



# "Math Is Cool" Masters – 2017-18

Sponsored by:

November 18, 2017

High School – Mental Math Contest

# Mental Math Contest

## High School

### GENERAL INSTRUCTIONS applying to all tests:

- *Good sportsmanship is expected throughout the competition by all involved, both competitors and observers. Display of poor sportsmanship may result in disqualification.*
- *Calculators or any other aids may not be used on any portion of this contest.*
- *Unless stated otherwise, all rational, non-integer answers need to be expressed as reduced common fractions except in case of problems dealing with money. In the case of problems requiring dollar answers, answer as a decimal rounded to the nearest hundredth (ie, to the nearest cent).*
- *All radicals must be simplified and all denominators must be rationalized.*
- *Units are not necessary as part of your answer unless it is a problem that deals with time and in that case, a.m. or p.m. is required. However, if you choose to use units, they must be correct.*
- *Leave all answers in terms of  $\pi$  where applicable.*
- *Do not round any answers unless stated otherwise.*
- *Record all answers on the colored cover sheets in the answer column only.*
- *Make sure all answer sheets have all the information (name, team number, etc.) at the top of the sheet filled out.*
- *Tests will be scored as a 0 if answers are not recorded on the answer sheets.*
- *Blank answer sheets and answer sheets with no name will be scored as a 0.*

### Mental Math Contest – 30 sec per question

#### **8 problems read orally to everyone - approximately 8% of Individual Score - 25% of team score**

*When it is time to begin, the proctor will read the first question twice. You may not do any writing or talking while arriving at a solution. Once you have a solution, record it on the sheet in front of you. **You may not change or cross out answers once you have written an answer down. If there are eraser marks, write-overs, or crossed-out answers, they will be marked wrong.** Once all students have laid their pencils on the desk, another question will be asked. If a student doesn't lay his/her pencil down, the maximum wait time is 30 seconds after completion of the second reading of the question before another question is asked. You may continue to work on a problem while the next question is being read. The value of each question is a one or zero. Each student will be asked the same eight questions. Individual scores used to determine individual placing will be determined by the sum of the Mental Math score and the Individual Test score for each individual. In addition, the top three Mental Math scores from one team will be totaled and doubled and will contribute to 25% of the team score.*



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#	Problem
1	What is the sum of the roots of $X^2 + 5X + 4 = 0$ ?
2	How many ways can two winners be picked from a group of ten people?
3	What is the sum of an infinite geometric series which starts with seven and has a common ratio of one-third?
4	What is the remainder when one-thousand-twenty-three is divided by ninety-eight?
5	If $X$ has a supplementary angle of one-hundred-twenty degrees, calculate $10X$ in degrees.
6	What is the angular speed of the minute hand of a clock, in degrees per minute?
7	Calculate the area of a circle whose center is at the origin and which is tangent to the line $Y = 8$ .
8	How many zeroes are at the end of one-hundred-factorial?



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## High School – Mental Math Contest

School: \_\_\_\_\_ Team # \_\_\_\_\_

Name: \_\_\_\_\_ Room #: \_\_\_\_\_

### GENERAL INSTRUCTIONS applying to all tests:

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- Calculators or any other aids may not be used on any portion of this contest.
- Unless stated otherwise, all rational, non-integer answers need to be expressed as reduced common fractions except in case of problems dealing with money. In the case of problems requiring dollar answers, answer as a decimal rounded to the nearest hundredth (ie, to the nearest cent).
- All radicals must be simplified and all denominators must be rationalized.
- Units are not necessary as part of your answer unless it is a problem that deals with time and in that case, a.m. or p.m. is required. However, if you choose to use units, they must be correct.
- Leave all answers in terms of  $\pi$  where applicable.
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### Mental Math Contest High School – 30 sec per question

#### 8 problems read orally to everyone - Approximately 8% of Individual Score - 25% of team score

When it is time to begin, the proctor will read the first question twice. You may not do any writing or talking while arriving at a solution. Once you have a solution, record it on the sheet in front of you. **You may not change or cross out answers once you have written an answer down. If there are eraser marks, write-overs, or crossed-out answers, they will be marked wrong.** Once all students have laid their pencils on the desk, another question will be asked. If a student doesn't lay his/her pencil down, the maximum wait time is 30 seconds after completion of the second reading of the question before another question is asked. You may continue to work on a problem while the next question is being read. The value of each question is a one or zero. Each student will be asked the same eight questions. Individual scores used to determine individual placing will be determined by the sum of the Mental Math score and the Individual Test score for each individual. In addition, the top three Mental Math scores from one team will be totaled and doubled and will contribute to 25% of the team score.

	Answer	1 or 0	1 or 0
1			
2			
3			
4			
5			
6			
7			
8			



# “Math Is Cool” Masters – 2017-18

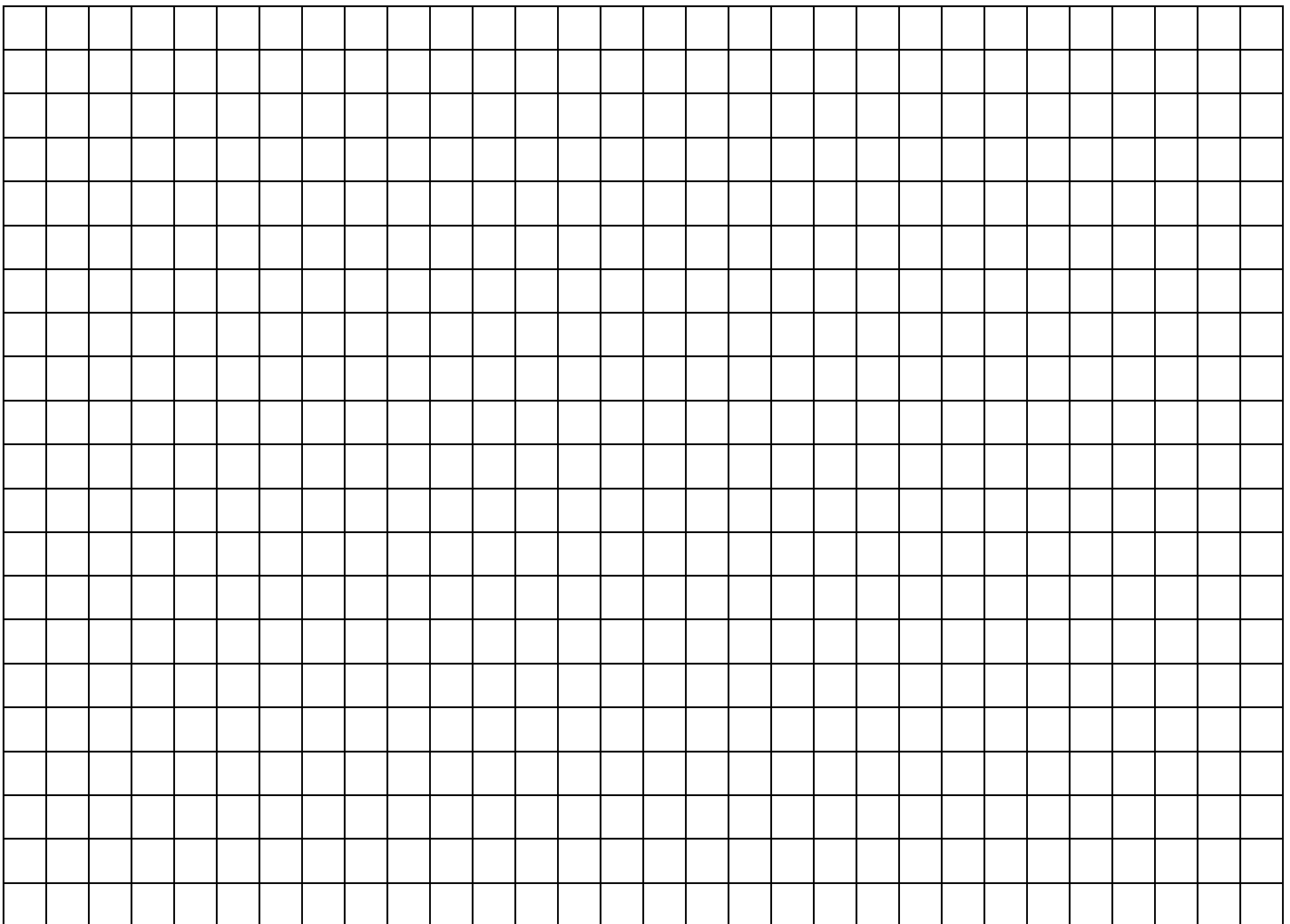
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## PRE-CALCULUS – Individual Contest

**Tear this cover sheet and scratch paper off and fill out the top of the colored answer sheet prior to the start of the test. The graph below is for your use, if needed.**

### **Individual Contest – PRE-CALCULUS - 35 minutes**

*You may NOT be seated next to anyone from your school. If you are MOVE NOW to avoid being disqualified! When you are prompted to begin, tear off the colored sheet and begin testing. Make sure your name and school are recorded on the answer sheet. The raw score will be 2 points for correct answers to problems 1-30 and 3 points for 31-40. Record your answers on the score sheet. No talking during the test. You will be given a 5 minute time warning.*





# “Math Is Cool” Masters – 2017-18

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## PRE-CALCULUS – Individual Contest

### Questions 1-30: 2 points each

1	If Cherie takes twelve hours to travel three-hundred miles, what is her average speed in miles per hour?
2	What is the measure, in degrees, of an interior angle of a regular nonagon?
3	Simplify: $(x^{225})^{\frac{1}{15}}$
4	What is the minimum perimeter, in meters, of an isosceles triangle with sides measuring seven and twenty-seven meters?
5	Express the domain of the real-valued square-root function in interval notation.
6	What are the coordinates, in the form $(x, y)$ , of the x-intercept of the line $2x - 3y = 12$ ?
7	Simplify in terms of $i (= \sqrt{-1})$ : $(i^{2017})^{1020}$
8	What is the quotient when one-third is divided by four-fourteenths?
9	Express 0.0032 in scientific notation.
10	What is the area, in square meters, of a right triangle with a hypotenuse of six meters and an interior angle measuring sixty degrees?
11	When $(17x^5 - 14x^3 + 2x^2 - 3)^4$ is expanded and written in polynomial form, what is the sum of the coefficients?
12	What number is one-third of the sum of forty-nine and four-hundred-four?
13	Given the equation $-4x^2 - 3x + 2 = -3$ , find the sum of the possible values of x.
14	What is the hexadecimal form of the binary number $(1101\ 1010\ 1011)_2$ ?
15	What value(s) of $u$ satisfy $u - 7 = 3u + 6$ ?
16	How many distinguishable ways can you put four identical balls into two different boxes?
17	What is $\langle 2, 0, 0 \rangle \cdot \langle 0, 0, 3 \rangle$ ?
18	Calculate the solutions for the equation: $\frac{(x+4)(x-3)(x+1)(x+9)}{x-2} = 0$
19	What is the remainder when $x^{2017} - 4x^3 + 5x - 3$ is divided by $x - 1$ ?
20	Given $2\sin(x)\cos(x) - 1 = 0$ over the real interval $[0, 2\pi)$ , find the sum of the solutions.
21	What is the sum of the sequence $\sum_0^{20} (2x - 1)$ ?
22	$g(x) = 8x^3 + 1$ is the inverse of $f(x)$ . Find the value of $f(28)$ .
23	What are the coordinates, in the form $(x, y)$ , of the focus point of the parabola $y = x^2 + 4x - 10$ ?
24	A number $n$ is "squishy" if there are no numbers of the form $p^p$ that divide $n$ , where $p$ is a prime number. How many squishy numbers are factors of 10 factorial?

25	Uncle Kishan is excited for Christmas shopping, and wants to give all of his nieces and nephews gifts. He visits 7 different stores. In each store, he spends \$1 less than one-half of what he spent at the previous store, spending all of his money. If he spends \$26 at the seventh store, how many dollars did Uncle Kishan start with?
26	What is the length, in meters, of the altitude to the longest side of a triangle with sides measuring fifteen, twenty, and twenty-five meters?
27	If Dick can do the job in twelve hours and Jane can do it in eight hours, how many <b>minutes</b> would it take them to do the job together?
28	If $\log_9 bc = 2$ and $\log_b c = 3$ , then what is $b + c$ ?
29	Find the solution(s) to the equation: $9x^2 - 16x - 4 = 0$
30	Let $a, b, c, d,$ and $f$ be natural numbers. If $d$ is a multiple of $b$ , $f$ is equal to both $b+10$ and $c-d$ , $a$ is equal to twice the quotient of $d/b$ , $a \leq 10$ and $c=31$ , what is $a+b+c+d+f$ ?

## Challenge Questions: 3 pts each

31	Solve for $x$ : $\frac{96}{x-4} - \frac{96}{x} = 4$
32	What is the area, in square meters, of a triangle with sides measuring eight, six, and twelve meters?
33	One bag of trail mix has 3 M&Ms, 5 peanuts, 6 raisins, and 1 almond. A different bag of trail mix is composed of 3 M&Ms, 2 peanuts, 4 raisins, and 3 almonds. You randomly draw two pieces from each bag of trail mix and eat them. What is the probability that you eat only M&Ms?
34	The sum of two prime numbers, $a$ and $b$ , is 300. What is the smallest possible value of $a$ ?
35	If $2x$ is a square number, which of the following could be $x$ for some natural number $n$ ? 1) $n$ squared 2) $n$ cubed 3) $n$ to the sixth power 4) $2n+1$ 5) $6n$
36	What quadruple of consecutive integers $(w, x, y, z)$ satisfies the equation $zy - xw = 54$ ?
37	If I have nine green socks, twelve red socks, and six blue socks, what is the probability that I draw one green sock, then one red sock, then one green sock?
38	Jack and Jill will both arrive at the bus stop at random times between eight and nine A.M. If Jack waits for ten minutes after arrival, and Jill waits for five minutes, what is the probability that they will be at the station at the same time?
39	A dog is leashed to the inside southwest corner of a fenced $9' \times 20'$ rectangular back-yard using a $10'$ leash. The $20'$ side runs east and west. At the north-west corner of the enclosure there is a 1-foot gap in the fence because the north fence is only $19'$ in length, and the dog can fit through this gap. What is the farthest east that the dog can get from the west wall outside of the fence? Disregard the dimensions of the dog.
40	If set $A$ is the set of all even numbers, set $B$ is the set of all odd numbers, and set $C$ is the set of the first twenty numbers of the Fibonacci Sequence, what is the positive difference between the sum of the elements in $A \cap C$ and the sum of the elements in $B \cap C$ ?



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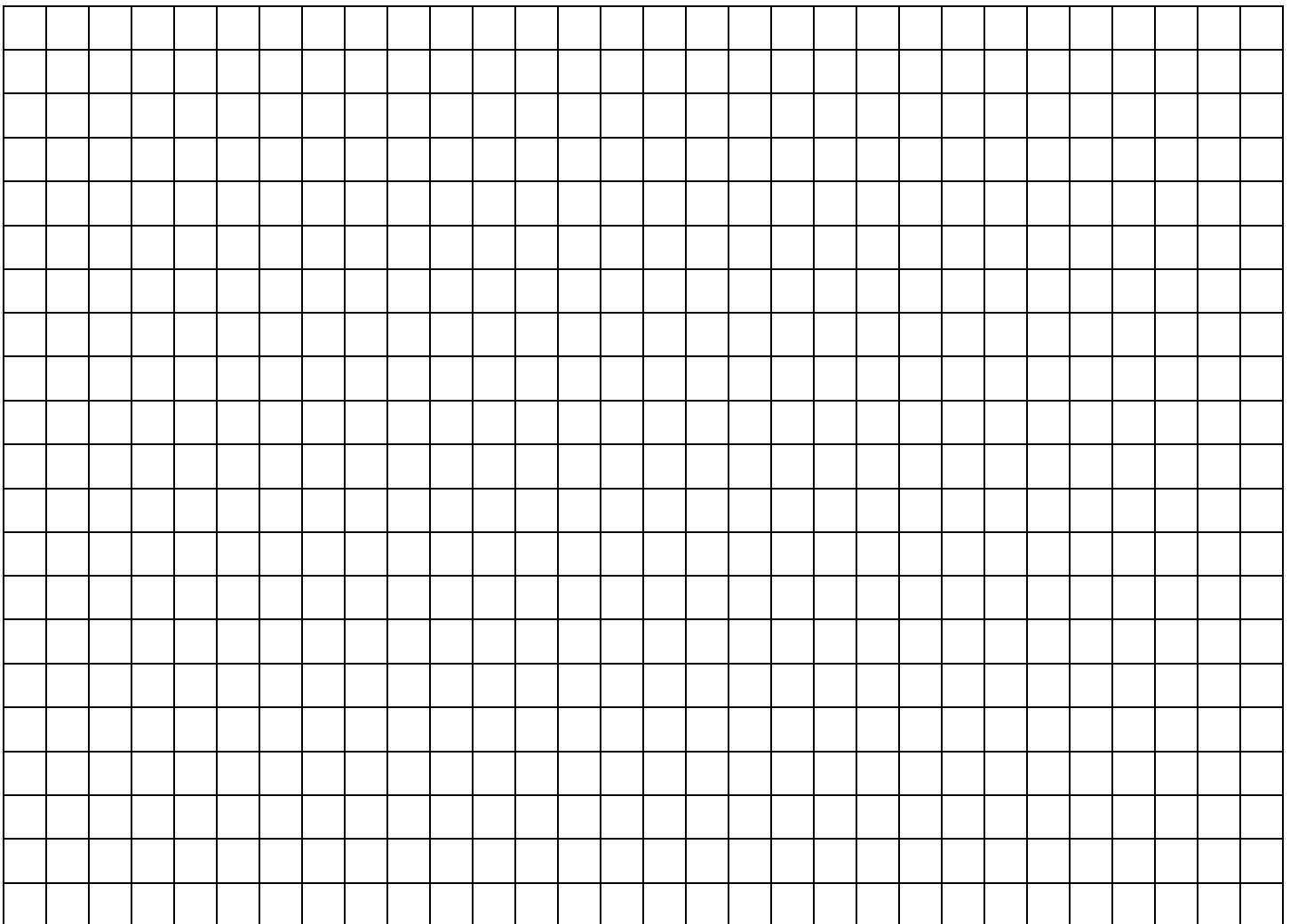
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## CALCULUS – Individual Contest

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## CALCULUS – Individual Contest

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4	What is the minimum perimeter, in meters, of an isosceles triangle with sides measuring seven and twenty-seven meters?
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6	What are the coordinates, in the form $(x, y)$ , of the x-intercept of the line $2x - 3y = 12$ ?
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20	Given $2\sin(x)\cos(x) - 1 = 0$ over the real interval $[0, 2\pi)$ , find the sum of the solutions.
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27	If Dick can do the job in twelve hours and Jane can do it in eight hours, how many <b>minutes</b> would it take them to do the job together?
28	If $\log_9 bc = 2$ and $\log_b c = 3$ , then what is $b + c$ ?
29	Compute the following integral: $\int_{-3}^1 (6x^2 - 5x + 2) dx$
30	Let $a, b, c, d,$ and $f$ be natural numbers. If $d$ is a multiple of $b$ , $f$ is equal to both $b+10$ and $c-d$ , $a$ is equal to twice the quotient of $d/b$ , $a \leq 10$ and $c=31$ , what is $a+b+c+d+f$ ?

## Challenge Questions: 3 pts each

31	What is the sum of the eigenvalues of $\begin{bmatrix} 1 & 4 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & -3 \end{bmatrix}$ ?
32	What is the sum of the absolute value of the relative extrema y-values of the equation $y = (x^2 - x - 1) * e^x$ ?
33	One bag of trail mix has 3 M&Ms, 5 peanuts, 6 raisins, and 1 almond. A different bag of trail mix is composed of 3 M&Ms, 2 peanuts, 4 raisins, and 3 almonds. You randomly draw two pieces from each bag of trail mix and eat them. What is the probability that you eat only M&Ms?
34	The equation $\frac{z^2}{4} - \frac{x^2}{9} - \frac{y^2}{25} = 1$ expresses a hyperboloid of two sheets. The intersection of a plane with the hyperboloid of two sheets gives a cross section that is an ellipse with major axis 10 and minor axis 6. What is the equation of one such plane?
35	Bri is swimming around the edge of a circular pool of radius one hundred meters at a constant speed of five meters per second. Her friend Ella is standing two hundred meters away from the center of the pool. How fast is the distance between Ella and Bri changing when they are two hundred meters apart?
36	What quadruple of consecutive integers $(w, x, y, z)$ satisfies the equation $zy - xw = 54$ ?
37	If I have nine green socks, twelve red socks, and six blue socks, what is the probability that I draw one green sock, then one red sock, then one green sock?
38	Jack and Jill will both arrive at the bus stop at random times between eight and nine A.M. If Jack waits for ten minutes after arrival, and Jill waits for five minutes, what is the probability that they will be at the station at the same time?
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# “Math Is Cool” Masters – 2017-18

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11th & 12th Grades – Individual Multiple Choice Contest

Student Name: \_\_\_\_\_ Room #: \_\_\_\_\_

SCHOOL NAME: \_\_\_\_\_ Team #: \_\_\_\_\_

## **Individual Multiple Choice Contest - 15 minutes – 10 problems – 20% of team score**

*This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. **Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet.** No talking during the test.*

**DO NOT WRITE IN SHADED REGIONS**

	Answer	-1, 0 or 2	-1, 0 or 2
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			



# “Math Is Cool” Masters – 2017-18

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## 11th & 12th Grades – Individual Multiple Choice Contest

1	What is the sum of the elements of the intersection of the two sets $\{1, 5, 17, 6\}$ and $\{5, 6, 16, 2, 3\}$ ? A) 11      B) 39      C) 50      D) 61      E) Answer not given.
2	Evaluate the expression $6 \cdot \sqrt{3} \cdot \lim_{x \rightarrow 0} \frac{\sin x}{x}$ . (If the limit does not exist, choose Does Not Exist.) A) 0      B) $6\sqrt{3}$ C) 9      D) Does Not Exist      E) Answer not given.
3	Two coins have values of three cents and five cents. What is the largest number of cents that can't be created using a combination of these coins? A) 7      B) 11      C) 17      D) 19      E) Answer not given.
4	Given: $(2x - 2)(x + 3) = (2x + 6)(ax + b)$ , give the value of $a+b$ . A) -1      B) 0      C) 1      D) 2      E) Answer not given.
5	If the probability of event A occurring is $1/3$ , the probability of event B is also $1/3$ , and events A and B are independent, calculate the probability that both event A and B occur. Treat that result as $c$ . Then find the value of $x$ that satisfies $c^{-1}x + 81c = 126$ . A) 10      B) 11      C) 12      D) 13      E) Answer not given.
6	The value of $n$ (a natural number) is evenly divisible by 2, 3, 4, 5, 6, 8, 10, 12 and 15. Find the least possible value of $n$ and then give the measure in degrees of one interior angle of a regular polygon which has $n$ sides. A) 168      B) 174      C) 177      D) 178      E) Answer not given.
7	Evaluate the following: $(123,463)(123,449) - (123,467)(123,445)$ A) 72      B) 82      C) 92      D) 102      E) Answer not given.
8	Today, I am ten years older than Frank. In twelve years, I will be ten years less than twice his age on that day. Frank and I were born on January 1 <sup>st</sup> , at the same time of the day, in our respective years. How old will I be, in years, five years from now? A) 19      B) 21      C) 23      D) 25      E) Answer not given.
9	Let $V_1$ be a vector with each of its components being a solution to $y = x^3 - 6x^2 + 5x + 12$ , in order from least to greatest. Let $V_2$ be a vector with each of its components being a solution to $y = x^3 + 4x^2 + 4x$ , in order from greatest to least. What is $V_1 \bullet V_2$ ? A) -14      B) -4      C) 4      D) 14      E) Answer not given.
10	Calculate the sum: $\sum_{x=0}^5 \frac{x}{x+1}$ A) $\frac{7}{2}$ B) $\frac{71}{20}$ C) $\frac{18}{5}$ D) $\frac{73}{20}$ E) Answer not given.



# “Math Is Cool” Masters – 2017-18

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11th & 12th Grades – Team Contest

SCHOOL NAME \_\_\_\_\_ Team # \_\_\_\_\_ Room # \_\_\_\_\_

**Team Contest - 15 minutes – 30% of team score**

*When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as **1 or 0**. Record all answers on the colored answer sheet.*

**DO NOT WRITE IN SHADED REGIONS**

	Answer	1 or 0	1 or 0
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			



# “Math Is Cool” Masters – 2017-18

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11th & 12th Grade – Team Contest

1	Evaluate the determinant: $\begin{vmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 6 & 4 & 2 \end{vmatrix}$
2	If $\log_5(x) = \log_5(y) + 3$ , express $y$ in terms of $x$ for positive $y$ and $x$ values.
3	Express the solution to $\frac{x-2}{x-3} \geq 2$ in interval notation.
4	Given an equilateral triangle, I randomly pick a point on its circumcircle. What is the probability that a quadrilateral formed using this point and the vertices of the triangle has at least two angles that are greater than 100 degrees?
5	When eight distinct integers are added together in pairs, their possible sums are -7, -6, -5, -3, -2, -1, 1, 2, 3, 4, 6, 7, 8, 10, 11, 12, 15, 16, 20. What are the eight distinct integers?
6	Express $13_6 + 137_8 + 11111_2$ in base 4.
7	How many lattice points satisfy $(2x - 2)^2 + (y + 5)^2 = 25$ ?
8	What is the solution, in the form $(h, s, u)$ , of the system of equations $11(h + s) = u$ , $8h + 23s = u - 12$ , and $\left(\frac{5u}{11}\right) - 8h = 7s$ ?
9	A farmer attaches his cow via a rope to a post on the outside of a regular hexagon with side length of three feet. If the rope is nine feet in length, what is the area, in square feet, that the cow is allowed to roam?
10	By pointing somewhat upstream, a boat crosses a river perpendicular to the current. The current is flowing south at 3 m/s, and the boat's speed in still water is 6 m/s. The river is 96 meters wide. How long, to the nearest second, will it take for the boat to cross the river?



# "Math Is Cool" Masters – 2017-18

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11th & 12th Grades – Pressure Round Contest

1	The mean, median, unique mode, and range of eight integers are all equal to 8. What is the largest integer that can be an element of this collection?
2	Find a 4-digit number whose first digit is how many 0's are in the number, the second digit is how many 1's are in the number, the third digit is how many 2's are in the number, and the fourth digit is how many 3's are in the number.
3	A trusted friend rolls two 100-sided dice (numbers from 1-100) behind a screen and tells you the sum of the numbers shown is 36. What is the probability that the product of the numbers shown is greater than 240?
4	What is the sum of the rational root and the reciprocals of the irrational roots of the equation $x^3 + x^2 - 11x - 3 = 0$ ?
5	Suppose that for less than 1 year, both my younger nephew and I are teenagers (recall that a teenager is someone whose age is a number ending in "teen"). What will my age be, in years, once my nephew is no longer a teenager?



# "Math Is Cool" Masters – 2017-18

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11th & 12th Grades – College Bowl Contest

# College Bowl Contest

## 11th & 12th Grades

SETS 1-6 (with Extra Questions at the end)

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### COLLEGE BOWLS INSTRUCTIONS

**Read these to the competitors before the first round:**

**College Bowl Contest - up to 10 minutes per round – 10 problems per round – 10% of team score**

- 1. All competitors must be facing the front of the room in one row. All spectators need to be behind the competitors.*
- 2. A maximum of ten questions per round will be scored. It is okay for both teams to score the same number of points! The proctor will record the points earned on each team's score sheet.*
- 3. You may use scratch paper and pencil. You may talk with your teammates while arriving at a solution. An electronic College Bowl Apparatus (CBA) will be used to identify the first team to have an answer.*
- 4. During these rounds, the questions will be read twice and a maximum time of 45 seconds will be allowed for you to answer after the second reading of the question is complete. If a team buzzes in after the second reading and gives an incorrect response, the other team has the remainder of the 45 seconds to respond. You may interrupt (buzz in) while a question is being read, however, if you do, the proctor will stop and an immediate response is needed. If the correct response is given, a new question will be asked. Otherwise, the question will be reread for the other team, making sure it has two full readings. Forty-five seconds will be given for the team to respond from the completion of the last reading. If an immediate response is not given after a team pulls the string, their lack of an answer in a timely manner is considered incorrect. In the event that only one team is competing in a round (i.e., one team is absent), the team competing will have a maximum of 30 seconds in which to buzz in.*
- 5. You do not need to wait to be acknowledged by the proctor; however, it is your right to do so if you would like to be acknowledged.*
- 6. If two students from the same team answer at the same time with different answers, the answer will be considered incorrect.*
- 7. If a problem arises with one of the questions, an extra question will be asked to replace that question. If the round finishes early, you need to stay in the room for the remaining time.*



# “Math Is Cool” Masters – 2017-18

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November 18, 2017

11th & 12th Grades – College Bowl Contest

## College Bowl Contest ROUND #1 – SET 1

#	Problem	Answer
1	What value(s) of W satisfy five-W-plus-five-equals-fifty?	9
2	Find the next term in the sequence one, three-fourths, five-ninths, seven-sixteenths, and so on.	$9/25$
3	What is the base-nine logarithm of two-hundred-forty-three?	$\frac{5}{2}$
4	Mrs. Z says that she will dismiss her class early if she draws either a Queen, a Three, or a diamond from a standard 52-card deck of playing cards. What is the probability that her class gets dismissed early?	$\frac{19}{52}$
5	Two concentric circles have radii of nine and two meters. What is the length, in meters, of a chord of the larger circle that is tangent to the smaller circle?	$2\sqrt{77}$
6	There are seventy-five kids at Camp Mathzone. If fifty of the kids can play tennis and thirty of the kids can play soccer, how many of the kids can play both tennis and soccer if every kid plays at least one sport?	5
7	What is the derivative of X-squared-L-N-X?	$2x \ln(x) + x$ Two-X-L-N-X-plus-X
8	What is the largest number of regions into which a triangle and a quadrilateral can divide a plane?	10
9	If I flip a coin until either a sequence of four heads comes up or a sequence of a tail immediately followed by three heads comes up, what is the probability that the sequence of coin flips ends such that no tails appeared?	$1/16$ ,
10	What is the sum of the factors of six-hundred?	1860





# “Math Is Cool” Masters – 2017-18

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11th & 12th Grades – College Bowl Contest

## College Bowl Contest ROUND #2 – SET 2

#	Problem	Answer
1	In which quadrant does the point negative-six-point-four-COMMA-negative-one-hundred-fifty-two-point-five-one lie?	24 [cm <sup>2</sup> ]
2	In the Land of Lala, the license plates are six characters long. If the first two characters must be distinct numbers and the next three must be distinct vowels, and the last character must be an R, how many license plates are possible?	$36\sqrt{3}$ [m <sup>2</sup> ]
3	What is the equation, in slope-intercept form, of the line tangent to the curve Y-equals-three-X-squared-plus-two-X at the point negative-two-COMMA-eight?	504 [ways]
4	What is the sum of the number of sides in a heptagon, the number of positive two-digit prime numbers, the number of doughnuts in a baker’s dozen, and the number of months that have at least twenty-eight days?	$\frac{15}{64}$
5	Evaluate sixteen to the negative-seven-fourths power.	7.5 [minutes]
6	What is the sum of the sequence two-sevenths [PAUSE] plus two-sevenths times ten-elevenths [PAUSE] plus two-sevenths times ten-elevenths squared [PAUSE] and so on?	12 [integers]
7	Karin kicks a ball, and the subsequent path of the ball follows the parabola H-equals-negative-four-T-squared-plus-ninety-six-T, where t is the time in seconds. After how many seconds does the ball touch the ground again after being kicked?	$6x + 2$
8	Two lines are initially parallel, but one is rotated sixty-one degrees clockwise while the other is rotated seventy-two degrees counter-clockwise. What is the measure, in degrees, of the larger angle between these two lines?	4
9	If five chickens lay nine eggs in six days, how many eggs would four chickens lay in five days?	250 [points]
10	How many integers from one to one-thousand inclusive do not have a one, a four or a six in them?	1664



# “Math Is Cool” Masters – 2017-18

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11th & 12th Grades – College Bowl Contest

## College Bowl Contest ROUND #3 – SET 3

#	Problem	Answer
1	What is the area, in square meters, of a parallelogram with sides measuring four meters and seven meters, and one height measuring six meters?	24
2	Billy goes to Las Vegas to test his luck at the casinos. He pays five dollars to play a game in which he tosses a fair coin. He gets ten dollars if he gets a head, but he must pay another four dollars if he gets a tail. What is the expected value of his winnings if he plays once?	-\$2
3	If the sum of two numbers is fifty-nine and their difference is sixty-five, what is the smaller of the two numbers?	-3
4	A triangle with sides measuring eight, nine, and six meters is similar to a triangle with sides measuring nine, twelve, and $Q$ meters. What is the value of $Q$ ?	$\frac{27}{2}$
5	The Villain and The Hero see one another at the same time when they are two-hundred meters apart. The Villain flees at a speed of twenty-five meters per second using his jetpack, while The Hero flies after him at a speed of thirty-three meters per second. How many seconds will it take The Hero to catch The Villain?	25
6	What is the area, in square meters, of an ellipse with axes measuring twenty-three and thirty-six meters?	$207\pi$
7	How many digits are in two-to-the-eighteenth-power?	6
8	A grocery store that sells chocolate bars for one dollar each has a rewards program that gives you a chocolate bar in exchange for three wrappers. How many chocolate bars can Sarah get with twenty-four dollars, including the ones from the rewards program?	35
9	Sarah has an eighty-eight-percent average on four tests in her Algebra class. There will be a fifth test and all tests are weighted equally in the final average. What must she earn on the fifth test in order to average exactly ninety-percent on the five tests?	98
10	What is the equation of the line normal to the curve $Y = -x^2 + 8x$ at the point two-COMMA-eight?	$x = 2$



# “Math Is Cool” Masters – 2017-18

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11th & 12th Grades – College Bowl Contest

## College Bowl Contest ROUND #4 – SET 4

#	Problem	Answer
1	What is the smallest integer that could be the length in meters of the third side of a triangle with two sides measuring fifty-seven and fifty-one meters?	7
2	I have a sequence of eight numbers in increasing order, whose median is twelve. If I were to split the sequence in half, the median of the first half would be four and the median of the second half would be twenty-eight. Given that the sum of the first and last entry in the sequence is thirty-two, what is the mean of the sequence?	15
3	Calculate the dot product of the vectors three-COMMA-five-COMMA-nine and eight-COMMA-one-COMMA-four.	65
4	Ian is trying to spell the word “Alpaca”, and he knows that it contains three As, one P, one L, and one C. However, he is not sure of the order, so he decides to guess. How many different ways can he guess to spell “Alpaca”?	120
5	A is the set of all integers between one and one-hundred inclusive. If a number in set A is randomly chosen, what is the probability that the number is prime?	$\frac{1}{4}$
6	What value(s) of S satisfy S-squared-plus-S-minus-ten-equals-zero?	$\frac{-1 \pm \sqrt{41}}{2}$
7	What is the area, in square meters, of a thirty-degree sector of a circle with a radius of six meters?	$3\pi$
8	What is the product of one-hundred-nine and two-hundred-eighty-three?	30847
9	Hayley walks around the edge of a regular hexagonal pond with three-meter sides. After she has walked a total of twelve meters around the pond, how far away is she from her original position?	$3\sqrt{3}$
10	X is a natural number with exactly six positive integer factors. If you use the least possible value for X, what the value of three-X-squared?	432



# “Math Is Cool” Masters – 2017-18

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11th & 12th Grades – College Bowl Contest

## College Bowl Contest ROUND #5 – SET 5

#	Problem	Answer
1	What is the area, in square meters, of an equilateral triangle with sides measuring sixty meters?	$900\sqrt{3}$
2	What is the product of three-point-five and two-point-two-six?	7.91
3	Evaluate one-hundred-twenty-six plus seventy-two plus two-hundred-fifteen minus eighty-three minus one-hundred-twenty-three.	207
4	What is the surface area, in square meters, of a right circular cylinder with a base radius of five meters and a height of eighteen meters?	$230\pi$
5	A right triangle has a hypotenuse measuring eighteen meters and a leg measuring three meters. What is the length of the other leg, in meters?	$3\sqrt{35}$
6	Seven two-sided keys (congruent bumps on both sides) fit around a circular ring. How many DISTINCT arrangements of the key ring can exist?	360
7	What value(s) of T satisfy T-squared-plus-T-minus-six-equals-zero?	2, -3 (either order)
8	What are the coordinates, in the form X-COMMA-Y, of the center of the hyperbola with equation X-squared-minus-four-Y-squared-plus-two-X-plus-twelve-Y-equals-one-hundred?	$\left(-1, \frac{3}{2}\right)$
9	A wooden cube is painted blue and then cut into sixty four smaller, identical cubes. If Jodi picks one of these small cubes at random, what is the probability that it has exactly two sides painted blue?	$\frac{3}{8}$
10	What are the coordinates, in the form X-COMMA-Y, of the point of intersection of the lines X-plus-Y-equals-negative-three and two-X-minus-Y-equals-six?	(1, -4)



# “Math Is Cool” Masters – 2017-18

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11th & 12th Grades – College Bowl Contest

## College Bowl Contest ROUND #6 – SET 6

#	Problem	Answer
1	What is the area, in square meters, of a right triangle with legs measuring ten and nineteen meters?	95
2	A regular polygon has vertices labeled in clockwise order from A to J. If a line is drawn through vertex G and the center of the polygon, which other vertex does it pass through?	B
3	Evaluate seven-thousand-five-hundred-forty-four divided by forty-six.	164
4	Eight swimmers just finished a race. I know that Alice, Bob, and Charlie were in the top three, but I don't know which places they won, or which places the other swimmers were in the race. How many possible arrangements of these eight swimmers are there, given this information?	720
5	What are the coordinates, in the form X-COMMA-Y, of the reflection of the point negative-one-COMMA-negative-six through the line X-equals-nine?	(19, -6)
6	Mrs. Z decides that she will cancel her class's quiz today only if, when she rolls two fair six-sided dice, the sum of the rolls is a prime number. What is the probability that her quiz gets cancelled?	5/12
7	What is the sum of the edges and vertices of an icosohedron?	42
8	When ten liters of a twenty-five percent acid solution are mixed with five liters of a ten percent acid solution, what percent of the final solution is NOT acid?	80
9	Sasha is five years older than Olsen. In two years, Sasha will be twice Olsen's age. How old will Sasha be in twelve years?	20
10	What is the volume of a cube with a diagonal of nine?	$81\sqrt{3}$



# "Math Is Cool" Masters – 2017-18

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11th & 12th Grades – College Bowl Contest

## College Bowl Contest EXTRA – EXTRA

#	Problem	Answer
1	Express the base-seven numeral three-four-base-seven as a base-ten numeral.	25
2	What is the largest integer less than one-hundred that is congruent to 2 in mod 3 and congruent to 4 in mod seven?	95
3	How many positive integers are factors of one-hundred-fifty?	12
4	What is the greatest common factor of forty and one-hundred-eight?	4
5	What is the largest palindromic multiple of nine less than twenty-thousand?	19791
6	What is the units digit of twenty-three-to-the-forty-fifth-power?	3



# “Math Is Cool” Masters – 2017-18

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## High School – Mental Math Contest

KEY

KEY

KEY

### GENERAL INSTRUCTIONS applying to all tests:

- Good sportsmanship is expected throughout the competition by all involved, both competitors and observers. Display of poor sportsmanship may result in disqualification.
- Calculators or any other aids may not be used on any portion of this contest.
- Unless stated otherwise, all rational, non-integer answers need to be expressed as reduced common fractions except in case of problems dealing with money. In the case of problems requiring dollar answers, answer as a decimal rounded to the nearest hundredth (ie, to the nearest cent).
- All radicals must be simplified and all denominators must be rationalized.
- Units are not necessary as part of your answer unless it is a problem that deals with time and in that case, a.m. or p.m. is required. However, if you choose to use units, they must be correct.
- Leave all answers in terms of  $\pi$  where applicable.
- Do not round any answers unless stated otherwise.
- Record all answers on the colored cover sheets in the answer column only.
- Make sure all answer sheets have all the information (name, team number, etc.) at the top of the sheet filled out.
- Tests will be scored as a 0 if answers are not recorded on the answer sheets.
- Blank answer sheets and answer sheets with no name will be scored as a 0.

### Mental Math Contest High School – 30 sec per question

#### 8 problems read orally to everyone - Approximately 8% of Individual Score - 25% of team score

When it is time to begin, the proctor will read the first question twice. You may not do any writing or talking while arriving at a solution. Once you have a solution, record it on the sheet in front of you. **You may not change or cross out answers once you have written an answer down. If there are eraser marks, write-overs, or crossed-out answers, they will be marked wrong.** Once all students have laid their pencils on the desk, another question will be asked. If a student doesn't lay his/her pencil down, the maximum wait time is 30 seconds after completion of the second reading of the question before another question is asked. You may continue to work on a problem while the next question is being read. The value of each question is a one or zero. Each student will be asked the same eight questions. Individual scores used to determine individual placing will be determined by the sum of the Mental Math score and the Individual Test score for each individual. In addition, the top three Mental Math scores from one team will be totaled and doubled and will contribute to 25% of the team score.

	Answer
1	-5
2	45
3	$\frac{21}{2}$
4	43
5	600[°]
6	6 [deg./min.]
7	64 $\pi$
8	24



# “Math Is Cool” Masters – 2017-18

Sponsored by:  
November 18, 2017  
PRE-CALCULUS – Individual Contest

**KEY**

**KEY**

**KEY**

## PRE-CALCULUS - Individual Contest – Score Sheet

	Answer
1	25
2	140
3	$x^{15}$
4	61
5	$[0, \infty)$
6	(6,0)
7	1
8	$\frac{7}{6}$
9	$3.2 \times 10^{-3}$
10	$\frac{9\sqrt{3}}{2}$
11	16
12	151
13	$\frac{-3}{4}$
14	(DAB) <sub>16</sub>
15	$-\frac{13}{2}$

	Answer
16	5
17	$\langle 0, -6, 0 \rangle$
18	-4,3,-1,-9
19	-1
20	$\frac{3\pi}{2}$
21	399
22	$\frac{3}{2}$
23	$(-2, \frac{55}{4})$
24	36
25	[\$]3542[.00]
26	12
27	288
28	30
29	(-2/9, 2)
30	73

	Answer
31	12 or -8
32	$\sqrt{455}$
33	1/770
34	7
35	2 and 5.
36	(12, 13, 14, 15)
37	$\frac{16}{325}$
38	$\frac{67}{288}$
39	$11 - \sqrt{82}$
40	10946

**31-40**

## PRE-CALCULUS

DO NOT WRITE IN SHADED REGIONS

**1-15**

**16-30**





# “Math Is Cool” Masters – 2017-18

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CALCULUS – Individual Contest

**KEY**

**KEY**

**KEY**

## CALCULUS - Individual Contest – Score Sheet

	Answer
1	25
2	140
3	$x^{-15}$
4	61
5	$[0, \infty)$
6	(6,0)
7	1
8	$\frac{7}{6}$
9	$3.2 \times 10^{-3}$
10	$\frac{9\sqrt{3}}{2}$
11	16
12	151
13	$\frac{-3}{4}$
14	$(DAB)_{16}$
15	$-\frac{13}{2}$

	Answer
16	5
17	$<0, -6, 0>$
18	-4,3,-1,-9
19	-1
20	$\frac{3\pi}{2}$
21	399
22	$\frac{3}{2}$
23	$(-2, \frac{55}{4})$
24	36
25	[\$13542;.00]
26	12
27	288
28	30
29	84
30	73

	Answer
31	-3
32	$\frac{5+e^3}{e^2}$ or $5e^{-2} + e$
33	1/770
34	$z = \pm 2\sqrt{2}$ (either or both)
35	$\frac{5\sqrt{15}}{4}$
36	(12, 13, 14, 15)
37	$\frac{16}{325}$
38	$\frac{67}{288}$
39	$11 - \sqrt{82}$
40	10946

**31-40**

## CALCULUS

DO NOT WRITE IN SHADED REGIONS

**1-15**

**16-30**



# “Math Is Cool” Masters – 2017-18

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11th & 12th Grades – Individual Multiple Choice Contest

KEY

KEY

KEY

## **Individual Multiple Choice Contest - 15 minutes – 10 problems – 20% of team score**

*This test is the only test where you will be penalized for incorrect responses. You will receive 2 points for a correct letter response, 0 points for leaving it blank and -1 point for an incorrect response. It is not necessary to write your personal name on the test, but you may put it at the bottom of the test so your coach will be able to give you back the correct test. This test is taken individually, but it is part of your team score, including zeros for missing team members. Your team score will be calculated by taking the mean of your four team members' scores. When you are prompted to begin, tear off the colored sheet and begin testing. **Since this is a multiple choice test, ONLY a letter response should be indicated as an answer on the answer sheet.** No talking during the test.*

**DO NOT WRITE IN SHADED REGIONS**

	Answer
1	A
2	B
3	A
4	B
5	D
6	C
7	A
8	C
9	A
10	B



# “Math Is Cool” Masters – 2017-18

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November 18, 2017

11th & 12th Grades – Team Contest

**KEY**

**KEY**

**KEY**

**Team Contest - 15 minutes – 30% of team score**

*When you are prompted to begin, tear off the colored sheet and give a copy of the test to each of your team members and begin testing. Each problem is scored as **1 or 0**. Record all answers on the colored answer sheet.*

**DO NOT WRITE IN SHADED REGIONS**

	Answer	1 or 0	1 or 0
1	0		
2	$[y =] \frac{x}{125}$		
3	(3,4]		
4	$\frac{2}{3}$		
5	-5, -2, -1, 0, 3, 4, 8, 12		
6	2213		
7	6		
8	$(\frac{4}{7}, -\frac{6}{7}, -\frac{22}{7})$		
9	$69\pi$ [ft <sup>2</sup> ]		
10	18 [s]		



# “Math Is Cool” Masters – 2017-18

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11th & 12th Grades – Pressure Round Contest

KEY

KEY

KEY

## PRESSURE ROUND

### 11th & 12th Grades

**Pressure Round Contest - 10 minutes - 5 problems - 5 rounds - 15% of team score**

*When it is time to begin, you will be handed a packet of five problems. There is a copy of the problems for each team member. Two minutes after the start of the test you are expected to submit an answer for one of the problems (it can simply be a guess). The maximum value of this answer is 1 point. In another two minutes you are expected to submit another answer to one of the four remaining problems; its maximum value is two points. This process will continue until all the problems are answered and each consecutive problem's worth will go up by one point. You must submit your answers on the colored sheets given to you. If you do not have an answer at the end of a two minute period, you must still submit an answer sheet with an identified problem number on it. Failure to do so will result in loss of points. This event is timed, and you will be given a verbal 5 second warning and told to hold your answer sheet up in the air. You may keep working as the sheets are collected. If a team answers the same question more than once, only the first answer will be scored and the other attempts will be ignored.*

	Answer
1	14
2	1210 or 2020 (either one)
3	$\frac{19}{35}$
4	-1
5	26 [years old]