“Math is Cool” Championships -- 2018-19

High School

Mental Math Solutions

	Answer	Solution
1	51	
2	44 [miles]	12+12+20
3	14π [cm]	R=root(49)=7 C=2piR=14pi
4	8 [cups]	A scaled unit corresponds to that scale squared on the square unit.
5	5/36	Memorize, or 2,6; 3,5; 4,4; 5,3; 6,2
6	3	1/(1-2/3)=3
7	36	X^2=48*27=16*81 Multiplying 48*27 directly is not recommended.
8	3	Memorized 11 Rule: 1-2+3-4+5=3

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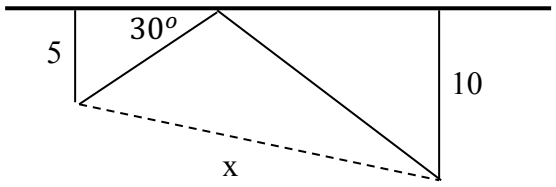
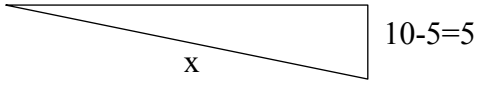
Individual Test Solutions

	Answer	Solution
1	[x =] 7	
2	$\frac{2}{9}$	
3	6 [lines] [lines of symmetry]	From vertex to opposite vertex (6/2=3) and center-of-edge to center of opposite edge (6/2=3).
4	24/7	Definition
5	13 [units]	$D = \sqrt{\left(11-(-1)\right)^2 + \left(-10-(-5)\right)^2} = \sqrt{144+25} = \sqrt{169} = 13$
6	20 [posts]	60/15=4, 90/15=6. 4+6+4+6=20
7	(0,1)	In this form, the function is guaranteed to intersect the y-axis at the constant term.
8	360	=12*6*5
9	35	Mean = 7 Median = 5
10	120	=5!
11	18π [square units]	The full circle is πr^2, and the shaded area is one eighth of the circle. $\frac{12^2\pi}{8} = 6 * 3 * \pi = 18\pi$
12	8/11	The line has slope -11/8, perpendicular line has the negative reciprocal slope.

13	25	250/4*2/5=250/10=25
14	3/8	Two coins must be heads, and the third must be tails. There are three ways to choose which coin is tails, hence $$\binom{3}{1}\left(\frac{1}{2}\right)^3 = \frac{3}{8}$$
15	1/2	$$\frac{9-5}{5-(-3)} = \frac{4}{8} = \frac{1}{2}$$
16	1072	$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$ $$9^3 + 7^3 = 729 + 343 = 1072$$
17	135 [degrees]	Interior angles are complementary to exterior angles, which are often easier to calculate. $$A = 180 - \frac{360}{n} = 180 - 45 = 135$$
18	8	Equivalent to asking for the number of positive factors of 42. $42 = 2 * 3 * 7$, and so has $(1+1)(1+1)(1+1) = 8$ factors. They could also be enumerated manually since 42 is not large.
19	211	$$= \frac{u_1(1-r^n)}{1-r} = \frac{16\left(1-\left(\frac{3}{2}\right)^5\right)}{1-\frac{3}{2}} = \frac{16\left(1-\frac{243}{32}\right)}{-\frac{1}{2}}$$ $$= 16 * \frac{211}{32} * 2 = 211$$
20	405	5+16+6*64=21+384=405
21	[$] 1.71	$$9.5 * (3.629 - 3.449) = 9.5 * 0.18 = 19 * 0.09 = 1.8 - 0.09 = 1.71$$
22	$2^{24} * 3$	$$= 2^{20+21-15} - 2^{6+47-29} = 2^{26} - 2^{24} = 2^{24}(2^2 - 2^0) = 2^{24} * 3$$
23	60	Recurse down from 5 like so: $$f(5) = 20 = f(4) + 2 \rightarrow f(4) = 18$$ And get 12+14+16+18=30+30=60.
24	3	Adding the first and third equations gives the desired relation almost immediately.
25	4 [hours]	T= 4q/hr, J = 6.3 q/hr. T+J=10.3 q/hr. $4 * 10.3 = 41.2$.
26	400[Bales]	The volume of the smaller bale is $\pi r^2 h = 16\pi$, and the volume of the larger bale is 25π. Proportion: $$\frac{volume * quantity}{cows * day} = \frac{16 * 1}{21 * 1} = \frac{25n}{75 * 175} = \frac{n}{3 * 175}$$ $$21n = 16 * 3 * 175$$ $$n = 16 * 25 = 400$$

27	$3\sqrt{3}$	The third point is (0,y), and the distance from that point to either of the others is 6. Using (3,0): $6^2 = 3^2 + y^2$ $36 - 9 = y^2 = 27$ $y = 3\sqrt{3}$
28	-64	=2i^6=-64
29	1.005	6% yearly rate corresponds to 6%/12=0.5% per month. Divide percents by 100 to get decimals.
30	$\frac{16}{81}$	$2[\,]3[\,]4 = \left(\frac{8}{9}\right)[\,]4 = \frac{\left(\frac{8}{9}\right)^2}{4} = \frac{16}{81}$
31	4	Multiplying it out and combining coefficients gets an x coefficient as below, which must be zero for the roots to sum to zero. $5 - 5m + 3m + 3 = 0$ $2m = 8 \rightarrow m = 4$
32	$2\sqrt{13}$	The x,iy plane is isomorphic to the x,y plane, so, effectively, we can ignore the i terms. After that, it's a straightforward application of the distance from a point to a line formula. $\frac{\lvert Ax + By + C\rvert}{\sqrt{A^2 + B^2}} = \frac{\lvert 2(4) + 3(-7) - 13\rvert}{\sqrt{2^2 + 3^2}} = \frac{26}{\sqrt{13}} = 2\sqrt{13}$
33	180	$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = 216$ $216 - 3ab(a+b) = a^3 + b^3$ $216 - 3 * 2 * 6 = 216 - 36 = 180$
34	224π [square units]	$SA = \pi r^2 + \pi rl, l = \sqrt{r^2 + h^2} = 25$ 7, 24, 25 being one of a pattern of Pythagorean triple. $SA = 49\pi + 175\pi = 224\pi$
35	2	Let x equal the expression. Since it repeats infinitely, we can say $x = \log_2(2 + x)$ Then by definition we get $2^x = 2 + x$ And from there x=2 is a reasonably fast logical deduction.
36	14	9 equally likely possibilities for the number of dice in each hat (note that the dice can be rerolled back into the hat from whence they came) A 3 1 1 3 5 3 3 5 3 B 4 6 4 4 2 2 4 4 6 C 5 5 7 5 5 7 5 3 3 The average of the possible numbers of dice in B is 4, and the expected value of 4 6-sided dice is 4*3.5=14.

37	$10\sqrt{7}$ [m]	 $5\tan(30)+10\tan(30)=15\sqrt{3}$  $3(225) + 25 = x^2 = 675 + 25 = 700, x = 10\sqrt{7}$
38	$[f(x) =]$ $-2x^2 + 6x + 1$ $[\forall\, x \in \mathbb{N}]$	$\begin{matrix} 5 & 5 & 1 & -7 & -19 \\ 0 & -4 & -8 & -12 & \\ -4 & -4 & -4 & & \end{matrix}$ Differences check (above) confirms that f is order 2 (quadratic), as the differences stabilize two 2 below the original values. Then solve a system of 3 equations acquired from the general $y = ax^2 + bx + c$ with the given information. $5 = a + b + c$ $5 = 4a + 2b + c$ $1 = 9a + 3b + c$ Can subtract (1) from (2) to get 0=3a+b, which can be combined with (3) to get c=1. Using that information with (1) and (2) straightforwardly produces a=-2 and then b=6.
39	21/2	189/18 We want to stack n with high-value digits, but keep the hundreds place small (look at 189/19 vs 789/24).
40	62	For distributions of n objects to m recipients, if 1 object is guaranteed to each recipient the number of ways is $(n-1)C(m-1)$, while without that guarantee the number of ways is $(n+m-1)C(m-1)$. $\frac{15C11}{27C11} = \frac{\frac{15!}{4!\,11!}}{\frac{27!}{16!\,11!}}$ $= \frac{15 * \dots * 5}{27 * \dots * 17}$ $= \frac{15 * 14 * 13 * 12 * 11 * 10 * 9 * 8 * 7 * 6 * 5}{27 * 26 * 25 * 24 * 23 * 22 * 21 * 20 * 19 * 18 * 17}$ $= \frac{2^8 * 3^5 * 5^3 * 7^2 * 11 * 13}{2^5 * 3^6 * 5^3 * 7 * 11 * 13 * 17 * 19 * 23}$ $= \frac{2^3 * 7}{3 * 17 * 19 * 23}$

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Multiple Choice Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	B	The maximum is achieved by drawing all diagonals from one vertex. N-3.
2	**99**	C	42*3=126
3	**3**	C	E[1d8]=4.5 4.5 * 4 = 18.
4	**4**	C	Alg1 – M – 6 a^2-b^2=(a-b)(a+b)
5	**99**	B	Just classic long division.
6	**6**	C	Cowboy problem! If all were full-sized, they'd be 20*23=460, which is 460-372=88 too much. Every compact is 8 less than a full, so there are 88/8=11 of them.
7	**7**	D	A and C are factually incorrect. B is nonsense. D is accurate.
8	**99**	D	A has positive slope. B has plausible slope, but would pass below the origin (x=0 gives y=-1) C would also pass below the origin (x=0 gives y=-4) D has a negative slope and passes above the origin (x=0 gives y=4).
9	**9**	A	It's two interwoven sequences. The odd-number indexed one is a simple arithmetic sequence 1 + 8(2)=17. The other one is x^2.

10	**10**	A	Law of cosines with 6, 7, and 45 degrees. $6^2 + 7^2 - 2 * 6 * 7 * \cos(45) = 85 - 84\frac{\sqrt{2}}{2}$
99	**2**	B	3, -2, 1, -1, 0, -1
99	**5**	A	Definition
99	**8**	B	The numerator is $(x - y)^2$, which goes to zero as the denominator goes to a nonzero quantity.

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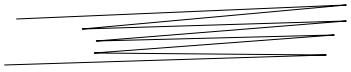
Team Test Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	21 [handshakes]	7*6/2
2	**99**	112	Given information can imply only a 7-4-4 triangle.
3	**3**	121	11^2
4	**4**	900	Total – adjacent O (OO is treated as if it was one letter). $\frac{7!}{2!\,2!} - \frac{6!}{2!} = 1260 - 360 = 900$ One option for evaluating this expression, which reduces the number of factorials to calculate, is to factor out a $\frac{6!}{2!} = 360$, leaving $\frac{6!}{2!}\left(\frac{7}{2} - 1\right) = 360 * \frac{5}{2} = 180 * 5 = 900$.
5	**99**	8	AB/99=0.ABABABAB…. In this case, 0.181818…
6	**6**	15[o]	If the center of the clock is point P, then angle APC is straightforwardly calculated as 1/12 of a circle: 30o. Therefore arc AC measures 30o, and the rule for an angle with a vertex on a circle intersecting an arc of that circle is that the angle is half the arc. 30/2=15.
7	**7**	120 [intersections]	Not only do we want our shapes to be irregular, we want them to be very concave. As one possibility, imagine the head of a 4-tonged fork as an octagon – that's the kind of shape we want the hexagon and icosagon (picture below, with the left edges intersecting some far distance to the left). At a near-right angle to each other, each side of one polygon can intersect every side of the other. The answer is then 6*20=120. 

8	**99**	$\frac{5\sqrt{14}}{2}$ [units]	The pyramid's slant edge and half its base diagonal form a right triangle with the height. Using Pythagoras: $10^2 = \left(\frac{5}{\sqrt{2}}\right)^2 + h^2$ $h^2 = 100 - \frac{25}{2} = \frac{175}{2}$ $h = \frac{5\sqrt{7}}{\sqrt{2}} = \frac{5\sqrt{14}}{2}$
9	**9**	$\frac{140}{179}$	Conditional probability $P(tart\|rejected) = \frac{P(rejected\|tart)P(tart)}{P(rejected)}$ $= \frac{P(rejected\|tart)P(tart)}{P(rejected\|tart)P(tart) + P(rejected\|sweet)P(sweet)}$ $= \frac{1 * \frac{7}{20}}{1 * \frac{7}{20} + \frac{3}{20} * \frac{13}{20}} = \frac{140}{140 + 39} = \frac{140}{179}$
10	**10**	$\frac{47}{9}$ [points per leaf]	As a weighted average, we have E(points)= $0.9 * 5 + 0.09 * 7 + 0.09 * 9 + 0.009 * 11 + \cdots$ $= \frac{9}{10}\left(5 + \frac{7}{10} + \frac{9}{100} + \frac{11}{1000} + \cdots\right)$ This is an infinite arithmetic-geometric series. Let $S = \frac{9}{10}\left(5 + \frac{7}{10} + \frac{9}{100} + \frac{11}{1000} + \cdots\right)$ Then $10S = \frac{9}{10}\left(50 + 7 + \frac{9}{10} + \frac{11}{100} + \frac{13}{1000} + \cdots\right)$ So $10S - S = 9S = \frac{9}{10}\left(52 + \frac{2}{10} + \frac{2}{100} + \frac{2}{1000} + \cdots\right)$ 9S contains an infinite geometric sequence. We apply the relevant sum formula thereto. $9S = \frac{9}{10}\left(52 + \frac{2}{9}\right)$ $9S = \frac{9}{10} * \frac{470}{9} = 47$ $S = \frac{47}{9}$
99	**2**	-2, $-\frac{1}{3}$, 5 [Any order]	Knowing that one of the solutions is 5 allows us to divide the polynomial by (x-5), turning it into a quadratic. Factoring the quadratic is straightforward.
99	**5**	-1	Since the det(product) = product(det), we only have to look at the first matrix, which straightforwardly has det(0) if x is -1.
99	**8**	1024	Found by letting x=y=1. 4^5=2^10=1024 Can be made to look more familiar by factoring out the 2 as so: $2^5(x + y)^5$.

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Pressure Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	35 [questions]	Since 40% missed corresponds to 60% correct, we set up the proportion below. $$\frac{30}{0.6} = \frac{x}{0.7}$$
2	**2**	7/12	C must be greater than 5.
3	**99**	48	Distance $= 5 = \sqrt{(x-1)^2 + (y-3)^2}$ $$25 = (x-1)^2 + (y-3)^2$$ The only sums of squares that equal 25 are 16+9 or 25+0. For the former, the following x,y pairs work: 5,6; -3,6; 5,0; -3,0 4,7; -2,7; 4,-1; -2,-1 And the latter: 6,3; -4,3 1,8; 1,-2 Adding all coordinates: 2(5+6-3+0+4+7-2-1+3+1)+6-4+8-2= 2(20)+8=48. Alternatively: Noting that there are 12 possible points and that they are equally spaced on a circle around (1,3) lets one look at pairs of opposite points, which will average to the center. Then the total is 12(1+3)=12*4=48.
4	**99**	$\frac{3}{2}$	$$f(x) = ax(x-4) + 2 = ax^2 - 4ax + 2$$ $$-4 = 4a - 8a + 2$$ $$4a = 6$$ $$a = \frac{3}{2}$$ Or $f(x) = a(x-2)^2 - 4$

5	**5**	64	Since it's an arithmetic sequence, y+z=x+w=15+49, which is 64.
99	**3**	$\sqrt{110}$	$A = \frac{1}{2}\left\|\begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix} \times \begin{bmatrix} -2 \\ 2 \\ -4 \end{bmatrix}\right\| = \frac{1}{2}\left\|\begin{matrix} 18 \\ 10 \\ -4 \end{matrix}\right\|$ $= \frac{1}{2}\sqrt{324 + 100 + 16} = \frac{1}{2}\sqrt{440}$ $= \sqrt{110}$
99	**4**	$\frac{3}{2}$	Let $\theta = 2\phi$ $\frac{\sin(4\phi)\cos(\phi)}{\cos^2(\phi) - \sin^2(\phi)}$ $= \frac{2\sin(2\phi)\cos(2\phi)\cos(\phi)}{\cos(2\phi)}$ $= 4\sin\phi\cos^2\phi = 4\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)^2$ $= \frac{3}{2}$ Or just plug in; it's easy and probably just as quick.

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College Bowl Round #1 Solutions

	Answer	Solution
1	20 [%]	
2	360[^{0}]	
3	15	
4	25	median is 7, mean is 7, mode is 11
5	114	2,4,8,16,32,64,128 and 2,4,6,8,10,12,14
6	98 [square units]	diagonal of 14, implies a side of length $14/\sqrt{2}$
7	8 [factors]	This is the same as asking for the number of factors in the GCD of 144 and 168, which is 24. Since $24 = 2^3 * 3$, the answer is $(3 + 1)(1 + 1) =$ $4 * 2 = 8$
8	2 [hours]	
9	8 [cubic yards]	
10	83 [square feet]	72*24 is 1728, 1728-1645 is 83

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College Bowl Round #2 Solutions

	Answer	Solution
1	196	
2	1080 [degrees]	
3	25 [%]	
4	37	19 is the first, 37 is next
5	(8,3)	complete the square
6	30240 [ways]	$\frac{9!}{3! * 2!}$
7	39 [points]	(n1-1)(n2-1) - 1
8	1/2	
9	1680	5280-3600
10	144[°]	

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College Bowl Round #3 Solutions

	Answer	Solution
1	48 [sq. feet]	
2	$\frac{25\sqrt{3}}{2}$ [square units]	
3	180	
4	18	sum of all 42 numbers is 903, 903-866 is 37
5	10 [units]	
6	(-4, -3)	(3, -4) to (-3, -4) to (-4, -3)
7	247	n(3n-1)/2
8	6[factors]	1,2,7,14,49,98
9	7	
10	16	8*32 is 256, √256 = 16

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College Bowl Round #4 Solutions

	Answer	Solution
1	[$]8.75	
2	20 [feet]	
3	18[o]	
4	10.35	
5	4	1,8,27, 216
6	48180	n(n+1)=219*220
7	36 [sq. cm]	
8	-159	ad-bc
9	5	
10	-25	5 and -5, which follow from values of x that cause division by zero.

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College Bowl Round #5 Solutions

	Answer	Solution
1	675 [pounds]	
2	126 [cubes]	
3	60	Vaguely disguised LCM question.
4	(2,1)	$6x - 11 = y = -\frac{3}{4}x + \frac{10}{4}$ $24x - 44 = 10 - 3x$ $27x = 54 \rightarrow x = 2 \rightarrow y = 1$
5	$6\sqrt{2}$ [units]	The leg length will be the square root of twice the area, and the hypotenuse is that times the square root of 2.
6	3	1!=1, 5!=120, 11!=39916800
7	440	largest 4 digit base 5 number is 4444, which is 624 in base 10
8	6, -4 (Order not important)	
9	54 [sq. cm]	
10	15	

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College Bowl Round #6 Solutions

	Answer	Solution
1	45[degrees]	
2	1/435	
3	8	5/7 is 0.714285 repeating
4	37	
5	96 √3 [square units]	area of a hexagon is (3/2)root3 times side^2
6	42 [hours]	Together they write 1/7 + 1/6 of a paper in an hour. Divide 13 by that sum to get the time taken.
7	676	673 and 3 are the only prime factors of 2019
8	81	
9	96[%]	
10	27	6*6*6*6*6*6=46656

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College Bowl Round (Extra) Solutions

	Answer	Solution
1	10	5 evens, 3 faces, and the ace are 9 fail cases. 10 thus guarantees.
2	7	3, 9, 7, 1, repeats ad infinitum for powers of 3. The ten’s place of thirteen is a red herring.
3	48 pi [meters squared]	64pi * 3/4
4	560	27^2-13^2=(27+13)(27-13)=40*14=560
5	6 [factors]	-1,-2,-4,-7,-14,-28
6	10	Median 1, range 9