance from (0,0) to (4,6) equals $\sqrt{4^2+6^2}=\sqrt{52}=2\sqrt{13}$. The base is 8 and the other side is the same as the first. Perimeter = $8+4\sqrt{13}$
3	3	11 / 18	The number 34 in base 6 = 22 in base 10 with a denominator of $6^2=36$. $\frac{22}{36}=\frac{11}{18}$.
4	4	68	With 8 kids there are a total of $\binom{8}{4}=\frac{8!}{4!4!}=70$ committees of size 4. There two committees (all girls and all boys) have to be eliminated for a total of 68.
5	5	8	$$\left(\sqrt[4]{16}\right)^3=2^3=8$$
6	6	9.2	$9^2=81$, $9.5^2=90.25$ so 9.2 is a good guess. Checking it verifies it.