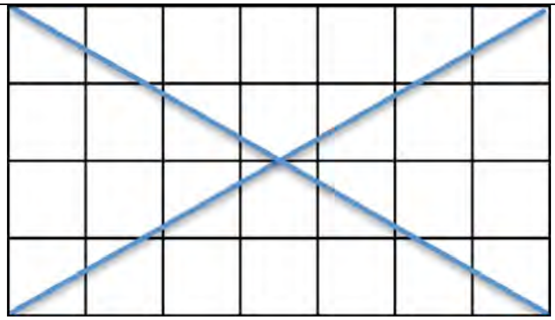
Mental Math

9/11	Answer	Solution
1	30	There are 5 such even numbers and the formula is n(n+1)=5(6)=30.
2	3 [ways]	HHT, HTH, THH.
3	6	1x60, 2x30, 3x20, 4x15, 5x12, 6x10.
4	9	$37 = 3x + 10,\ \ 27 = 3x, x = 9.$
5	18 [sq in]	Even though not the same size, all 4 triangles have the same area as the diagonals bisect each other. The same would be true if the shape were a general parallelogram.
6	65	The numbers from -10 to +10 cancel so all we need to do is add 11+12+13+14+15 = 50+15 = 65.
7	3	If the equation of the parabola is $4(y-k) = (x-h)^2$, the distance is p=3.
8	78 [ways]	There are 13 different values or ranks in a deck each with 4 cards. So, there are 13 ways to choose a rank and 4 choose 2 equals 6 ways to get a pair. 13(6) = 78.

Individual Test

9/11	Answer	Solution
1	11236	$(106^2) = (100+6)^2 = 10000 + 12(100) + 36 = 11236$
2	3	$\left\{4, \sqrt{4}, \frac{1}{7}\right\}$
3	[x=] 7	$4(x+5) - 6 = 2(3x-1) + 2;$ $4x + 20 - 6 = 6x - 2 + 2;\ \ 14 = 2x;$ $x = 7$
4	25 [%]	During the sale the price would be $p\left(\frac{4}{5}\right)$. If the increase is y, then $p\left(\frac{4}{5}\right)y = p; y = \frac{5}{4}$, so a 25% percent increase.
5	2	$17 - 14 - ((5-3)^2 - 6) - \frac{9}{3} = 17 - 14 - (4-6) - 3 = 17 - 14 - (-2) - 3 = 3 + 2 - 3$ $= 2$
6	3	$x^2 - 1 = 3(x+1);\ \ x^2 - 3x - 4 = (x-4)(x+1)$ Solutions: $4 + (-1) = 3\ or\ -\frac{-3}{1} = 3.$
7	13	$2^{12} - 1 = (2^6 - 1)(2^6 + 1) = (2^3 - 1)(2^3 + 1)(2^2 + 1)(2^4 - 2^2 + 1) = 7(9)(5)(13)$
8	36 [sq cm]	The radius of the circle is $\sqrt{18} = 3\sqrt{2}$ making the diameter of the circle and diagonal of the square is $6\sqrt{2}$; the side length is then 6 making the area equal 36 sq cm.
9	48 [ways]	There are 3! = 6 ways to order the couples and 2 ways of ordering each couple. The total is: $6\,(2^3) =\ 48.$
10	21 [instructors]	463 / 28 = 16.54 so 17 classes needed to be run during each of the 6 periods. 17 * 6 = 102 and since each instructor teaches 5 classes means 21 instructors are needed.
11	52 [$units$]	The exterior tangents are each of length 20. The outer pieces of the circles make a whole circle of diameter 2(6)=12. The total length is then: $2(20) + 12\pi$. $40 + 12 = 52$.
12	49	The side of 7 must be a short leg. $7^2 = 49$ is the difference between two squares. Let n and $n + k$ be the two numbers with a sum of $2n + k$. $49 = (n+k)^2 - n^2 = 2nk + k^2 = k(2n+k)$ k can only be 1, 7 or 49 and can easily be seen to 1 so the sum is 49.
13	26 [cm]	To get the largest perimeter, choose sides as different as possible, 1 and 12. These gives a perimeter of 2(1+12)=26.

9/11	Answer	Solution
14	92	The last test will be count double. The six scores must total to 6(90) = 540. These 4 scores add to 356 meaning the last score needs to be: $\frac{540-356}{2}=92$.
15	20 [vertices]	There are a total of 12*5=60 vertices to the pentagons but a vertex of the dodecahedron is where three faces meet. 60/3=20.
16	20	Starting with 115, add multiples of 2(23)=46. 20(46)>900 so that is too much; 19 more multiples make 20 total.
17	6	The numbers from 10(1010_2) to 15 (1111_2) inclusive; a total of 6 numbers.
18	1	The equation will be: $3x-2y=C$. Since it goes through (2,4); $C=3(2)-2(4)=-2$. $3+(-2)+(-2)=-1, and\ \lvert -1\rvert = 1$
19	3 [units]	$4\pi r^2=\frac{4}{3}\pi r^3;\ \ 1=\frac{1}{3}r;\ \ r=3$
20	4	There are two ways to get $15 and there are 4 choose 2 = 6 possible pairs. The probability is then 1/3 and multiply by 12 = 4.
21	2	$\frac{(1+i)^2}{i}=\frac{1+2i+i^2}{i}=\frac{2i}{i}=2$
22	0	$If\ a>c, the\frac{1}{a}<\frac{1}{c}\ , the\ first\ is\ false.$ $If\ a=1, the\ second\ is\ also\ false$ $If\ a>b>c\ then\ a^2>bc,\qquad \frac{a}{c}>\frac{b}{a}\ and -\frac{a}{c}<-\frac{b}{a}$
23	5 [deg]	The interior angles are just 180 minus the external angles. 360/8 – 360/9 = 45 – 40 = 5 degrees
24	12	For the parabola ax^2+bx+c, the minimum will occur at $x=-\frac{b}{2a}$. So, $x=-2$, $(-2)^2+4(-2)+16=12$ or write the polynomial as $(x+2)^2+12$
25	2	The graph of the equation is a hyperbola that is horizontally oriented. Therefore, it intersects the x-axis in 2 places: (3,0) and (-3,0).
26	6	The geometric mean of n numbers is: $\sqrt[n]{a_1a_2\ldots a_n}=\sqrt[3]{3(9)(8)}=3(2)=6$

9/11	Answer	Solution
27	18	 If there is one diagonal. $num = n + m - \gcd(m, n)$
28	2	The point will be the linear combination: $.4(2,3) + .6(7,-7) = (.8 + 4.2, 1.2 - 4.2) = (5,-3)$. So $5 + (-3) = 2$
29	3 [points]	One needs to look for common slopes, let s_{mn} be the slope between the m^{th} and n^{th} points. $s_{12} = 3, s_{13} = \frac{1}{3}, s_{14} = 2, s_{15} = \frac{1}{3},$ $s_{23} = -1, s_{24} = -\infty, s_{25} = 1,$ $s_{34} = -\frac{1}{2}, s_{35} = \frac{1}{3}, s_{45} = \frac{3}{4}$ So, three points are collinear.
30	43	$\left(\sqrt{x} + \sqrt{y}\right)^2 = x + 2\sqrt{xy} + y = 7^2 = 49$ $x + y + 2\sqrt{9} = 49, \quad x + y = 43$
31	5 [primes]	$p = a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ Since p is prime *a-b=1*, *a=b+1*. $p = (b+1)^2 + (b+1)b + b^2 = 3b^2 + 3b + 1$ Try b=1, p=7; b=2, p=19, b=3, p=37, b=4, p=61, b=5, p=91X, b=6, p=127, b=7, p=169X. So a total of 5 primes.
32	148	$\begin{vmatrix} 1 & 3 & 6 \\ 8 & 0 & 4 \\ 0 & 5 & 3 \end{vmatrix} =$ $1[0(3) - 4(5)] - 8[3(3) - 5(6)] + 0[\ ] = -20 + 168 = 148$
33	44	There are 12 such numbers each has the 4 digits appearing three times in the ones and tens column. The average in column is then (1+3+4+8)/4 = 4. Actually it doesn't matter if you permit repeated digits or not, the average is still 44.

9/11	Answer	Solution
34	18 [days]	A crew of x workers can do 1/3 of the work in one day while x+3 workers can do ½ of the work. Therefore 3 workers can do ½-1/3 =1/6 of the work in one day so 1 worker would 1/18 of the job in one day. 18 days.
35	0	2 divides 10!+2, 3 divides 10!+3, etc. There are no prime numbers in the given range.
36	1	Use Descartes rule of signs. Since there are no sign changes in the coefficients, there are no positive real solutions. Substituting (-x) for x, yields $-3x^5 - x^3 + 5x^2 + 6 = 0$ which has 1 sign change and there is 1 negative real solution.
37	84	This is equivalent to distributing the 6 "powers" between a, b, c and d where there is no guarantee that any variable gets any power. The solution is to add 4, line up the powers 1 2 3 … 10; then choose 3 of the 9 gaps. 9 choose 3 = 9(8)(7)/6 = 84.
38	4	$\cos(2\theta) > \cos(\theta) \ \rightarrow \cos(2\theta) - \cos(\theta) > 0.$ $2\cos^2\theta - \cos(\theta) - 1 = (2\cos(\theta) + 1)(\cos(\theta) - 1) > \ 0$ Need both positive or both negative; this happens on $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$ or $\frac{1}{3}$ of the total interval. $\frac{1}{3}\,(12) = \ 4.$
39	5	Let the altitude divide the side into lengths x and y. We have: $x + y = 16$ and using the Pythagorean formula equating the two expressions for the length of the altitude: $10^2 - x^2 = 14^2 - y^2$. Substituting: $x = \frac{10^2+16^2-14^2}{2(16)} = \frac{160}{32} = 5$, clearly the shorter segment.
40	29	Solving the equation for y, one gets: $y = 12 + \dfrac{144}{x-12}.$ $144 = 2^4 3^2$ has $(4+1)(2+1) = 15$ positive factors so if we include the negative values, we get 30. However, we need to exclude -12 since x would be 0; leaving 29.

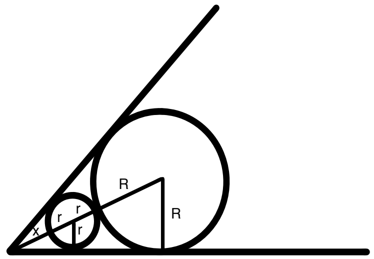
Individual Multiple Choice

9	11	Answer	Solution
1	1	B	$3^x + 3^x + 3^x + 3^x + 3^x + 3^x = 6 * 3^x = 2 * 3^1 * 3^x = 2 * 3^{x+1}$
2	2	C	$169^{17} = 13^{34}$ = K $3^{68} = 9^{34}$ = L 11^{34} = J $L\ J\ K$ is smallest to largest.
3	3	D	The third side is 11. $\sec\theta = \frac{H}{A} = \frac{61}{11}$
4	50	B	$\binom{12}{4} = \frac{12!}{8!\,4!} = 11 * 5 * 9 = 495$
5	5	D	Powers of 7 end in 7, 9, 3, 1, 7, … Powers of 8 end in 8, 4, 2, 6, 8, … 7^{49} *end in* $7, 8^{102}$ *ends in* $4,$ $7 * 4 = 28,$ *which ends in* 8
6	6	D	$x_{10} = 0.2\overline{4}_{12};\ \ 12x_{10} = 2.\overline{4}_{12}$ $11x_{10} = 2.2_{12} = 2\frac{2}{12} = \frac{13}{6}.$ $x = \frac{13}{66}$
7	50	C	4, 1, 1 * 3 1, 2, 3 * 6 2, 2, 2 * 1 10 total out of 216

9	11	Answer	Solution
8	8	B	$4x^2+9y^2-16x+18y=11$ $4(x^2-4x+4)+9(y^2+2y+1)=11+16+9=36$ $\frac{(x-2)^2}{9}+\frac{(y+1)^2}{4}=1$ $3*2*\pi=6\pi$
9	9	B	Add 5 pies to the total and make sure everyone gets one. There are 9 choose 4 ways to do this. Then take one back from each. $\binom{9}{4}=\frac{9(8)(7)(6)}{4(3)(2)}=126$
10	50	D	Let x = 1, the right hand side is the sum that we want. The left hand side is $(7+3-6)^4=4^4=256$
50	4	D	$\sin^4\theta-\cos^4\theta=$ $(\sin^2\theta+\cos^2\theta)(\sin^2\theta-\cos^2\theta)$ So, $\sin^2\theta-\cos^2\theta-2\sin^2\theta$ $=-\cos^2\theta-\sin^2\theta=-1$
50	7	B	First consider lines that do not go through the center of the grid so that the line must be on one of the faces. Be careful not to double count a line. Top and Bottom faces: 8 lines each Left and Right: 6 lines (exclude top/bottom) Front and Back: 4 lines (exclude top, bottom and sides) Now, through the center there are 3 lines that go through the center of each face to the other side. Finally, there are 4 long diagonal lines going through the center. 8+8+6+6+4+4+3+4= 43.
50	10	C	$S_n=\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\cdots$ $2*S_n=1+\frac{2}{2}+\frac{3}{4}+\frac{4}{8}+\cdots$ $S_n=1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$ $S_n=2$

Team Test

9	11	Answer	Solution
1	1	8 [integers]	Brute force, or totient function multiplication properties: $$\left(\frac{2-1}{2}\right)\left(\frac{3-1}{3}\right)\left(\frac{5-1}{5}\right)30 = 8$$ 1, 7, 11, 13, 17, 19, 23, 29
2	-	20 [days]	The loss expectation each day (12 hours) is: $$12\left(\frac{1}{2}\right)\frac{5+10}{2} = 45$$ So we expect 45 pounds of loss and 50 pounds of gain giving a gain of 5 pounds each day. 20 days.
3	3	4536	2016=2*2*2*2*2*3*3*7 Sum of factors is then $(1+2+4+8+16+32)(1+3+9)(1+7) = (63)(13)(8) = 6552.$ 6552 - 2016 = 4536
4	4	7	The sum of the roots (-b/a) over the number of roots (3).
5	5	1	Solve the radical first: $$x = \sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}$$ $$x = \sqrt{2+x},\ \ x = 2$$ Then the fraction stair substituting x=2: $$\frac{-1}{y-2} = y.\ \ y = 1$$
6	-	30	An icosahedron has 3 edges per face, 20 faces, and two faces per edge (3*20/2=30)
7	7	1	ax+by+cz=4,2,3 4a+2b+c=4 | a-3b=5 | -2a+b-c=-3 2a+3b=1 | a-3b=5 a=2, b=-1, c=-2

9	11	Answer	Solution
8	8	3	 The angle through the centers is 30 degrees (half of 60), thus: $\sin(30°) = \frac{r}{r+x} = \frac{1}{2}, \quad x = r.$ $\frac{R}{R+2r+x} = \frac{1}{2}, \; R = 3r, \; \frac{R}{r} = 3$
9	9	5	Factor out 13/3. $\frac{13}{3}\left[1 + \frac{2}{3i} - \frac{4}{9} - \frac{8}{27i} + \cdots\right] = \frac{13}{3}\left[\frac{1}{1 - \frac{2}{3i}}\right]$ $= \frac{13}{3}\left[\frac{3i}{3i-2}\right]\left[\frac{3i+2}{3i+2}\right] = \frac{13}{3}\left[\frac{-9+6i}{-9-4}\right]$ $= \frac{13}{3}\left(\frac{-9+6i}{-13}\right) = 3 - 2i$
10	-	32	Suppose Able eats *i* nuggets, then Bobo can at most eat 20-*i* nuggets. Since they are independent, this has probability: $\frac{1}{21}\left(\frac{21-i}{21}\right)$. Summing these over all possibilities: $\sum_{i=0}^{20}\left(\frac{1}{21}\right)\left(\frac{21-i}{21}\right) = \frac{1}{21^2}\sum_{i=0}^{20} 21 - i$ $= \frac{1}{21^2}\left[21^2 - \frac{20(21)}{2}\right] = \frac{1}{21}(21-10) = \frac{11}{21}. \; 11 + 21 = 32$ You might also set up a grid of lattice points: $0 \le x \le 20 \;\; and \; 0 \le y \le 20$ and count the number on or below the main diagonal $1 + 2 + 3 + \cdots + 20 = \frac{20(21)}{2}$. Dividing by the total number of points 21^2 gives the same answer.

9	11	Answer	Solution
-	2	100	The first two coordinates are polar coordinates in the xy-plane and the last is the z-coordinate. $$d^2 = (5^2) + \left(5\sqrt{3}\right)^2 = 25 + 75 = 100$$
-	6	36	Effectively asks the integral of the derivative over the range, divided by the length of the domain. $$\frac{537 - 285}{7} = \frac{252}{7} = 36$$
-	10	71	Let the origin be the base of the flagpost. Slope information at (0, c) $$y'(x) = -2x + b;$$ $$y'(0) = b = \tan\left(\frac{3\pi}{4}\right) = -1$$ $$y(x) = -x^2 - x + c.\ y(8) = 0$$ $$0 = y(8) = -64 - 8 + c;\ \ c = 72$$ $$c + b = 72 - 1 = 71$$

Pressure Round

9	11	Answer	Solution
1	9	4010	The sum of the integers from 1 to n is $\frac{n(n+1)}{2}$. The elements in the nth set are then: $$\frac{1}{2}n(n+1),$$ $$\frac{1}{2}n(n+1)-1,\ldots,$$ $$\frac{1}{2}n(n+1)-(n-1)$$ $$T_n=\frac{1}{2}n^2(n+1)-\frac{1}{2}n(n-1)=\frac{1}{2}n(n^2+1).\ T_{20}=\frac{1}{2}(20)(20^2+1)=4010.$$
2	2	-1	$$(r+s+t)^2=r^2+s^2+t^2+2(rs+rt+st).$$ From the polynomial. $$r+s+t=-\frac{-3}{1}=3$$ $$rs+rt+st=\frac{5}{1}=5$$ So, $3^2=r^2+s^2+t^2+2(5)$, $9-10=-1$.
3	3	3	Use the fact that the remainder when a number is divided by 9 is determined by the sum of the digits of the numbers. Since any rearrangement of the digits will have the same remainder, subtracting them will always give a multiple of 9. Multiplying by a positive integer will be another multiple of nine whose digits must total to a multiple of 9. This means the remaining digit must be 3 since 8+1+2+9+9+4+3 is a multiple of 9.
4	9	4	The difference between the boys and the girls average is (89-77)=12 and the difference between class average of 81 and the girls is (89-81)=8 or 2/3 of that difference. So 1/3 of the class are girls. The answer is then 1+3=4.
5	5	162	Four steps in the sequence added 3, so 8 steps must add 6 making $a_{15}=21$. The sum will then be $9\frac{(15+21)}{2}=9(18)=162$
9	1	30	Consider the 6 remainders calculated by dividing the 6 integers by 5. Since there are only 5 possible remainders (0,1,2,3,4); at least two of the remainders must be equal, by the Pigeon Hole principle. Those numbers will then differ by a multiple of 5, p=1 and $30p=30$.

9	11	Answer	Solution
9	4	60 [degrees]	$$(a+b+c)(a+b-c)=(a+b)^2-c^2=3ab$$ Isolate c^2. $$c^2=a^2+b^2-2ab\left(\frac{1}{2}\right), \qquad so\cos C=\frac{1}{2}.$$ Therefore the measure of C is 60 degrees.

College Bowl Round 1

9	11	Answer	Solution
1	1	32	$512 = 8^3, 4(8) = 32$ Have to assume 512 is a perfect cube, since 512 ends in 2, the cube root must end in 8.
2	2	6 [factors]	1 2 4 17 34 68, or $68 = 2^2 \cdot 17^1. \quad (2+1)(1+1) = 6$
3	50	24	$\sqrt{64} = 8, \sqrt{1000} = 31+.\ So\ 8\ to\ 31.$ A total of 24 number
4	4	200 [people]	$(100+25)\left(\frac{8}{5}\right) = 125\left(\frac{8}{5}\right) = 200$
5	50	$413_{[7]}$, four-one-three [base 7]	$136_7 + 244_7 = 413_7$ Add normally but carry at 7. For example, 6 base 7 plus 4 base 7 is 3 base 7 and carry a 1., etc.
6	6	143	1 1 2 3 5 8 13 21 34 55, total is 143. Find the next two numbers, 89, 144; the total is 144-1.
7	7	1680 [ways]	$8P4 = 8(7)(6)(5) = 1680$ It's not combination since it matters who gets which hat.
8	8	73	2 3 5 7 11 += 28 13 17 19 23 29 += 101
9	50	15	1+8+27+64+125=225. Or, use the formula for the sum of cubes: $\left(\frac{n(n+1)}{2}\right)^2$ So, the square root is just n(n+1)/2.
10	10	10 [polygons]	Triangles: 4, connect every 4th vertex to make the triangle, move by 1 vertex to make another. Square: 3, Hexagon: 2, Dodecagon: 1. 1+2+3+4 = 10
50	3	8 pi [sq un]	Divide by 64. $\frac{x^2}{2} + \frac{y^2}{32} = 1.\ A = \sqrt{2(32)}\pi = 8\pi$ An ellipse centered at the origin.
50	5	1101_4 or one-one-zero-one base 4.	1001 base 4 is 9 base 10; squared it is 81 and 81=64+16+1 = 1101 base 4. Or convert to base 4, $(1001_2)^2 = (21_4)^2 = 1101_4$

9	11	Answer	Solution
50	9	4 [points]	One needs $x^2 + y^2 = 36^2$. Of course, x=36, y=0 works. It turns out that there are no x>0, y>0 solutions to the equation above. There are then only 4 points, those on the axes that will work.

College Bowl Round 2

9	11	Answer	Solution
1	1	50 [legs]	8 + 5*6 + 3*4 = 50
2	2	168	$89 = 2^1 \cdot 3^1 \cdot 13^1$, $total = (1+2)(1+3)(1+13) = 168$
3	50	9	$8! = 8(7)(720) = 8(5040) = 40320.$ $4+0+3+2+0 = 9$
4	4	5 [factors]	$120 = 2^3(3^1)(5^1)$, $n = (3+1)(1+1)(1+1) = 16$ $16 = 2^4 and\ has\ 5\ factors.$
5	50	20 [palindromes]	1 2 3 4 5 6 7 8 9 11* 22* 33* 44* 55* 66* 77* 88* 99* 101 111 121* 131 141 151 161 171 181 191 202 212
6	6	5040 [ways]	$\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{2 \cdot 2 \cdot 2} = 7! = 5040$
7	7	14 [students]	$\frac{20(.75) + 100n}{20+n} \geq 85$ $1500 + 100n \geq 1700 + 85n$ $15n \geq 200, \quad n \geq 14$
8	8	13 [chickens]	Let c be chickens. $2c + 4(18 - c) = 46$ $72 - 46 = 2c, c = 13.$
9	50	12	Can factor but easier to use Euler method. 516-372=144 so GCF must be a factor of 144 and 372-2(144)=84, 144-84=60, 84-60=24 and 60-2(24)=12.
10	10	19 [units]	Drop the altitude to the longer base by one of the upper vertices. (21-5)/2 = 8 and the height of the 30-60-90 triangle created is $8\sqrt{3}$. The diagonal of the trapezoid is the hypotenuse of a right triangle and $d^2 = (8\sqrt{3})^2 + (21-8)^2 = 192 + 169 = 361$. So d = 19 units.
50	2	1 [point]	The only time is in the first quadrant.

9	11	Answer	Solution
50	5	$\frac{9\sqrt{2}}{4}$ nine root 2 over 4. [cu units]	In an equilateral triangle with side length 3, the median will be $\frac{3\sqrt{3}}{2}$, which will be $\frac{3\sqrt{3}}{2}\left(\frac{2}{3}\right) = \sqrt{3}$ from each vertex. Dropping an altitude from the top vertex of the tetrahedron, then to a vertex in the base is a right triangle having height. $h^2 = 3^2 - \sqrt{3}^2 = 6$. The volume is 1/3 times the height times the base area. $$V = \left(\frac{1}{3}\right)(\sqrt{6})\left(\frac{3^2\sqrt{3}}{4}\right) = \frac{9\sqrt{18}}{3(4)} = \frac{9\sqrt{2}}{4}$$
50	9	line	$\sin\theta = \frac{5}{r}$. $r\sin\theta = 5$; $y = 5$.

College Bowl Round 3

9	11	Answer	Solution
1	1	0	$4x^3 + 0x^2 - 3x + 9 = 0$ Sum of roots is $-\frac{b}{a} = 0$.
2	2	6	Raising a number ending in 4 to powers just alternates between 4 and 6. Since 16 is even, the answer is 6.
3	50	35	Half the numbers are 15 above the mean, so the other half are 15 below the mean. 50-15=35.
4	4	288 [sq units]	The surface area of a sphere is $4\pi r^2 = 144\pi,\ r^2 = 36, r = 6$. The diameter is then 12. If s is the side length of the cube, then $\sqrt{s^2 + s^2 + s^2} = 12, or\ s^2 = \frac{144}{3} = 48.\ SA = 6s^2 = 6(48) = 288$.
5	50	10302	$\frac{102!}{100!} = 102(101) = 10302$
6	6	41	Just change the sign of the imaginary portion to get the conjugate. $(5 + 4i)(5 - 4i) = 25 - 16i^2 = 41$
7	7	17 and 1/7	$\frac{5!\,6!}{7!} = \frac{5!}{7} = \frac{120}{7} = 17\ \frac{1}{7}$
8	8	-21	$x^2 + y^2 = 58, (x + y)^2 = 16$ $16 = x^2 + 2xy + y^2 = 58 + 2xy$ $2xy = 16 - 58 = -42,$ $xy = -21$
9	50	84	Clearly the digits have to be distinct and not include 0. Each choice of 3 digits can only make 1 good number. $\binom{9}{3} = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2} = 3(4)(7) = 84$
10	10	20/21	The right triangle will be 20-21-29 and the cotangent is 20/21.
50	3	0	The numerator remains 100 but the denominator grows without bounds. The limit is 0.
50	5	3/2	$\log_{16}\left(\frac{2}{3}\right) + \log_{16}(96) = \log_{16}\left(\frac{2}{3}\ 96\right) = \log_{16} 64 = \frac{3}{2}$ Since $16^{\frac{3}{2}} = 16(\sqrt{16}) = 64$.
50	9	35 [cu units]	The volume of the parallelepiped is the "triple product" $u \cdot (v \times w) = \begin{vmatrix} 0 & 4 & 3 \\ 0 & 3 & 4 \\ 5 & 5 & 5 \end{vmatrix} =$ $5[4(4) - 3(3)] = 35$.

College Bowl Round 4

9	11	Answer	Solution
1	1	84	Need 2 2's, a 3 and a 7. $$2^2(3)(7) = 84.$$
2	2	51	The mean of the positive integers is (1+50)/2=25.5 and the negative is -25.5 for a difference of 51.
4	4	30	$$\frac{584}{999} = .584\overline{584}$$ The sum is $5 + 8 + 4 + 5 + 8 = 30$
3	50	10 [points]	If the radius of the circle is slightly smaller than the distance from the center to a vertex, the circle will intersect each side twice for 10 total.
5	50	-55	The average is: $-\frac{162}{3} = -54$ is the middle number and -55 is the smallest.
6	6	90 [ounces]	The glass capacity is: $\frac{1}{8}c = 15, c = 15(8) = 120.$ $$w = \frac{3}{4}(120) = 90.$$
7	7	6/49	6 1 | 5 2 | 4 3 and reverse. 6 ways, 49 total possibilities
8	8	[$] 105	This means Bert spent 3/5 of his money so, total is (5/3) 324 = 540. 540-435 = 105.
9	50	14 [people]	It takes 4(7)=28 people hours to paint the fence so we need 28/2 = 14 people.
10	10	8	$$\log_b 32 = \frac{5}{3}, \qquad b^{\frac{5}{3}} = 32,$$ $$b = 32^{\frac{3}{5}} = 2^3 = 8.$$
50	3	13	$$\frac{7+x}{9+x} = \frac{10}{11}.\ 77 + 11x = 90 + 10x, x = 13.$$
50	5	Square root of 110	These values are excluded: n= 1, 4, 9, 16, 25, 36, 49, … 100 | ten of them.

9	11	Answer	Solution
50	9	17 pi over 12	$\sin x \cos x = \frac{1}{4}$ $2 \sin x \cos x = \frac{1}{2} = \sin 2x$ $2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}.$ $x = \frac{17\pi}{12}$

College Bowl Round 5

9	11	Answer	Solution
1	1	-38	$4+5-10(5)+3=12-50=-38$
2	2	5.5	$\frac{50(55)}{20(25)}=\frac{110}{20}=\frac{11}{2}=5.5$
3	50	120 [ways]	It doesn't matter the first person sits, the other 5 can be arranged in 5! Ways. 120.
4	4	4	$\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}=\frac{20}{5}=4$
5	50	32	There are 15 terms from the second to the 17^{th}. $5+15(1.8)=5+15+12=32$
6	6	93	$186+x=.6(465)=279$ $x=279-186=93$
7	7	7	$f(x)=18-4x=-10$ $28=4x, \quad x=7.$
8	8	1 OR 1 to 1.	The volumes will be the same since 2(1/2)=1.
9	50	2 [units]	$d=\frac{\lvert Ax+By+C\rvert}{\sqrt{A^2+B^2}}=\frac{4(3)-3(2)+4}{\sqrt{4^2+(-3)^2}}=\frac{10}{5}=2$
10	10	32 pi [cu units]	$\frac{x^2}{4}+\frac{y^2}{16}=1-\frac{z^2}{9}$ This is an ellipsoid, an extension of a sphere, with volume: $V=\frac{4}{3}\pi\, abc=\frac{4}{3}(2)(4)(3)\pi=32\pi$
50	3	Negative 3 comma negative 57	$y=3x^2+18x-30$ $=3(x+3)^2-57$ Minimum is $(-3,-57)$.
50	5	72 [minutes]	At 6am, A has already traveled half the distance so has 2 hours left. The ratio of speeds is 3:2 so A will go 3/5 of the distance before they meet. $\left(\frac{3}{5}\right)2=\frac{6}{5}\ hours=72\ min$
50	9	-1	You can use the double angle formula for cosine to get: $\log_2(\cos^2(30°)-\sin^2(30°))=\log_2(\cos(60°))=\log_2\left(\frac{1}{2}\right)=-1$

College Bowl Round 6

9	11	Answer	Solution
1	1	13049	6574 + 6475 = 13049.
2	2	48 [sq units]	This is a trapezoid with height $4-1=3$ and bases $4(1)+6=10$ *and* $4(4)+6=22$. $$A=\left(\frac{1}{2}\right)3(10+22)=3(16)=48$$
3	50	80	2*5*8=80
4	4	2401	$$7^4=49^2=(50-1)^2=2500-100+1=2401$$
5	50	Saturday	560 is a multiple of 7. We want 565 days in the future so 5 days past Monday is Saturday.
6	6	9 [sq units]	This is a trapezoid that has bases of 6 and 3 and a height of 2. $$A=\left(\frac{1}{2}\right)2(6+3)=\ 9$$
7	7	4320	The term will be: $$\binom{6}{3}(2x)^3(3y)^3=(20)(8)(27)x^3y^3=4320x^3y^3$$
8	8	3x minus 2 times 2x plus 3 OR reversed	$$6x^2+5x-6$$ $$=6x^2+9x-4x-6$$ $$=3x(2x+3)-2(2x+3)$$ $$=(3x-2)(2x+3)$$
9	50	440 pi	Drawing a figure, one can see that it can reach 3/4 of a circle with radius 24 and 2(1/4) of a circle with radius 4. $$A=\left(\frac{3}{4}\right)\pi(24^2)+\left(\frac{1}{2}\right)\pi(4^2)=432\pi+8\pi=440\,\pi$$
10	10	-2	$$f(x)=x^3-6x^2+9x-6\ on\ [0,3]$$ $$f'(x)=3x^2-12x+9=3(x^2-4x+3)$$ $$=3(x-1)(x-3)$$ local max at x=1, min at x=3 $$f(1)=1-6+9-6=-2$$
50	3	241	It is a right triangle and the longest median will be to the shortest side. The median is the hypotenuse of a right triangle with legs 4 and 15. $$h^2=4^2+15^2=241$$

9	11	Answer	Solution
50	5	65 pi [sq un]	The slant height is found by Pythagoras: $\sqrt{5^2+12^2}=13$. The lateral area is $5(13)\pi = 65\pi$.
50	9	Two comma negative 6.	In standard form: $x^2-4x+12y+40=0$ $(x-2)^2=4(-3)(y+3)$ A parabola open downward with vertex at $(2,-3)$ and whose directrix 3 units up and focus three units down. $(2,-6)$.

College Bowl Extra Questions

9	11	Answer	Solution
1	1	122 [degrees]	The minute hand moves 6 degrees every minute so it is at 264 degrees. The hour hand starts at 120 degrees and moves 1/2 degree every minute so it is at 142 with a difference of 122 degrees.
2	2	30 [numbers]	5 choices for the tens place (1, 4, 6, 8, 9) and 6 choices for the ones place (include 0).
3	3	40 [%]	$38 = 95x, x = \frac{38}{95} = \frac{2}{5} = 40\%$
4	4	[$] 5600	$3500 + \left(\frac{3}{5}\right)3500$ $= 3500 + 2100 = 5600$
5	5	56250	Square 75 and add a 0. 75 squared is 7(8)=56 followed by 5(5)=25.
6	6	10800	$3(60)(60) = 3(36)(100) = 10800$
7	7	Twenty seven over five. $\frac{27}{5}$	The harmonic mean is the reciprocal of the average of the reciprocals so. $HM = \frac{2xy}{x+y} = \frac{2(3)(27)}{3+27} = \frac{27}{5}$