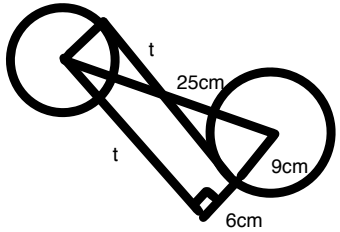
Mental Math

9/11	Answer	Solution
1	Answer	Comment / Hint
2	14	$7x - 5y = 8$ Slope = -y/x, as the equation is in standard form. $10m = 10\left(\frac{7}{5}\right) = 14$.
3	41	$441 - 400$ or $20+21$
4	60 [ways]	The letter T is repeated. $5!/2=60$.
5	Incenter	The angle bisectors meet at the center of the inscribed circle called the "incenter".
6	135 [degrees]	Use the exterior angle of 360/8=45. The interior angle is then 180-45=135 degrees.
7	6	There is a 5/9 chance the first digit is odd. In this case exactly one of the other two digits must be odd, which happens ½ of the time. If the first digit is even, the other two must both be even or odd, which happens ½ of the time. 5/18 + 4/18 = 1/2. 12(1/2)=6.
8	34	Sum all sixteen numbers and then divide by 4. $\frac{1}{4}\left(\frac{16(17)}{2}\right) = 2(17) = 34.$

Individual Test

9/11	Answer	Solution
1	0	$0 = (x+2)^2 - 4(x+2) + 4 = (x+2-2)^2 = x^2$
2	[$] 3040	328/45 = 7+ so 8 buses are needed. Then, 8*175 + 5*328 = 3040.
3	2	The slope $m = \frac{-6-2}{3-1} = -\frac{8}{2} = -4$, $y = -4x + b, \quad 2 = -4(1) + b, b = 6.$ $y = -4x + 6. \quad -4 + 6 = 2$
4	108 [sq in]	The altitude to the 24 inch base forms a 9-12-15 right triangle. The area is then (2)(1/2)(9)(12) = 108 sq inches.
5	81	The 3 scores that we know total 13+13+10=36 points above the average of 87. So, the others must total 36 below the average of 87 and 87 – 36/6 = 81.
6	[x=] 1	$3(2x-1) - 2(x-2) = 6x - 3 - 2x + 4 = 4x + 1 = 5., x = 1$
7	8075	$95 \times 85 = (90+5)(90-5) = 90^2 - 5^2 = 8100 - 25 = 8075$
8	11	There is a pentagon in the middle and 10 triangles around it.
9	5	$\frac{(7-(4-3^2)+8)}{4} = \frac{7-(-5)+8}{4} = \frac{20}{4} = 5$
10	[$] 33	The final cost is $\$32 \times 125\% \times 75\% \times 110\% = \$32 \left(\frac{5}{4}\right)\left(\frac{3}{4}\right)\left(\frac{11}{10}\right) = \$32 \left(\frac{33}{32}\right) = \$33.$
11	4 [π] [sq un]	As the hexagon can be divided into equilateral triangles of side length 2, the radius of the circle is also 2. The area of the circle is then: $\pi(2)^2 = 4\pi$
12	6	$1 + 2 + 4 + 8 + \cdots + 512 = 2^{10} - 1 = 1023.$ The remainder when divided by 9 is 1+0+2+3=6.
13	−1	$f(-2) = \frac{(-2)^2 + (-2) - 1}{(-2) + 1} = \frac{4 - 2 - 1}{-1} = -1$
14	36	There are 4 vowels (A-I-I-E) with 4!/2!=12 permutations, and 3 consonants (R-N-R) with 3 permutations. In total, there are $12 \times 3 = 36$ permutations.
15	12 [cm^4]	The short diagonal divides the rhombus into 2 equilateral triangles each has area: $\frac{s^2\sqrt{3}}{4} = \frac{4\sqrt{3}}{4} = \sqrt{3}.$ The total area is then: $2\sqrt{3}$ *sq cm.* $\left(2\sqrt{3}\right)^2 = 12.$
16	41	$3^8 - 1 = (3^4 - 1)(3^4 + 1) = (3^2 - 1)(3^2 + 1)(2)(41)$

9/11	Answer	Solution
17	3	$4^x - 2^{x+2} = 32.\quad (2^x)^2 - 4(2^x) - 32 = 0$ $(2^x - 8)(2^x + 4) = 0.\ x = \log_2 8.$
18	-1080	The appropriate term is: $\binom{5}{2}(2x)^2(-3y)^3 = 10(4)(-27)x^2y^3 = -1080x^2y^3$
19	5 [mm]	The perpendicular line will bisect the chord (the chord and 2 radii form an isosceles triangle) forming a pair of 13-12-x right triangles. x must be 5.
20	20 [cm]	 $(6+9)^2 + t^2 = 25^2; t = 20cm$
21	12 [ways]	Girls and boys must alternate around the table. The first girl can sit anywhere. To her right, there are 3 choices of boys, then there are 2 choices of girls, etc. 1*3*2*2*1*1=12 ways.
22	4	Substitute the equation into itself: $1 = \frac{5}{x+1}$ x + 1 = 5, x = 4
23	22 [ways]	Treat the letters WIN as one letter X. There are 24 ways of arranging XWIN. Subtract 2 for XWIN and WINX which would be 2 occurrences of WINWIN leaving 22.
24	8 [deg]	$21n = \frac{n(n-3)}{2}$ $42 = n - 3 : n = 45$ The external angle measure is 360/45 = 8.
25	35	The differences are 3, 6, 10… The differences of the differences are 3, 4… So the next difference will be 10+5=15, and the next term 20+15=35.

9/11	Answer	Solution
26	749	5+24+108=137 6+35+196=237 7+48+320=375 137+237+375=749
27	8	Six equilateral triangles of area $\frac{s^2\sqrt{3}}{4}$, where s = 1. $6\left(\frac{\sqrt{3}}{4}\right)=\frac{3\sqrt{3}}{2}$. $3+3+2=8$.
28	249	The expression simplifies to 1000!. The number of zeroes in the quantity is equal to the number of factors of five it has (as a 5 and a 2 make a 10, and there are more 2s). Number of fives = $1000/5 = 200$. Number of additional fives from 25s: $200/5 = 40$. Number of additional fives from 125s: $40/5 = 8$. And one additional 5 from 625. 200+40+8+1=249.
29	155	$n-2, n, n+2$ $n^3-4n=15(3n)=45n$ $0=n^3-49n=n(n-7)(n+7);\ \ n=7$ $5^2+7^2+9^2=155.$
30	78 [ties]	In each division there will be 2 (10C2) = 90 games played. So a total of 2(90)=180 games with a potential of 180(3)=540 points possible. A tie results in only two points being given so there are 540-462 = 78 ties.
31	64	$7\times\overbrace{777\ldots77}^{k}=5\ \overbrace{444\ldots4}^{k-2}\ 39$ The digits on the right add to 265. So, $265=5+4(k-2)+3+9;\ \ k=64.$
32	4 [π] [sq un]	Area= Outer sectors– inner sectors. Consider the central angles. The 4 inner arcs have a central angle of 135 degrees. The outer arcs then have angle 360 – 135 = 225 degrees. In total, 4(225) – 4(135) = 4(90) = 360. The difference in area is then a complete circle of radius 2 (half the side length). $\pi 2^2=4\pi$.

9/11	Answer	Solution
33	15	First, form two vectors by subtracting $(-2, 1, 2)$ from the other points and take the cross product. $$\begin{vmatrix} i & j & k \\ 3 & 0 & -1 \\ 6 & 2 & 1 \end{vmatrix} = 2i - 9j + 6k$$ The vector is normal to the plane with equation: $2x - 9y + 6z + 1 = 0$. The answer is then $6 - -9 = 15$
34	2	$$\left(\sqrt{10 + \sqrt{84}}\right)^2 = \left(\sqrt{a} + \sqrt{b}\right)^2.$$ $$10 + 2\sqrt{21} = a + b + 2\sqrt{ab}.$$ $$a = 7, b = 3 \; and \; \sqrt{7 - 3} = 2.$$
35	76	The smallest the sum can be is 1+2+3+4+5=15 and the largest is 16+17+18+19+20 = 90. Any number in between is possible as well for a total of 90-15+1 = 76.
36	15	One way to do this problem is to consider the isosceles triangle AED with 2 sides of length 2 and an included angle of 150 degrees. Law of cosines: $$c^2 = 2^2 + 2^2 - 2(2)(2)\cos(150°) = 8 - 8\left(-\frac{\sqrt{3}}{2}\right) = 8 + 4\sqrt{3}$$ $$8 + 4 + 3 = 15$$ OR Put the figure on the rectangular coordinate system with A at the origin. D is then at $(0, -2)$and E at $\left(1, \sqrt{3}\right)$ and use the distance formula. $$d^2 = (1 - 0)^2 + \left(-2 - \sqrt{3}\right)^2 = 8 + 4\sqrt{3}$$
37	$\left\{9, \frac{1}{9}\right\}$	$$\frac{\log_3 3}{\log_3 x} = \frac{\log_3 x}{\log_3 81}$$ $$1(4) = (\log_3 x)^2$$ $$\log_3 x = \pm 2$$ $$x = \left\{9, \frac{1}{9}\right\}$$
38	3	There is a pattern. Powers of six end in: 06, 36, 16, 96, 76, 56, 36… A repeating sequence of 5 tens digits starting at the second power. The twelfth power will be the first in the sequence, or three.

9/11	Answer	Solution
39	8	In an ellipse the sum of the distance to the foci is constant. The point (3,4) is 5 units from the origin and 13 units from (8,-8) for a total of 18. Let the y-intercept(s) be $(0, y)$. Consider the distances from $(0, y)$ to the foci: $$\lvert y\rvert + \sqrt{8^2 + (y+8)^2} = 18$$ If $y > 0$. $$64 + (y+8)^2 = (18 - y)^2$$ $$y^2 + 16y + 128 = 324 - 36y + y^2$$ $$52y = 196, \qquad y = \frac{49}{13}$$ If $y < 0$ $$64 + (y+8)^2 = (18 + y)^2$$ $$y^2 + 16y + 128 = 324 + 36y + y^2$$ $$-20y = 196, \qquad y = -\frac{49}{5}$$ $$\frac{y_1}{y_2} = -\frac{5}{13}. \quad \lvert -5 + 13\rvert = 8.$$
40	18 [%]	The rectangle is 5x5 with an area of 25. Need to find the area that is closer to the origin. The perpendicular bisector of the segment from (0,0) to (4,4), that is, the points that are equidistant, goes through (2,2) with a slope of -1. The bisector intersects the rectangle at (3,1) and (0,4). The right triangle (area closer to origin) is 3x3 with an area of 9/2. The probability is then: $\frac{\frac{9}{2}}{25} = \frac{9}{50} = 18\%$.

Individual Multiple Choice

9	11	Answer	Solution
1	1	B	31+37+41+43+47 = 199.
2	2	C	Subtract first and then multiply: $13\ (17) = \ 15^2 - 2^2 = 225 - 4 = 221.$
3	3	A	The minute hand will be pointing straight down at 180 degrees and the hour hand will be halfway between two and three at 75 degrees as each hour is 30 degrees. 180-75 = 105 degrees.
4	4	E [0]	The area of the entire circle is $\pi r^2 = 16\pi$. If the smallest area of the 4 sections is 4π, all four must be the same meaning the chords meet at the center. P is the center of the circle.
5	50	C	$\frac{9}{8} - \frac{8}{9} = \frac{81-64}{72} = \frac{17}{72}$. $17 + 72 = 89$.
6	6	A	Just looking at the shorter length, we have 1, 2, 3, 4, and 6 for a total of 5 rectangles.
7	50	D	There are two possibilities: a) sum of one of the 4 ones with one of the 2 fours for 8 ways and b) one of 3 twos with the three for 3 more ways. A total of 11 ways.
8	8	D	The numbers form 5 consecutive even numbers that must all be multiples of 2. At least two of them are also multiples of 4 and at least one is also a multiple of 8 for a minimum total of 8 factors of 2. Further, at least one must be a multiple of 3 and one a multiple of 5. The largest factor is then $2^8(3)(5) = 3840 = 2(4)(6)(8)(10)$.
9	9	B	When expanding $(x^2+1)^3$, the coefficient of x^2 is $\binom{3}{1}\ 1^1 1^2 = 3$ and the constant coefficient is $1^3 = 1$. The other factor expands to $(2x)^3 + \binom{3}{1}(2x)^2(1) + \binom{3}{2}(2x)(1^2) + 1$. The only ways to get x^2 is $3(1) + 1(3)(4) = 15$.
10	10	E (6)	$p^3 + 6p - 252 = 0$. One by one the roots, 1, 2, 3, 4 and then 6 can be tested.
50	4	C	The amplitude of the function is the coefficient of cosine, 3. The period is $\frac{2\pi}{3}$, where 3 is the coefficient of the argument. The product is $3\left(\frac{2\pi}{3}\right) = 2\pi$.
50	7	D	$x^4 - 256 = 0 = (x^2 - 16)(x^2 + 16)$. These have roots of ± 4 and $\pm\ 4i$.
50	10	A	2015=1024+512+256+128+ 64+16+8+4+2+1, i.e. every power of two except 32. Since those are all x terms, the only multiplied integer in the term of interest is 32, or 2 to the fifth.

Team Test

9	11	Answer	Solution
1	1	22	6 to 28, obtuseness is irrelevant since the minimum and maximum lengths allowed under the triangle inequality produce obtuse triangles.
2	20	2880	$\begin{bmatrix}-1 & 5\\ 3 & -4\end{bmatrix}\begin{bmatrix}8 & -5\\ 4 & -3\end{bmatrix}=\begin{bmatrix}12 & -10\\ 8 & -3\end{bmatrix}$ 12*10*8*3=2880
3	3	72	n(n+1)=5256. The square root of 5256 is roughly 70. The units digit of 6 is 2 times 3 so 72(73). 5256=2*2*2*3*3*73=72*73
4	4	40	Sum of the first n consecutive perfect squares = $\frac{n(n+1)(2n+1)}{6}$ Squares to 299: 289 = 17^2. 17*18*35/6 = 51*35=1785. Squares to 199: 196=14^2 14*15*29/6 = 35*29=1015 Squares to 99: 81 = 9^2 9*10*19/6 = 15*19=285 1785-1015-1015+285 2070-2030=40
5	5	37	Distance formula for line Ax+By+C=0 and point (x,y): $d=\frac{\lvert Ax+By+C\rvert}{\sqrt{A^2+B^2}}=\frac{20+12+0}{\sqrt{16+9}}=\frac{32}{5}$. $32+5=37$
6	20	6	The repeating fraction can be written in base ten as $\frac{2b^2+2b+1}{b^3-1}$. Successively trying b=3, 4, 5, 6; it can be seen that in base 6, the fraction reduces to $17/43$.

9	11	Answer	Solution
7	7	4 [feet]	Let d be the distance from the spot the other turtle is spotted to the sun patch. It takes a total of 4 seconds (3+1) longer at ¼ f/s than ½ f/s. $$\frac{d}{\frac{1}{4}} = \frac{d}{\frac{1}{2}} + 4.\ d = 2.$$ Add the original 2 feet for a total of 4 feet.
8	8	24325	$$n = 1000q + r = 999q + q + r = (37)(27)q + q + r.$$ So, $q + r$ is divisible by 37 if n is. There are $\lfloor 999999/37 \rfloor = 27027$ multiples less than 1M and $\lfloor 99999/37 \rfloor = 2702$ less than 100K. The number of 6-digit multiples is then 27027-2702=24325.
9	9	284	Pick any spade. The second stack will have three cards (one per suit) that match it. The probability that both cards drawn from the second stack do not match the spade is then $\frac{36}{39} \times \frac{35}{38} = \frac{6}{13} \times \frac{35}{19} = \frac{210}{247}$. The complementary probability is $\frac{37}{247}$.
10	20	11	The large face is an equilateral triangle with side length s. The other faces are isosceles triangles with base lengths s. Using the area relationship and short face altitude a, $\frac{s^2\sqrt{3}}{4} = 2 \times \frac{sa}{2} \rightarrow a = \frac{s\sqrt{3}}{4}$. The other side length of the small face is then the hypotenuse of a right triangle with legs of $\frac{s\sqrt{3}}{4}$ and $\frac{s}{2}$. $$\frac{3s^2}{16} + \frac{s^2}{4} = x^2 = \frac{7s^2}{16}$$ $$x = \frac{s\sqrt{7}}{4}.\quad 7 + 4 = 11.$$
20	2	146 [cu un]	Vector triple product: $$\begin{bmatrix}2\\7\\5\end{bmatrix} \cdot \begin{bmatrix}5\\3\\2\end{bmatrix} \times \begin{bmatrix}1\\7\\0\end{bmatrix} = \begin{bmatrix}2\\7\\5\end{bmatrix} \cdot \begin{bmatrix}-14\\2\\32\end{bmatrix} = -28 + 14 + 160 = 146$$
20	6	567	The distinguishable packages each have three possible destinations: 3 * 3 * 3 = 27 ways. The indistinguishable packages distribution is modeled by choosing two gaps in a line of 5+3=8 objects (adding three accounts for the possibility that a destination receives none). $\binom{7}{2} = 21.$ 27*21=567.

9	11	Answer	Solution
20	10	13	The coefficient of x^2 will be the sum of all possible products of two of the multiplied x coefficients. The products involving the first term are in order $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}$ …, a series that sums to $-\frac{1}{3}$. The products involving the second but not first term similarly form a series that sums to $-\frac{1}{12}$. The products involving the third but not second or first sum to $-\frac{1}{48}$. These sums of products form an infinite geometric series with first term $-\frac{1}{3}$ and ratio $\frac{1}{4}$, summing to $-\frac{4}{9}$. $$|-4| + |9| = 13$$ Alternatively, as the x coefficient and constant are simple to calculate, one could write $P(x) = 1 - \frac{2}{3}x + Cx^2 + R(x)$, where R contains the here irrelevant x cubed and higher terms. It follows that $P(-x) = 1 + \frac{2}{3}x + Cx^2 + R(-x)$, and that $P(x)P(-x) = 1 + \left(2C - \frac{4}{9}\right)x^2 + Q(x)$, where Q's smallest x power is cubic. We also have from the original expression of P that $$P(x)P(-x) = (1-x)(1+x)$$ $$\left(1+\frac{1}{2}x\right)\left(1-\frac{1}{2}x\right)\left(1-\frac{1}{4}x\right)$$ $$\left(1+\frac{1}{4}x\right)\left(1+\frac{1}{8}x\right)\left(1-\frac{1}{8}x\right)\ldots$$ $$= (1-x^2)\left(1-\frac{1}{4}x^2\right)$$ $$\left(1-\frac{1}{16}x^2\right)\ldots$$ This polynomial has by infinite series sum an x^2 coefficient of $-\frac{4}{3}$. Equating coefficients yields $$-\frac{4}{3} = 2C - \frac{4}{9}$$ $$-\frac{4}{9} = C, \quad |-4| + |9| = 13$$

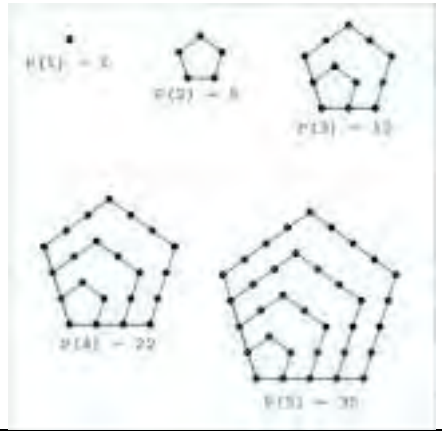
Pressure Round

9	11	Answer	Solution
1	9	146	Looking at all possibilities: 6611 6 6521 24 6431 24 6422 12 6332 12 5531 12 5522 6 5441 12 5432 24 5333 4 4442 4 4433 6 total 146
2	2	5	One needs to find x such that: $73^x = 2073071593$. We must assume it is an integer, since otherwise we won't be able to do it without a calculator. 73 has 2-digits and 2073071593 has 10 so 10/2=5 seems likely. Also, the units digits of 3^5 ends in 3 so that adds evidence that 5 is the answer.
3	9	162	Arbitrarily assign a variable to each number: $a+b+c=6, a+b+d=9$ $a+c+d=14, b+c+d=19$ Adding all equations: $3(a+b+c+d)=6+9+14+19=48$ $a+b+c+d=16$ This gives the sum of all 4, now subtract each equation in turn. $a=-3, b=2, c=7, d=10$. Now, sum the squares to get: $9+4+49+100=162$.
4	9	13608	$987^2 = 974169$ and $9\times7\times4\times1\times6\times9 = 13608$.
5	5	6	Hexadecimal is base 16 and includes the digits A,B,C,D,E,F. The base ten number $100 = 64_{16}$. The numbers with an A are then: A, 1A, 2A, 3A, 4A, 5A for a total of 6.

9	11	Answer	Solution
9	1	7	Minimize the squared distance from the point to the parabola. $P(a) = (a-5)^2 + (a^2 - a + 3 - 4)^2 = (a^2 - 10a + 25) + (a^4 - 2a^3 - a^2 + 2a + 1)$ $= a^4 - 2a^3 - 8a + 26$ $P'(a) = 4a^3 - 6a^2 - 8 = 2(2a^3 - 3a^2 - 4) = 2(a-2)(2a^2 + a + 2) = 0, when\ a = 2.$ $(y(a) = y(2) = 5$. The point is (2,5) and $2 + 5 = 7$.
9	4	6	$\left(\frac{\csc\frac{13\pi}{4}}{\cot\frac{\pi}{3}}\right)^2 = \left(\frac{\frac{1}{\sin\frac{13\pi}{4}}}{\frac{\cos\frac{\pi}{3}}{\sin\frac{\pi}{3}}}\right)^2 = \left(-\sqrt{2}\times\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)^2 = \left(-\sqrt{2}\,\sqrt{3}\right)^2 = \left(-\sqrt{6}\right)^2 = 6$

College Bowl Round 1

9	11	Answer	Solution
1	1	390	Since they are relatively prime, the least common multiple is the product and $26\times15 = 13\times30 = 390$.
2	2	1120	$47^2 - 33^2 = (47-33)(47+33) = 14(80) = 1120$
3	50	11 and $\frac{7}{18}$	$3\frac{5}{9} + 7\frac{5}{6} = 3 + 7 + \frac{10}{18} + \frac{15}{18} = 11\frac{7}{18}$
4	4	306	$2 + 4 + \cdots + 34 = 2(1 + 2 + \cdots 17) = 17(18) = 306$
5	50	39	$\left(1\frac{19}{20}\right)\left(\frac{2}{5}\right)50 = \left(\frac{39}{20}\right)\left(\frac{100}{5}\right) = 39.$
6	6	32	Each factor of 10=2x5 adds a 0 to the end. There are plenty of factors of 2. Each multiple of 5 contributes a 5, multiples of 25 contribute an extra 1 and multiples of 125 another. $130 = 5(26) = 5(25) + 5 = 125(1) + 5$ $26 + 5 + 1 = 32$
7	7	61	7, 43, **61**
8	8	20	$\frac{15}{2} = x - d, \frac{25}{2} = y + d; \frac{15}{2} + \frac{25}{2} = x + y = 20.$
9	50	20	$1 + 2 + 3 + 4 + 6 > 12,$ $1 + 2 + 3 + 6 + 9 > 18,$ $1 + 2 + 4 + 5 + 10 > \mathbf{20}$
10	10	2	3^2 has remainder 2, *so* 3^4has remainder 4 *and* 3^8 has remainder 16, remainder $= 2$. Using mod's. $3^2 \equiv 2\ mod\ 7, 3^8 \equiv 16 \equiv 2\ mod\ 7.$
50	3	4096	When expanded $(x+1)^{12} = x^{12} + 12x^{11} + \cdots + 1$. The sum of the coefficients is the same as the expansion evaluated at x=1. $(1+1)^{12} = 4096$.

9	11	Answer	Solution
50	5	35	$p(n) = n(3n-1)/2$ 
50	9	25	$9^{\log_3 5} = 3^{2\log_3 5} = 3^{\log_3 5^2} = 5^2 = 25$

College Bowl Round 2

9	11	Answer	Solution
1	1	210	$210 = 2{\times}3{\times}5{\times}7$
2	2	30	$18 = \frac{3}{5}x.\ x = 18\left(\frac{5}{3}\right) = 30.$
3	50	14641	Manually, or it is just the digits of the 4th row of Pascal's triangle.
4	4	950	$5 + 10 + 15 + \cdots 95 = 5(1 + 2 + \cdots + 19) = 5\left(\frac{19(20)}{2}\right) = 5(190) = 950$
5	50	-20000	$(2x-5)^6 = \cdots + \binom{6}{3}(2x)^3 5^3 + \cdots$ $\binom{6}{3}(2x)^3(-5)^3 = 20(8)(-125)x^3 = -20000$
6	6	729	There are 9 choices for hundreds place, 9 for the tens and 9 for the unit digit. $9{\times}9{\times}9 = 729$
7	7	15	$60 = 15{\times}4$. Four is already a perfect square so multiplying by 15 gives 30-squared.
8	8	144	Each element is the sum of the two previous ones. $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144$
9	50	16	$\begin{vmatrix} 3 & 5 \\ 4 & 12 \end{vmatrix} = 3(12) - 4(5) = 16$
10	10	320	The sum of 9 terms is 45 so the average is 5 which is the 5th term. $a + 4d = 5 = -3 + 4d; d = 2$. The sum of 20 terms is: $20\left(\frac{-3 + \ -3 + 19(2)}{2}\right) = 320$
50	3	11/4 or eleven fourths	$\log_4\left(32\sqrt{2}\right) = x$ $2^{\frac{11}{2}} = 2^5 2^{\frac{1}{2}} = \ 32\sqrt{2} = 4^x = (2^2)^x = 2^{2x}$ $x = \frac{11}{4}$
50	5	181	Factor $1432 = 2^3(179)$. $2 + 179 = 181$. For 179; check 2, 3, 5, 7, 11, 13.
50	9	6 0 [base 8] or six zero [base 8]	$14B = 256 + 64 + 11 = 331$ $100011011 = 256 + 16 + 8 + 2 + 1 = 283$ $331 - 283 = 48 = 60_8$

College Bowl Round 3

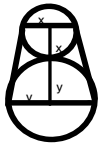
9	11	Answer	Solution
1	1	51	One could add and divide by 4 or take difference from 50. -4, -3, 3, 8 totaling 4 or an average distance from 50 of 1 = 51.
2	2	5040	Seven factorial = 5040.
3	50	5/13 (Five over thirteen)	There are 13 ranks and 5 (2, 4, 6, 8, 10) show even numbers. 5/13.
4	4	40	There are 4 choices (2,4,6,8) for the hundreds digit and 10 choices for the tens place. The unit digit must be the same as the hundreds in order to be a palindrome.
5	50	median	The very low values that occur in left skewed data lower the mean but the median is unaffected by the extreme values.
6	6	5/26	There are 13 marbles total. $\frac{6}{13}\left(\frac{5}{12}\right)=\frac{5}{26}$
7	7	6 [factors]	$176=2^4\times11,\ \ 132=2^2\times3\times11$ So, any number $\{1,2,4\}\times\{1,11\}$ will be a common factor. 3(2)=6 factors.
8	8	13	Considering the letters G-R-E-N, there are 4x3=12 ways. Now include the EE ordering for a total of 13.
9	50	7 / 32	Need to count the number of ways. (x can be either). HHHxx 4 ways THHHx 2 ways TTHHH 1 way Total of 7 / 32.
10	10	C-M-A-R-Y	There are 120 total permutations with 24 starting with each letter and 12 with any two-letter combo. There are 24 that start with 'A' and 12 more starting with 'CA', the next one, 37th, is CMARY.
50	3	1 / 2	Need to choose an angle in the 2nd or 4th quadrants. 2/4 = 1 / 2.
50	5	2 / 9	There are three choices for the image of 'one' and two choices for 'two' and 'three' gets what's leftover, so there are 6 one-to-one functions. However, there are three cubed possible functions. The probability is then 6/27 = 2/9.
50	9	1 / 4	Need the area under the line from 0 to 1. It is a right triangle with width 1 and height 1/2 for an area (probability) of 1/4.

College Bowl Round 4

9	11	Answer	Solution
1	1	67	The total needs to be 4(40)=160. $160-(45+32+16)=67.$
2	2	35 (combinations)	Seven choose three = $\frac{7(6)(5)}{3(2)(1)}=35.$
3	50	1/8 (One Eighth)	Equivalent to rolling three evens. $\left(\frac{1}{2}\right)^3=\frac{1}{8}$
4	4	1	The mean is 18 and the median is 19 for a difference of 1.
5	50	40(cards)	You might get all 13 spades, clubs and diamonds for 39 cards before you drew the heart. 40 cards.
6	6	$\frac{3}{8}$ or three-eighths.	There are 6 ways of getting two heads and two tails: 6 = 4 choose 2. $\frac{6}{2^4}=\frac{6}{16}=\frac{3}{8}$
7	7	59	There are 3 ways to pick the two dice that match and 12 possible numbers; the other die has 11 possibilities. $\frac{3(12)(11)}{12^3}=\frac{11}{48}$ 11+48=59.
8	8	10	Putting the 6 balloons in a line, dividing the balloons is equivalent to choosing 2 of the 5 gaps between the balloons. 5 choose 2 equals 10.
9	50	$\frac{4}{7}$ or four sevenths.	Actually, the last marble has the same probability as the first marble OR one could fix red as the last marble and there are 6!/(3!3!) ways of ordering the other 6 while in total there are 7!/(3!4!) ways of ordering the 7 marbles. Dividing, one gets 4/7.
10	10	1 / 6	The total interval has width 6 and the interval of interest is of width 1, so a probability of 1/6.
50	3	1 / 3	Only tangent and cotangent are positive.
50	5	10	With independence the variance of the difference is the sum of the variances. $s=\sqrt{8^2+6^2}=10$

9	11	Answer	Solution
50	9	3 / 8	$P(M\|N) = \frac{4}{5}, P(MN) = \frac{1}{2}$ $P(N) = \frac{P(MN)}{P(M\|N)} = \frac{\frac{1}{2}}{\frac{4}{5}} = \frac{5}{8}$ $P(N^c) = 1 - P(N) = \frac{3}{8}$

College Bowl Round 5

9	11	Answer	Solution
1	1	3 [triangles]	1-4-4, 2-3-4, 3-3-3
2	2	$\frac{5}{3}$	Essentially equivalent to finding one-third of the distance between the points. $3k = \sqrt{4^2 + 3^2} = 5$ $k = \frac{5}{3}$
3	50	6840 degrees	The exterior angles measure $\frac{360}{40} = 9$. The total of the interior angles is then $171 * 40 = 684 * 10 = 6840$.
4	4	18 pi	The inner circle has radius 3, so the square has side length 6 and diagonal $6\sqrt{2}$, which is also the outer circle's diameter. The outer circle's area is then $\pi\left(3\sqrt{2}\right)^2 = 18\pi$
5	50	4th	The points given are in the first and third quadrants. A fairly simple diagram shows that it passes through the second but not the fourth.
6	6	50 [un]	The total circumference is then $5\pi\left(\frac{360}{18}\right) = 100\pi$. The radius is then 50.
7	7	60/13 [un]	One can calculate the area in two ways: $\frac{5(12)}{2} = \frac{13(x)}{2}, \quad x = \frac{60}{13}$
8	8	144 [sq un]	$x + y = 12$, the bases are $2x$ and $2y$.  $A = \left(\frac{1}{2}\right)12(2x + 2y) = 12(x + y) = 144$.
9	50	4 [un]	$x^2 + 2x + 1 + y^2 - 4y + 4 = 11 + 1 + 4 = 16 = r^2$. $r = 4$.
10	10	72	The area of the hexagonal face is $\frac{3s^2\sqrt{3}}{2} = 24\sqrt{3}$. Multiplying by the height gives 72.
50	3	$\frac{\pi}{6}$ or pi over 6.	The period is equal to 2π divided by the coefficient of the argument. $\frac{2\pi}{12} = \frac{\pi}{6}$

9	11	Answer	Solution
50	5	$\frac{\pi}{4}$ or pi over 4.	$\cos\theta = \frac{\boldsymbol{A}\cdot\boldsymbol{B}}{\lvert A\rvert\lvert B\rvert} = \frac{5}{\sqrt{5}*\sqrt{10}} = \frac{\sqrt{2}}{2}.$ $\theta = \frac{\pi}{4}$
50	9	3 [areas]	Graphically, subtracting $r = \sin\theta$ (a circle above the x-axis) from $r = \sin(3\theta)$ (a three-petaled curve with one petal below the x-axis) reduces the top two petals in size and increases significantly the size of the third. No additional areas are created, however.

College Bowl Round 6

9	11	Answer	Solution
1	1	30 [un]	The triangle is a multiple of the Pythagorean triple 15-8-17
2	2	5 [vertices]	Visually, the hexahedron looks like a pair of tetrahedrons stuck together.
3	50	10 [degrees]	The minute hand is pointing toward 4 but the hour hand has moved one-third of the way to 5. Since, the angle between hours is 360/12=30. The angle will be 10 degrees.
4	4	464	$\frac{n(n-3)}{2}=\frac{32(29)}{2}=16(29)=464.$
5	50	$(-2,3)$ or negative 2 comma 3.	$x=\frac{-b}{2a}=-\frac{4}{2}=-2.$ $y=(-2)^2+4(-2)+7=3$
6	6	(5, 1) or five comma one.	One needs to average the x-coordinates and the y-coordinates. $\left(\frac{2+4+9}{3},\frac{3+1-1}{3}\right)=(5,1).$
7	7	8 [sq un]	The diameter of the circle (4) is the diagonal square. As a rhombus, the area of the square is one-half the product of the diagonals (4)(4)(1/2) = 8.
8	8	12 / 5	Consider the right triangle enclosed by the line and the axes. Its area can be calculated either with the two bases, or with the hypotenuse and altitude (which is the distance of interest). $\frac{3(4)}{2}=\frac{5x}{2}, x=\frac{12}{5}$
9	50	2 [sq un]	The graph is a square going through (1,0), (0,1), (-1,0) and (0,-1). The area will be (2)(2)/2=2.
10	10	$60-20\sqrt{3}$ or 60 minus 20 root 3.	$x+2x+x\sqrt{3}=60$ $x=\frac{60}{3+\sqrt{3}}*\frac{3-\sqrt{3}}{3-\sqrt{3}}$ $x=\frac{180-60\sqrt{3}}{6}=30-10\sqrt{3},\quad hypotenuse=2x=60-20\sqrt{3}$
50	3	$4\sqrt{14}$ or 4 root 14.	$\sqrt{8^2+4^2+12^2}=\sqrt{224}=4\sqrt{14}$

9	11	Answer	Solution
50	5	5	Shoelace method (multiplying diagonals, adding columns) (1,4) 12 (3,0) 0 0 (−1,3) 9 3 (1,4) −4 15 5 Area = $\frac{15-5}{2} = 5$
50	9	5	The slope is $y' = -\frac{x}{y} = -\frac{2}{1} = -2$. If the line goes through (2,1) it also go through (1,3) and (0,5).

College Bowl Extra Questions

9	11	Answer	Solution
1	1	-251	$b^2 - 4ac = 3^2 - 4(13)(5) = 9 - 260 = -251.$
2	2	735	The gcd(x,y) * lcm(x,y) = xy = 21(35) = 735
3	3	9	There are eight ways it can be done: (1,8), (2, 7),…, (8,1). 8/64 = 1 / 8 and 8+1=9.
4	4	19	(7,12), (8,11), …, (12,7) totaling 19 is the most frequent.
5	5	-14	$0 = 6(-7) - 3n$ $n = -\frac{42}{3} = -14$
6	6	95 [degrees]	Sum of interior angles is (from exterior angle) $6\left(180 - \frac{360}{6}\right) = 6 * 120 = 720$. The given angles add to 625, and the difference is 95.
7	7	5	The ordered list is: -2, -1, 3, 4, 7. The median is 3 with 1st quartile -1 and 3rd quartile 4. $4 - (-1) = 5.$