Mental Math

9/11	Answer	Solution
1	30	Everything cancels but 6(5)=30
2	55	$1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55 = \frac{(5)(6)(11)}{6}$
3	[x=]5	$4x - 9 = 11; 4x = 20, x = 5.$
4	18	**5**+2>6, 6+2>**7**. 5+6+7=18
5	2	There are 6 ways of getting a seven and only 3 ways of getting a four. The ratio is 2.
6	5	To be divisible by 9, the sum of the digits must be divisible by 9. The total of the known digits is 13 so D must be 5.
7	4	To have three factors, the number must be the square of a prime number. Only $2^2, 3^2, 5^2$ *and* 7^2 are less than 100. (4)
8	144 [°]	The exterior angles sum to 360 degrees making each one is 36 degrees in a decagon. The interior angle is 180-36=144 degrees.

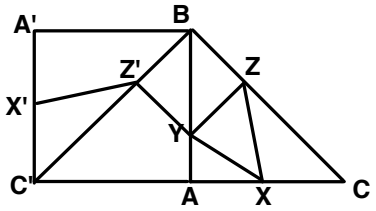
Individual Test

9/11	Answer	Solution
1	2916	Since 54 = 50 +4. $54^2 = 2500 + 4(2)(50) + 4^2 = 2916$
2	16	To find the midpoint, average the x and y coordinates. $\frac{3+17}{2} = 10, \frac{10+2}{2} = 6.\ 10 + 6 = 16$
3	41 [years]	$(B-5) = 3(M-5).\ B = 3M - 10$ $M = 17, B = 3(17) - 10 = 41.$
4	4	Only e^2 is irrational, the other 4 are rational.
5	1	The average is $\frac{12+10+16+19+13+13+15}{7} = 14$ and the median is 13 (in order they are 10, 12, 13, **13**, …). 14-13=1
6	4884	$66\times74 = (70-4)(70+4) = 70^2 - 4^2 = 4900 - 16 = 4884.$
7	[x=] 11	$4(x-2) + 3x + 2 = 6x + 5.\ 4x - 8 + 3x + 2 = 6x + 5.\ 7x - 6 = 6x + 5.\ x = 11$
8	[$] 112	$200(.8)(.7) = 200(.56) = \$112.$
9	4 [ways]	2 heads and 1 tail has 3 ways – the one tail could occur on any one of the 3 flips. All heads can occur in 1 way for a total of 4.
10	32 [meters]	1x60 is 122. 2x30 is 64. 3x20 is 46. 4x15 is 38, 5x12 is 34 and 6x10 is 32. Choose the shape closest to a square.
11	50	Take them in pairs. $(100-99) + (98-97) + \cdots + (2-1) = 50(1) = 50.$
12	28 [cu in]	$x^3 = \frac{7}{2}.\ (2x)^3 = 8x^3 = 8\left(\frac{7}{2}\right) = 28\ cu\ in.$
13	17	The next larger palindrome will be 1235321 and the sum of the digits is 17.
14	1115 [cents]	260+2(60) +3.5(210) =260+120+735=1115 cents
15	9	$\frac{9^4}{3^5 + 3^5 + 3^5} = \frac{3^8}{3(3^5)} = \frac{3^8}{3^6} = 3^2 = 9$
16	55 [sq inches]	The area of a rhombus is one-half the product of the diagonals. 10(11)/2 = 55 square inches.

9/11	Answer	Solution
17	13	$1001 = 7\times11\times13.$
18	14	The middle number of an arithmetic sequence is the average of the two. $\frac{11.8+16.2}{2} = 14.$
19	12 [ways]	There are 4!=24 ways total. Treating the 2 kids as one, there are 3!=6 of ordering them times 2 ways of ordering the kids or 12 ways the kids sit next to each other. 24-12 = 12 ways.
20	30 [edges]	Each triangle has 3 sides and each side is shared with one other face. $3(20)/2$ = 30 edges.
21	6 [un]	$4(9) = 6(x).\ x = 6$
22	83	$.7(93) + .3(x) = 90;\ \ 651 + 3x = 900;\ \ 3x = 249;\ \ x = 83$
23	54 [diagonals]	The external angle is 180-150=30 degrees and the number of sides is $360/30$ = 12 sides. The number of diagonals is $12(9)/2$ = 54.
24	1	$\log_2(\log_2(\log_2 16)). = \log_2(\log_2 4)) = \log_2 2 = 1$
25	$4104_{[5]}$	One could convert to base 10, namely 23. Then square it, 529 and convert it back to base 5. $529 = 4(125) + 1(25) + 0(5) + 4 = 4104_5.$
26	377	$(3-2i)(2+5i) = 6-4i+15i-10i^2 = 16+11i.$ The square of the magnitude will be $16^2 + 11^2 = 256 + 121 = 377.$ Or, one can note that the magnitude of the product is product of the magnitudes: $(3^2+2^2)(2^2+5^2) = 13(29) = 377.$
27	8	One could multiply it out but it is simpler to realize that if $x = y = z = 1$ then the expansion is just the sum of the coefficients. $(2x-y+z)^3 = (2-1+1)^3 = 8$
28	7	$n^2 - n - 56 = (n-8)(n+7)$ So, either n-8 or n+7 must be divisible by 13. $13 \mid n+7;\ \ n = 6, 19, 32, 45$ $13 \mid n-8; n = 21, 34, 47$ There are 7 integers.

9/11	Answer	Solution
29	9 [socks]	Let w be the number of white socks and b the number of black socks. Then $$\frac{w}{w+b}\,\frac{w-1}{w+b-1}=\frac{7}{12}=\frac{7k}{12k}$$ The smallest k such that 7k is the product of 2 consecutive numbers is k=6, with w=7. Looking at the denominators. $$(w+b)(w+b-1)=(b+7)(b+6)=72$$ Or b=2. Thus there are 7 white and 2 black socks for 9 total socks
30	180 [min]	Rate problem: F1 fills 1/6 per hour, F2 fills 1/8 per hour, and F3 fills 1/12 per hour, while L1 removes 1/24 per hour. $$\frac{1}{6}+\frac{1}{8}+\frac{1}{12}-\frac{1}{24}=$$ $$\frac{4+3+2-1}{24}=\frac{1}{3}$$ It then takes three hours exactly overall, or 180 min.
31	7	Substituting 1/x for x in the equations and multiplying by x^2 gives the equation: $$2-7x+x^2=0, with\ solutions \frac{1}{m} and \frac{1}{n}$$ The sum of which is $-\frac{b}{a}=-\frac{-7}{1}=7$
32	5	Clearly $(a,b,c)=(0,0,0),(1,1,1)$ are solutions. To find the others observe: $$ab=a^2c=c\ \Rightarrow a=\pm 1.$$ Similarly $b=\pm 1, c=\pm 1$ Checking, we see that if one of a, b or c is 1 and the others are -1, then it is a solution. There are then 5 total solutions.
33	33	The sum of the roots $a+b+c=0$ follows from the zero x^2 term. Substituting the individual roots into the expression gives the three relations below: $$a^3=-4a+11, b^3=-4b+11, c^3=-4c+11$$ Which can then be linearly summed: $$a^3+b^3+c^3=-4(a+b+c)+33=33$$

9/11	Answer	Solution
34	36 [sq un]	Extend perpendicular line segments to each of the sides of the rectangle. Let S be the intersection on AB and T be the intersection on AD. We have: $AS^2 + AT^2 = 49, \quad SB^2 + AT^2 = 4, \quad TD^2 + AS^2 = 81$ $PC^2 = TD^2 + SB^2$ $= (TD^2 + AS^2) + (SB^2 + AT^2) - (AS^2 + AT^2) = 81 + 4 - 49 = 36$ $PC = 6$
35	60 [%]	Let there be 100 students taking Algebra and Geometry and y be the number that said Yes. Yes No Total Algebra .6y 100-.6y 100 Geometry .4y 100-.4y 100 Total y 200-y 200 Now, $\frac{100-.4y}{200-y} = 0.8$. $\frac{500-2y}{200-y} = 4$. $500 - 2y = 800 - 4y$ $y = 150$ and the table becomes. Yes No Total Algebra 90 10 100 Geometry 60 40 100 Total 150 50 200 The percent of Geometry students saying Yes is 60.
36	75	We need the sum of the unit's digits to be 0 or 5. Consider the units digits of the powers of 1, 2, 3 and 4. 1 1 1 1 2 4 8 6 3 9 7 1 4 6 4 6 The first 3 columns add to a multiple of 5. With the digits repeating in cycles of 4, there are (3/4)(100)=75 multiples of 5.
37	-1	$a^3 + 1 = (a + 1)(a^2 - a + 1) = (a + 1)(0) = 0$ $a^3 + 1 = 0; \quad a^3 = -1$

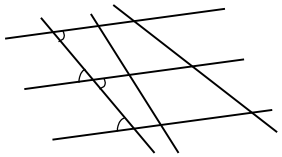
9/11	Answer	Solution
38	10	$10^{2017} = 10^{3(672)+1} = (10^3)^{672}\,(10) = (1001-1)^{672}\,(10)$ In the first expansion, 1001 will appear in all the terms except the last which is $(-1)^{672} = 1$ so that remainder will be 1. The remainder when 10 is divided by 1001 is 10 so the answer is (1)(10)=10.
39	84 [cubes]	First get the prime factorization. $7!\cdot 8!\cdot 9! = (7!)^3(8^2)(9)$ $= (2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7)^3(2^6)(3^2)$ $= (2^4\cdot 3^2\cdot 5\cdot 7)^3(2^6)(3^2) = 2^{18}\cdot 3^8\cdot 5^3\cdot 7^3$ All exponents must be multiples of 3 to be a cube; 2 has 7 choices (0,3,6,9,12,15,18), 3 has 3 choices, 5 and 7 each have 2 choices. The total is then 7(3)(2)(2)=84.
40	10 [un]	Start by reflecting the triangle about the segment AB, making a new point C'. Then reflect that triangle about segment C'B. Label corresponding points Z', Y' and X' as shown in the figure.  Since the reflections represent congruent triangles, the perimeter of XYZ is the sum of XY+YZ'+Z'X' with X and X' representing fixed endpoints. The shortest sum is the length of the straight line from X to X'. By Pythagoras. Note AC=2, AX=1, CX'=1 $(XX')^2 = 1^2 + 3^2 = 10$

Individual Multiple Choice

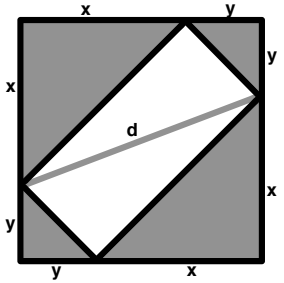
9	11	Answer	Solution
1	1	B	123 + 423 = 546
2	2	A	2 * 19 * 5 = 190
3	3	B	(36 + 12) in / (12 + 2) sec = 24/7 in/sec
4	50	B	The terms a_n are $\sum_{i=1}^{n} f_i$, where the f_i are the terms of the Fibonacci sequence.
5	5	B	The sequence is a, ar, ar^2, ar^3 so $(ar)(ar^2) = (a)(ar^3) = 4(7) = 28$ (B)
6	6	E $\left(\frac{13}{18}\right)$	Center of the circle must occupy a $(24-4)\times(30-4)$ rectangle within the 24×30 patch. $$\frac{20(26)}{24(30)} = \frac{2(26)}{3(24)} = \frac{13}{18}$$
7	50	C	For the first half of the file, we download $\frac{1}{2}+\frac{1}{4}+\frac{1}{5}=\frac{19}{20}$ of it per hour, or half of the file in $\frac{10}{19}$ hours. The second half of the file is twice as fast, so it only takes $\frac{5}{19}$ hours, for a total of $\frac{15}{19}$ hours.
8	8	C	The sum of the roots of $f(x)$ is $-\frac{b}{a} = 17$. Therefore, letting *m*, *n*, *p*, *q*, be the roots of *f*, $$\frac{b^2}{a^2} = 289 = (m+n+p+q)^2$$ The right side when multiplied out is the sum of the sum of the squares of the roots (which we want) and twice the sum of the roots taken two at a time, which is $\frac{c}{a} = 16$. Thus the sum of the roots' squares is $$289 - 32 = 257$$
9	9	C	The odd numbers from 1 to 17 add to $9^2 = 81$. But $2(47) = 94$ which counts the middle square twice. $94 - 81 = 13$.

9	11	Answer	Solution
10	50	D	Sum the angles in all the triangles. There are 360 degrees for each of the 20 interior points and 90 degrees more for each corner of the square. $20(360) + 4(90) = 40(180) + 2(180) = 42(180)$ So, there must be 42 triangles created. (D)
50	4	D	Determinant of the inverse is the inverse of the determinant. The determinant of the given, noninverted matrix is -45, so the answer is $-\frac{1}{45}$
50	7	C	$\frac{v_1 \cdot v_2}{\lvert v_1 \rvert \lvert v_2 \rvert} = \frac{5+5-7}{\lvert\sqrt{75}\rvert\lvert\sqrt{27}\rvert} = \frac{3}{45} = \frac{1}{15} = \cos(\theta)$ θ is then the larger acute angle of a 1--$4\sqrt{14}$ --15 right triangle and this has a cosecant (hypotenuse over opposite) of $\frac{15}{4\sqrt{14}} = \frac{15\sqrt{14}}{56}$
50	10	C	Grouping the polynomial expression allows the use of binomial expansion: $(x^4 - 3ix^3 - 2x + 6i)^4 = \left((x^3 - 2)(x - 3i)\right)^4$ $= (x^3 - 2)^4 (x - 3i)^4$ An x^5 term only occurs when the left exponent distributes once to the x^3 and the right exponent twice to the x. The coefficient is calculated: $\left(\binom{4}{1} 1 * (-2)^3\right)\left(\binom{4}{2} 1^2 * (-3i)^2\right)$ $= 4 * -8 * 6 * -9 = 32 * 54 = 1728$

Team Test

9	11	Answer	Solution
1	1	6	 Each line creates 8 angles, which by axioms includes 2 groups of four angles of the same size. 2*3=6.
2	20	75	11 and 7 are the same interval as 7 and 3, so the 11th term is most directly found by adding the difference between the given terms to the higher one. 47+47-19=75.
3	3	39 [mph]	Total distance over total time 12 miles at 30 mph = 2/5 hours 12(1.25)=15 miles at 50 mph = 3/10 So (12+15)=27 mi in 7/10 hr = 270/7 mph ~38.57 mph which rounds to 39 mph.
4	4	42 [minutes]	Each attempt can be described by the equation. $m = .05(2) + 0.1(1 + 2 + m) + 0.85\ (2 + m)$ where *m* is the expected number of minutes to success. $m = .1 + 0.3\ +\ 0.1m + 1.7 + 0.85m$ $m = 2.1 + 0.95m$ $0.05m = 2.1$ $m = 42\ minutes$
5	5	6 [feet]	The height of the intersection is independent of the distance between the poles. If m and n are the two pole heights, the intersection height is given by: $h = \frac{mn}{m+n} = \frac{10(15)}{10+15} = 6\ [feet]$

9	11	Answer	Solution
6	20	243	Synthetic division recommended. -2 | -7 4 0 -10 1 -3 14 -36 72 -124 246 -7 18 -36 62 -123 243
7	7	120 [ways]	Consider the letters MLGM, there are 4!/2!=12 ways to order them. Now consider one such permutation _M_L_G_M_. The three A's must occupy 3 of the blanks which can be done in 5C3=10 ways. The total is then 12(10)=120 ways.
8	8	3	As standard for recursive expressions, assign a variable to the whole thing, allowing the following relation: $w = \frac{y}{x+w} = \frac{x}{y+w}$ Multiplying out the denominators gives expressions for x and y individually: $y = wx + w^2$ $x = wy + w^2$ And doing the same with the original expression: $\frac{y}{x+w} = \frac{x}{y+w}$ $y^2 + wy = x^2 + wx$ Then, adding w^2 to both sides of the above equation and substituting: $y^2 + wy + w^2 = x^2 + wx + w^2$ $y^2 + x = x^2 + y$ $y^2 - y - x^2 + x = 0$

9	11	Answer	Solution
9	9	16 [un]	Label the segments of sides of the square as x and y.  Since the gray area is 128, $x^2 + y^2 = 128$. The sides of the rectangle are: $x\sqrt{2}\ and\ y\sqrt{2}$. By Pythagoras, $d = \sqrt{2x^2 + 2y^2} = \sqrt{256} = 16$.
10	20	49	$$\frac{2n^2 - 5n + 1}{n - 6} = 2n + 7 + \frac{43}{n - 6}$$ Since $2n$+7 is an integer $43/(n$-6) must be making n=49.
20	2	22029	First hex the binary by combing 4-digits into 1, then add in hex. 1011 0010 1101 4AE0 + B2D ---------- 560D =13+16*0+6*256 + 5*4096 =29 + 1536 + 20480=22029
20	6	2	$$= \cos^{-2}\left(-\frac{\pi}{4}\right) = 2$$
20	10	14	$$2S = 16 + \frac{7}{1} + \frac{6}{2} + \cdots$$ $$2S - S = S = 8 + 7 - \left(\frac{1}{2} + \frac{1}{4} + \cdots\right)$$ $$S = 15 - 1 = 14$$

Pressure Round

9	11	Answer	Solution
1	9	2018	$2^{2017} + 10^{2017} = 2^{2017} + 2^{2017}5^{2017} = 2^{2017}(1 + 5^{2017})$ 5^{2017} ends in 25, adding 1; $1 + 5^{2017}$ ends in 26; divisible by 2 but not by 4. 2017+1=2018.
2	2	43	$f(x) = \frac{x^2 + 7x - 30}{x - 3} = x + 10;$ $f(2) + f(5) + f(6) = 2 + 5 + 6 + 30 = 43$
3	3	6 [sq un]	$2(AM + NC) = 3MN.\ 2(AM + MN + NC) = 5MN$ $2(AC) = 5MN.\ MN = \frac{2}{5}AC$ $Area = \frac{2}{5}\left(\frac{1}{2}\right)30 = 6$
4	9	120 [ways]	This is equivalent to choosing 3 seats out of 6 and then inserting an empty seat to the right of the first two occupied ones. There are then 3! of ordering the teachers. $\binom{6}{3}(3!) = 6(5)(4) = 120$
5	5	15	$\lvert 3x + 5\rvert < 23;\ -23 < 3x + 5 < 23.$ $-28 < 3x < 18. -\frac{28}{3} < x < 6$ So, the integers from $-9\ to\ 5$ or 15 inclusive in total.
9	1	8 [pounds]	The non-water weight stays the same in both the plums and prunes. $(1 - .8)(36) = (1 - .1)x.\ x = \frac{.2(36)}{.9} = 8\ pounds$
9	4	36 [°]	Since θcannot be 0; multiply both sides by 4 $sin\theta$ and use the double angle formula. $\sin(\theta) = 4\sin(\theta)\cos(\theta)\cos(2\theta)$ $= (2)2\sin(\theta)\cos(\theta)\cos(2\theta)$ $= 2\sin(2\theta)\cos(2\theta) = \sin(4\theta)$ Since $0° < \theta \leq 90°, \sin(\theta) > 0.$ 4θ must be supplementary to $\theta.$ $\theta = 180 - 4\theta;\ \ \theta = 36°$

College Bowl Round 1

9	11	Answer	Solution
1	1	3575	$55(65) = (60-5)(60+5) = 60^2 - 5^2 = 3600 - 25 = 3575$
2	2	14[times]	$\frac{112}{8} = 14$
3	50	204	The numbers are 12 and 17 and 12(17) = 204.
4	4	5	3.14159…
5	50	234 [base 7]	$123 = 2(49) + 3(7) + 4\ so\ 234_7$
6	6	2 [quadrants]	The graph exists only in quadrants 1 and 2.
7	7	1	$9x^2 + 6x + 1 = (3x+1)^2$ so there is 1 repeated root.
8	8	9	$7! = (7)(6)(5)(4)(3)(2)$ is a multiple of 9 (factors of 3 and 6) so the sum of the digits is a multiple of 9, leading to a digit root of 9.
9	50	42	There are 90 two-digit numbers so there are 30 multiples of 3 and 18 multiples of 5 and 6 multiples of 15. $30 + 18 - 6 = 42$.
10	10	20	The 2's, 4's, 6's, 8's and 10's; four each. 4(5)=20.
50	3	93	$g(f(6)) = g(22) = 93.$
50	5	13 [meters per hour]	The resultant velocity is the length of third side of the triangle which can be found with Pythagoras. $\sqrt{12^2 + 5^2} = 13\ m.ph$
50	9	$\sqrt{2}$	$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}\ so\ \sec\frac{\pi}{4} = \frac{2}{\sqrt{2}} = \sqrt{2}$

College Bowl Round 2

9	11	Answer	Solution
1	1	96	Sum of 1st ten odd numbers = $10^2 = 100$ minus 1+3=4, $100 - 4 = 96$.
2	2	1.06	$\frac{17}{16} = 1\frac{1}{16} = 1.0625 \approx 1.06$
3	50	10 [ways]	5C3 = 5C2 = 5(4)/2 = 10
4	4	160 [sq un]	The graph is a line and the shape created is a trapezoid. The base along the x-axis is 12-4=8 and the two bases are of height 4+12=16 and 12+12=24 with an average of 20. The area is then 8(20)=160.
5	50	40	One zero for every multiple of 5 and an extra for each multiple of 25.
6	6	31 minus 27 i	The product is: $35 + 20i + 7i = 4i^2 = 31 + 27i$. The conjugate is: $31 - 27i$.
7	7	53	First divide by 9; 318 / 6 = 53. Or, since 50^2=2500, the answer will be close 50 and the units digit would be 3 since 3x4=12 to get the unit digit of 2.
8	8	10 [triangles]	Actually, all of the triangles that can be made are isosceles, 5C3 = 10 triangles
9	50	Tuesday	7 full weeks plus 4 days. Today is Friday, four days later would be Tuesday.
10	10	2	2017 is prime, so there are only 2 factors.
50	3	8 [elements]	Including the empty set, there are 8 subsets. Each of the three elements can be in or out of a subset: 2(2)(2)=8.
50	5	25	The product of all three solutions is -125 and one of them is -5. $-\frac{125}{-5} = 25$. $(\frac{5}{2}(1 - i\sqrt{3}))(\frac{5}{2}(1 + i\sqrt{3}))$
50	9	$\frac{10}{7}$	Use change of base with base 2. $\log_{128} 1024 = \frac{\log_2 1024}{\log_2 128} = \frac{10}{7}$

College Bowl Round 3

9	11	Answer	Solution
1	1	10	Triangle inequality **10**+8>17.
2	2	28 [times]	Consider the positive values: there is 3, 13, 23 and 11 3's in the thirties. 14 positive times 2 = 28 total.
3	50	156	$n(n+1) = 12(13) = 156$
4	4	0	The sum of the solutions is: -b/a = -0/5 = 0.
5	50	$\frac{23}{42}$ [liters]	$\frac{5}{6} - \frac{2}{7} = \frac{35}{42} - \frac{12}{42} = \frac{23}{42}\ liters$
6	6	15000	The b^2 term is: $(4C2)(5a)^2(10b)^2 = 6(25)(100)a^2b^2 = 15000\ a^2b^2$
7	7	44	1 1 **2** 3 5 **8** 13 21 **34**. 2+8+34 = 44.
8	8	990	This is simply 45 choose 2. 45(44)/2=45(22)=990.
9	50	6	There are gaps of 6 between 23 and 29, 31 and 37 – no larger gaps.
10	10	5/8	From the problem, the probability that the next computer sold is a Mac is ¾ and a PC at ¼ . $P(same) = \frac{3}{4}\left(\frac{3}{4}\right) + \frac{1}{4}\left(\frac{1}{4}\right) = \frac{10}{16} = \frac{5}{8}.$
50	3	4	$\log_3 5 = \log_x 5 \cdot \log_3 4$ $\log_3 5 / \log_3 4 = \log_x 5$ $\log_4 5 = \log_x 5.\ x = 4$
50	5	2 [quadrants]	Just the first and third quadrants.
50	9	11	The sum of the top faces and the opposite sides is 3(7)=21. Since the top faces add to 10, the sum of the opposite faces is always 11.

College Bowl Round 4

9	11	Answer	Solution
1	1	8	$14(7) - 15(6) = 98 - 90 = 8$
2	2	7	2 x 3, 5, 7, 11, 13 3 x 5, 7
3	50	91	There will be 36 1x1 squares, 25 2x2 squares, 16 3x3 squares, etc. $36 + 25 + 16 + 9 + 4 + 1 = 91$
4	4	-5	The sum of the roots is $-\frac{25}{142}$. The product is $\frac{5}{142}$. $\left(-\frac{25}{142}\right)\left(\frac{142}{5}\right) = -5$
5	50	100	All zeroes except for the 100 1's down the main diagonal. Sum = 100.
6	6	$\frac{28}{3}$	The harmonic mean is the reciprocal of the average of the reciprocals. $HM = \frac{3}{\frac{1}{6}+\frac{1}{12}+\frac{1}{14}}\frac{84}{84} = \frac{3(84)}{14+7+6} = \frac{3(84)}{27} = \frac{28}{3}.$
7	7	20 [minutes]	The net change is 3-2.4 = 0.6 liters per minute. The time is: $\frac{12}{0.6} = \frac{120}{6} = 20\ minutes$
8	8	29	The numbers are: $n, n+1, n+2, n+3$. The middle two terms add to the same as the outer two so the sum is 58/2=29.
9	50	17	Ten terms (3rd to 13th) adds 10(1.3)=13 so the 13th term is 4+13=17
10	10	15 [degrees]	Look at the difference in exterior angles instead: $\left(\frac{360}{6}\right) - \left(\frac{360}{8}\right) = 60 - 45 = 15\ degrees$
50	3	4 [integers]	Relatively prime means gcd(n,12)=1. n=1, 5, 7, 11.
50	5	8 [sq un]	The equation: $\frac{x^2}{4} + y^2 = 1$ is an ellipse with major axis 2(2)=4 and minor axis 2(1)=2. The rectangle is 2x4 with area 8.

9	11	Answer	Solution
50	9	12π	The period of $\cos\left(\frac{x}{2}\right)$ is $\frac{2\pi}{\frac{1}{2}} = 4\pi$, while the period of $\cos\left(\frac{x}{3}\right) = 6\pi$. The sum has a period = lcm(4,6) π =12π .

College Bowl Round 5

9	11	Answer	Solution
1	1	8	The total $9 + \cdots + 81 = \frac{81+9}{2}\ N = 360.\ N = \frac{360}{45} = 8$
2	2	7 [zeroes]	There are 6 factors of 5 and one of 25 and lots of factors of 2 – 7 zeroes.
3	50	7	Use the sum of solutions is –b/a. $-\left(-\frac{5}{1}\right)+\left(-\left(-\frac{4}{2}\right)\right) = 5 + 2 = 7$
4	4	4	100 mod 24 is simply the remainder when 100 is divided by 24. 100=4(24)+4.
5	50	13	This can be done recursively. Let S_n be the number ways to climb n steps. $S_{n+2} = S_{n+1} + S_n.\ S_1 = 1, S_2 = 2, S_3 = 1 + 2 = 3, S_4 = 5, S_5 = 8, S_6 = 13.$
6	6	6 [times]	The third row (1 3 3 1) has two three's. The seventh row has two 35's and the ninth row has two 36's. The total is 6 times.
7	7	0	All perfect squares end in 0,1, 4, 5, 6 or 9.
8	8	33 [ones]	Brute force counting of the ones. 1(1), 2(1), 3(2), 4(1), 5(2), 6(2), 7(3), 8(1), 9(2), 10(2), 11(3), 12(2), 13(3), 14(3), 15(4), 16(1) – total 33. OR. Consider the 4-digit binary numbers 0000 to 1111 a total of 4(16)=64 digits half of which are 1. Adding the one in 10000 gives a total of 64/2+1-33.
9	50	9 [times]	The side of the larger cube is 9 and the smaller is 3. The surface area ratio will be: $\left(\frac{9}{3}\right)^2 = 9.$
10	10	$12000	Count the number of cow-weeks: $500 + 450 + 400 + 350 + 300 = 2000.\ 2000(\$6) = \$12000.$
50	3	2 [radians]	$\frac{x}{2\pi}\ \pi(12^2) = 144;\ \ x = 2$

9	11	Answer	Solution
50	5	5π	Convert to rectangular coordinates. $r = 4cos\theta - 2sin\theta$ $r^2 = 4r\cos\theta - 2r\,sin\theta$ $x^2 + y^2 = 4x - 2y$ $(x-2)^2 + (y+1)^2 = 5 = r^2$ $A = 5\pi$
50	9	6 [peaks]	The range translates to $-20 < -6\pi < 4\pi < 15$. All even multiples of pi are peaks for a total of 6.

College Bowl Round 6

9	11	Answer	Solution
1	1	540	$12 = 2^2 3, 15 = 3(5); 18 = 3^2 2$ $gcd = 3, lcm = 2^2 3^2 5 = 180$ $3(180) = 540$
2	2	1	i raised to the power of 4 is 1 and thus is also 1 for any multiple of 4 including 6 factorial.
3	50	2	$\frac{V_{old}}{V_{new}} = \frac{\left(\frac{1}{3}\pi 4^2 6\right)}{\frac{1}{3}\pi 2^2 12} = \frac{4}{2} = 2$
4	4	11	Could use the Chinese Remainder Theorem but brute force is faster. Choose the highest divisor 4 rem 3. N = 3, 7, 11, 15. 3 and 7 do not work but 11 does.
5	50	1036	Have each new line intersect all the previous lines creating n new regions. $A_n = \frac{n(n+1)}{2} + 1$ $A_{45} = \frac{45(46)}{2} + 1 = 1036.$
6	6	1440 [ways]	There are eight ways (3 rows, 3 columns and 2 diagonals). There are 3! ways of picking them. There are six other squares that are filled by the O's. 6 choices for the first and 5 for the second. $8 \times 3! \times 6 \times 5 = 1440$
7	7	14	Want y to be as large as possible as it multiplied by 18. y=10, x=4 works but y=11 has no integer solution. 10+4 = 14.
8	8	16 [numbers]	There are six numbers each that begin with either a 3 or 4 and four more that begin with 23 or 24 for a total of 16.
9	50	N	The total number of letters up to the nth one is $\frac{n(n+1)}{2}$. For n=13, 13(14)/2=91. The 100th letter will be the 14th letter or 'N'.
10	10	36	This is a triangle with vertices at (0,5), (6, 11), (6, 23). $A = \frac{1}{2}(6)(12) = 36.$
50	3	253 [degrees]	Cosine is an even function so $360 - 107 = 253$ degrees has the same cosine.

9	11	Answer	Solution
50	5	12	Divide by 9 giving: $$\frac{x^2}{9^2}+\frac{y^2}{12^2}=1$$ An ellipse with semi-major axis of 12.
50	9	1	Since we are dividing by 7 into a base 8 number, we can use the sum of the digits. $$4+3+2+6=15_{10}=17_8$$ $$1+7=10_8.\ 1+0=1$$

College Bowl Extra Questions

9	11	Answer	Solution
1	1	12	$3\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n = 3\left(\frac{1}{1-\frac{3}{4}}\right) = 3\left(\frac{1}{\frac{1}{4}}\right) = 3(4) = 12$
2	2	9 [$sq\ un.$]	The base 9-3=6 and the height is 4-1=3. The area is: $A = \frac{1}{2}(6)(3) = 9.$
3	3	8 [cu un]	The diameter of the sphere is $2\sqrt{3}$ which is also the space diagonal of the cube. $2\sqrt{3} = s\sqrt{3};\ \ s = 2;\ \ s^3 = 8.$
4	4	10 [numbers]	One could brute force it or use Fermat's theorem: $n^7 \equiv n\ mod\ 7$. This means that 7 divides equally into $(n^7 - n) = n(n^3 - 1)(n^3 + 1)$ which implies all cubes are of one of those forms.
5	5	01 both digits are needed in that order.	The last 2-digits of the powers of 21 are: 21, 41, 61, 81, 01, repeat. So the fifteenth power is 01.
6	6	3/11	$Two\ reds: \frac{3}{11}\frac{2}{10}.$ $Two\ blue\ or\ green: \frac{4}{11}\frac{3}{10}$ Total: $\frac{6+12+12}{11(10)} = \frac{30}{11(10)} = \frac{3}{11}$
7	7	1 [statement]	X must be true. If Y is false, then Z can be false or true for the implication to be true.