"Math is Cool" Masters -- 2018-19

High School

Mental Math Solutions

	Answer	Solution
1	8	Cancel first: 4*2=8
2	2	Reflecting across y=x swaps X and Y, so the sum remains -5+7=2
3	-3/4	M=-A/B=-3/4
4	$16\sqrt{3}$ [square meters]	A=s^2*root3/4 =4^2*root3
5	15 [percent]	21/140*100
6	$\frac{5}{9}$	1*5/6*4/6
7	178 [degrees]	53+39=92, so the two lines barely cross at a 2^{o} angle. The larger angle will be 180-2=178
8	28 [dogs]	43 ducks would be 86 feet, which is 56 feet too few, so we need to turn 56/2 of the ducks into dogs.

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Individual Test Solutions

	Answer	Solution
1	1:26 P.M.	The wedding should originally have ended at noon. The groom showed up 16 minutes before this, and the ceremony lasted 102 minutes, so it ended $102 - 16 = 86$ minutes after noon.
2	[$]3.64	7.25+4.35+5.50=17.10 .08*17.10=1.368~1.37 17.10+1.37=18.47 18.47-14.83=3.64
3	3, -8, 7 (any order)	A function has at most one y-value for a particular x-value. Because y-values are already given for x-values of 3, -8, and 7, this new y-value cannot be for any of these x-values.
4	1440 [square feet]	6 * 15/4 * (6/(3/4))^2 =(6*15*6*6*4*4)/(4*3*3) =6*15*2*2*4 = 96*15
5	(0, -6)	The original intercept at (0,-2) is moved three times as far away.
6	16	(64^(1/3))^2 = 4^2
7	2	81=3^4, so 5x-6=4
8	12 [feet]	The average of the bases is $\frac{54}{3} = 18$. 24 is 6 more than 18, so the other base needs to be 6 less than 18.
9	B	The range is $27 - 7 = 20$, so σ is probably in the vicinity of $\frac{20}{6} = \frac{10}{3} \sim 3$.
10	-25	The intercepts are 4 and -6, so the axis is -1.
11	37[%]	$\frac{\frac{3}{5}+2}{3+4} * 100 = \frac{\frac{13}{5}*100}{7} = \frac{13*20}{7} = \frac{260}{7}$
12	12 [inches]	The area of a kite is twice the product of its diagonals. This sets up a straightforward equation.
13	5 [miles]	$\frac{625}{5^3} = \frac{625}{125}$
14	A, B, D (in any order)	A general understanding of rational & irrational should help you answer these. Note: C & D are the opposite of one another, so only one can be true.

15	1331	$\frac{11^7}{11^4} = 11^3$
16	$-5 + \frac{25}{2}i$	$(-8i + 9)\left(-1 + \frac{1}{2}i\right)$
17	1:3 or 1/3 or 1 to 3	You need two numbers that are twice as large when added as they are when subtracted. 3 & 1 fit the bill.
18	25	$\sqrt{5 \cdot 125}$
19	100	$r = \sqrt{25} = 5$, so $s = 2 \cdot 5 = 10$.
20	30[°]	The altitude perpendicular to AD is $\frac{160}{20} = 8$, so we can draw triangle ABE with leg 8 and hypotenuse 16, so it's a 30-60-90 triangle.
21	$\frac{12 - 2\sqrt{2}}{17}$	Multiply by $\frac{6-\sqrt{2}}{6-\sqrt{2}}$to get a denominator of 36-2=34, then cancel a 2.
22	128π	3/4 * 12^2 + 1/4 * 4^2 + 1/4 * 8^2
23	129	Their net speed is 4, so they'll meet every 1/16th of an hour, which gives 8*16+1.
24	-1, -2, 3	The factors could be +- 1, 2, 3, or 6. Trying 1 doesn't work, but -1 does, so factor out (n+1), leaving (n^2-n-6), which factors to (n-3)(n+2).
25	14	11, 12, 15, 22, 24, 33, 36, 44, 48, 55, 66, 77, 88, 99
26	306	The 9th term is 2+8*8=66, making outer pairs sum to $2 + 66 = 68$, for a total of 68*9/2.
27	CEBADF	A is by B, who is by E, who is to F's left, so I can write [ABE]-F. Then I can add C-[ABE]-F. C can't be by D, so it's C[ABE]DF, and A is to the right of E, so it's CEBADF.
28	$44,289.03	25000*1.1^6=25000*1.771561=25*1771.561.
29	12 [meters]	Draw a rectangle using the tangent and the center of the smaller circle, leaving a right triangle with a hypotenuse of 13 and a leg of 7-2=5.
30	72	There are 14c2=91 ways to give out the candies, but 1 way to give them all 4 and 3 ways to give two 6, 5, 3, 2, 1, 0.
31	60	It might be p^11, p^5*q, p^3*q^2, or p^2*q*r. 2^11 is 2048, 2^5*3=96, 2^3*3^2=72, and 2^2*3*5=60.
32	6501	The difference is 999* the difference of the first and last digits, plus 90* the diff of the middle two. 5445 is 5*999+5*90, so the outer two differ by 5, as do the inner two.

33	38	In order, the fourth number is 25 and the upper two are 60. To minimize the range, make the lower three numbers, 22, 23 and 24. Finally, verify that the sequence is possible by assigning the 5th value of 38 to make the mean 36. The range is 60 – 22 = 38.
34	$\sqrt{65}$	Unfold the face and an adjacent face (on the far sides from the near vertex), then do Pythagorean triangles. 7x4 will have the shortest hypotenuse of the options.
35	12:01:12 AM	Divide by 60 to get 453 minutes with a remainder of 16 seconds. Do it again to get 7 hours with a remainder of 33 minutes, then add.
36	$(-\infty, 5]$	The root could go from 0 to infinity, but it's subtracted from 5…
37	9	99, 99^2=9801, 99^3 will end in 99, 99^4 will end in 01, etc.
38	123552	There are 13c2 ways to pick the two doubled ranks, 11 ways to pick the other, 6 ways to pick each of the two pairs, and 4 ways to pick the single.
39	$27 - 18\sqrt{2}$	You can make a 45-45-90 triangle from the center of the big rug, through the center of the little rug, and to the corner. The hypotenuse will be 9root2, but is also 9+r+rroot2.
40	$24.00	C=np=(n+1)(p-.8)=(n-1)(p+1.2) p-.8n=.8 and -p+1.2n=1.2 n=5 and p=4.8
41	$-\frac{24}{25}$	Cos^2=1-sin^2 Pythagorean
42	$-\frac{28}{3}$	Basic power rule integral The symmetry of 2 & -2 means you can ignore the x^4 term you create.
43	$\begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$	Transposed matrix of cofactors, divided by the determinant.
44	$2\sqrt{7}$	Law of Cosines.
45	$(1, -58)$	Derivative, factor out 12, factor the cubic to get 1, -2, and -3, the mins are 1 and -3, focus on 1 because -2 is close to -3.

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Multiple Choice Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**99**	C	Complete the square by adding 16 to both sides of the equation.
2	**99**	D	Check each possibility for correct first term and correct rate of increase
3	**3**	B	A doesn't work because it implies a vertex with negative x coordinate. C implies two negative x intercepts. D has x-intercepts at 1 and -7. B has positive y-int (16-8=8) and vertex with positive x-coordinate, two positive x-intercepts
4	**4**	A	Math or Science = Math + Science – (Math and Science) = 51+44-38=95-38=57. There are then 57 out of 60 that like both. 57/60=19/20.
5	**5**	D	Draw in radius AB = 8+9=17. Then do Pythagorean Theorem to find that half of BD is 15.
6	**99**	B	The x^2 and y^2 terms have opposite signs, which indicates a hyperbola. Clearly not degenerate.
7	**7**	D	After the first fill, ¾ of the solution is lemonade. After the second fill, ¾ x ½ or 3/8 of what is left is lemonade. After the third drink, 1/8 of a gallon of lemonade is left, and then you add 2/3 of a gallon more. $\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)+\frac{2}{3}=\frac{1}{8}+\frac{2}{3}=\frac{3}{24}+\frac{15}{24}=\frac{19}{24}.$
8	**8**	B	Just count as the hundreds place is the same as the ones place: hundreds: tens 2: 5, 6, 8, 9 3: 0, 1, 2, 3, 5, 6, 8, 9 5: 0, 1, 2, 3, 5, 6, 8, 9 6: 0, 1 A total of 22.

9	**9**	D	The sum of factors for a number N that is factored as $N = p^a q^b$ is $(1 + p + \cdots + p^a)(1 + q + \cdots + q^b)$ and $372 = 12(31) = 6(62)$. There are several ways these factors can be put in the proper form. $12(31) = (1 + 11)(1 + 2 + 4 + 8 + 16)$ $12(31) = (1 + 11)(1 + 5 + 25)$ $12(31) = (1 + 2)(1 + 3)(1 + 5 + 25)$ $6(62) = (1 + 5)(1 + 61)$ These yield: $11(16) + 11(25) + 2(3)(25) + 5(61)$ $= 176 + 275 + 150 + 305 = 906$
10	**10**	A	They should have taken $24/5$ hours, so their time is $25/24$ what it should have been, so their speed was $24/25$ what it should have been. The missing 100 bricks is $1/25$ of what their speed should have been, so their speed was actually 2400 bricks per hour for 5 hours. $5(2400)$=12000
99	**1**	C	$< 4, -8 > \cdot < -9, -5 > = 4(-9) + (-8)(-5) = -36 + 40$ $= 4$
99	**2**	A	Factor numerator & cancel to get z+3.
99	**6**	D	$\cos(2t) = 2\cos^2(t) - 1 = 2\left(\frac{2}{3}\right)^2 - 1 = \frac{8}{9} - 1 = -\frac{1}{9}$

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Team Test Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	-5	Substitute x=4, y=3. $Ax + 3y = 17 = 4A + 9, A = 2$ $5x + By = -1 = 20 + 3B = -1, B = -7$ $A + B = -5$
2	**2**	$(3, -4)$	Complete the squares, the radius is clearly positive when -39 is moved to the right side. All that is needed is $-\left(-\frac{6}{2}\right) = 3, etc.$
3	**99**	34	Log definition. $\log_2 32 = 5 \Leftrightarrow 2^5 = 32.\ 2 + 32 = 34$
4	**99**	4	The vertex of the parabola is straightforwardly determined to be at $(1, -2)$, so the form is $a(x-1)^2 - 2$. Substituting $f(0) = 1$ gives $a = 3$, and expanding gives $c = 1$. However, it easier to use $-2 = f(1) = a - 6 + c.\ a + c = -2 + 6 = 4.$
5	**99**	$[\$]31.50$	Two possibilities 2x2x5 or 5x5x2, hence volume 20 or 50. The price sum will be $0.45(50 + 20) = \$31.50$
6	**6**	861937	C is 1, Neither A&D are 0. A can't be 9 or D would be 0, so try A=8 (we want a big number). D is then 9, so F is 7. For a big number, B can be 6 if E is 3. 861937
7	**7**	$\frac{47}{128}$	There are several permutations taken as to where the HHH first appears: HHHXXXX=16, THHHXXX=8, XTHHHXX=8, XXTHHHX=8, YYYTHHH=7. X stands for either H or T and YYY can be anything but HHH. 16+8+8+8+7=47 out of 128 total equally-likely possibilities.
8	**8**	$M = 3^{48}$	Either match base or exponent for clear comparison. $L = 80^{12} = (80^2)^6 = 6400^6,$ $M = 3^{48} = (3^8)^6 = 6561^6$ $N = 6000^6, so\ M$ is the largest.
9	**9**	$35 + 5\sqrt{47}$ [cm]	You can make a right triangle using the two centers as the hypotenuse. The vertical leg is drawn from the center of the small ball down and the horizontal leg is from the center of the larger ball towards the corner. It has hypotenuse 20+15, "base" $20\sqrt{2} - 15\sqrt{2} = 5\sqrt{2}$, so this height is $5\sqrt{47}$. Finally, add the two radii, 20+15=35..

10	**10**	39	Label the integers: $a < b < c < d < e$. (No sum is the same, so they must be distinct). $a + b = 4, a + c = 6, so\ c = b + 2$ $c + e = 45, d + e = 67$ If $a + d = 12$, then e would have to be very large and the other conditions could not be met. $b + c = 12, c = b + 2 \Rightarrow b = 5, c = 7\ and\ a = -1.$ $c + e = 45, so\ e = 38\ and\ d = 29.$ $e - a = 38 - (-1) = 39.$ One can check to make sure the others sums are correct.
99	**3**	$4e^{\frac{3\pi}{2}i}$	In the complex plane, the magnitude is clearly 4 and the angle is $270°\ or \frac{3\pi}{2}$.
99	**4**	-15	Expanding the first row: $0(\ ) - 1[3(0) - (-1)(1)] - 2[3(3) - (-1)(-2)]$ $= -1(1) - 2(7) = -15.$
99	**5**	4	Use the Product Rule, Power Rule, Chain Rule! $w'(v) = 3(2v - 1)^2(2)(v + 1)^2 + 2(v + 1)(2v - 1)^3$ $w'(0) = 3(1)(2)(1) + 2(1)(-1) = 6 - 2 = 4.$

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Pressure Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**99**	840/11 [mph]	280/60 + 1 = 280/x
2	**2**	3	Triple Venn Diagram. Total is 24, exterior is 0, center is 1. Subtotals are G = 6, T = 12, & C = 15. G&T&~C=2 and G&C&~T=2, so G-alone=6-1-2-2=1. We want to know C&T&~G (x), and the only other things we don't know are C-alone and T-alone. These three add to 24-6=18, so we can write (9-x) + x + (12-x) = 18.
3	**3**	0	For any particular value b_n, there is another value, $-b_n$, so everything cancels.
4	**99**	12	There is a formula for this type of question: for an NxM rectangle, the diagonal intersects the interiors of N + M – GCD(N, M) = 6+9-3=12.
5	**5**	$\sqrt{493}$	Reflect Camp across the River (which may be easier with a y-axis transform so that the line becomes y_1=x, but remember to undo it), then distance formula.
99	**1**	[5 -8 1]	Standard matrix multiplication, but somewhat unusual sizes.
99	**4**	$4 + 2\ln 3$	Product and chain rules, then evaluate using log rules.

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College Bowl Round #1 Solutions

	Answer	Solution
1	[quadrant] 2	Going down 7 units puts the point in quadrant 4 and moving to the right doesn't change it. Reflecting puts the point in quadrant 2.
2	86[°]	The angles of a triangle add up to 180 degrees. So the answer is 37+49=86.
3	1 [piece of candy]	400->200->100-99=1
4	15 [degrees]	360/24 = 15
5	$2\sqrt[3]{7}$ Two times the cube root of seven.	$\sqrt[3]{56} = \sqrt[3]{8 \cdot 7}$ $= \sqrt[3]{8} \cdot \sqrt[3]{7} = 2\sqrt[3]{7}$
6	13	221=13*17, 299=13*23 or use Euler's method. 299-221=78. 221-2*78=65. 78-65=13.
7	120	Treat SLI as a 'letter', then there are 5 letters and no repeats, which is 5! Ways.
8	6	Powers of 4 alternate units digits between 4 and 6.
9	365	The 10th term is 5+9(7)=68. The sum is: $\frac{10(5+68)}{2} = 5(73) = 365$
10	Three-fourths	Tangent=opposite / adjacent = 3/4

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College Bowl Round #2 Solutions

	Answer	Solution
1	One-third	
2	240[°]	The total should be $(5-2)180.$
3	1200 [ways]	$\frac{6\cdot 5\cdot 4}{3\cdot 2}\cdot\frac{5\cdot 4}{2}\cdot\frac{4\cdot 3}{2}$ $=5\cdot 4\cdot 5\cdot 2\cdot 2\cdot 3$
4	$\frac{55}{96}$	$\frac{12}{12}*\frac{11}{12}*\frac{10}{12}*\frac{9}{12}=\frac{11*5*3}{12*6*4}=\frac{11*5}{12*2*4}=\frac{55}{96}$
5	2 and $-\frac{7}{8}$ Both required	Factor by grouping. $8G^2-9G-14$ $=8G^2-16G+7G-14$ $=8G(G-2)+7(G-2)$ $=(8G+7)(G-2)=0.$ $G=2,-\frac{7}{8}$
6	56[%]	1.2*1.3 ~ 12*13 = 156
7	3	240-236=4 must be a multiple of the LCM.
8	240 [sq un]	Twice a 8-15-17 right triangle; other leg is 30. Area is 16(30)/2=240
9	1	The remainder by 5 is related to the last digit, which is 7->9->3->1.
10	60[°]	$C=2\pi r=100\pi$ $\frac{\left(\frac{50\pi}{3}\right)}{100\pi}(360°)=60°$

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College Bowl Round #3 Solutions

	Answer	Solution
1	35	Working backwards, 3*19=57, 57+13=70, 70/2=35
2	[$]1.24	2 * 3.41/5.5 =4*3.41/11 = 4*.31
3	39	$\frac{65}{100}(60) = \frac{13}{20}(60) = 13(3) = 39.$
4	459	153/17=9, 153*3=459
5	576	210=2*3*5*7, Sum of factors = (1+2)(1+3)(1+5)(1+7)=3*4*6*8=576
6	4	191, 193, 197, and 199. 2, 3, and 5 have quick checks. For 7, 11, and 13, faster to find the multiple in this range than try each of the four possibilities individually. For 7, 210-14=196. For 11, 220-22=198. For 13, 130+65=195.
7	7	The sum of the product of the roots, taken two at a time, is $-\frac{c}{a}$ (regardless of whether the roots are real or imaginary). In this case, a=3 and c=-21.
8	5	What power must 4 be raised to to get 1024?
9	8948	For a large answer, we'd like 8X4Y, then 894Y, then 8949. We want the second-largest, however.
10	60[°]	50/300 = 1/6 of the circle.

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College Bowl Round #4 Solutions

	Answer	Solution
1	15	There are 90 numbers, one-sixth of which count.
2	2/3	Essentially, what is the probability of getting an odd on the first die? 4/6
3	147	49/100 * 300 = 49 * 3=147
4	1/72	There are 3 ways to roll two 1s and a 2, out of $6^3 = 216$ total ways.
5	(0,6) or 0 comma 6.	Given the perpendicularity, the equation of this line is 2x+3y=c. Given the x-intercept, c=18. Therefore the y-int is found by 2(0)+3(y)=18: y=6.
6	$\frac{8}{9}$	R=d/t Ratio=$\frac{2}{3} / \frac{3}{4} = \frac{8}{9}$
7	5000	$5C3 * (5x)^3 * (-2)^2 = 10 * 125x^3 * 4 = 5000x^3$
8	24	FGHIJKLMNOPQ is halfway around, and is 12.
9	216	54 already has one power of 2 as a factor, the LCM needs 2 more. 4(54)=216
10	9	$-\frac{1}{2} = \frac{p-7}{4-8}$. $2 = p - 7, p = 9.$

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College Bowl Round #5 Solutions

	Answer	Solution
1	46	The center number will be $\frac{144}{3} = 48.$
2	22419	150^2-9^2=22500-81
3	16	Alg1 – E – 11 The net speed at which the distance is closed is $5 - 3 = 2.$
4	$\frac{60}{13}$ $[un]$	Area of a right triangle is either leg*leg/2 or hyp*alt/2. So 12*5=13x, and x=$\frac{60}{13}$
5	-9	-15=2n+3
6	10 [un]	Height = $\frac{V}{\frac{1}{3}\pi r^2} = \frac{3 \cdot 128\pi}{64\pi} = 3 \cdot 2 = 6.$ $s = \sqrt{6^2 + 8^2} = 10$
7	$322_{[8]}$	Every three base two digits become 1 base eight digit. 011 010 010 $=322_8$
8	$12\sqrt{5}$ $[sq\ m]$ Twelve times the square root of five	Heron's formula gives $s = \frac{9 + 7 + 14}{2} = 15.$ $A = \sqrt{15 \cdot 1 \cdot 6 \cdot 8}$ $= 3\sqrt{5 \cdot 2 \cdot 8}$ $= 3 \cdot 4\sqrt{5} = 12\sqrt{5}$
9	56 [ways]	$8c5 = 8c3 = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2} = 56$
10	1 [un]	$3x + 4 = 5x + 2, 2x = 2, x = 1$

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College Bowl Round #6 Solutions

	Answer	Solution
1	3	Dilation preserves slope. You can also check points.
2	2/15	X=.13333 10X=1.333=4/3
3	8 [ways]	The first kid could get 0-7 candies.
4	$(1,-2)$	x-coord = -b/2a =6/6=1. Then y-coord = $3(1)^2-6(1)+1=-2$
5	$\frac{60}{143}$	$\frac{6C2*8C2}{14C4}=\frac{15*28}{\frac{14*13*12*11}{4*3*2}}=\frac{420}{7*13*11}=\frac{60}{143}$
6	$96\sqrt{3}$	$6\left(\frac{8^2\sqrt{3}}{4}\right)$
7	267	200 multiples of 5 83 multiples of 12 16 multiples of 60 283-16=267
8	10 [feet]	This is twice a 5-12-13 right triangle.
9	$(-1,-6)$ Negative-one-COMMA-negative-six	Alg1-M-31 The X will be 1/3 the way from -8 to 13 (21 apart, so add 7). Y is similar.
10	3	60 seconds per minute, 60 minutes per hour, 24 hours per day, and 365 days/year. Question is how many zeroes are on the end of $60*60*24*365$ 60 and 365 have one factor of 5, 24 has none. Total is 3 zeroes.

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College Bowl Round (Extra) Solutions

	Answer	Solution
1	$\frac{3}{8}$	There are $2^3 = 8$ total ways, three of which we want (TTH, THT, HTT).
2	4	(3+2+7)/3
3	16pi [un]	The whole circle's area is $8 \cdot 8\pi = 64\pi$, so $r = 8$.
4	(4, 2)	Average the Xs, then Ys.
5	71	You'd have to memorize this. 7(13)-7-13=71
6	$\frac{16\pi}{3}$ $[sq\ un]$	Altitude $2\sqrt{3}$, therefore radius $\frac{4\sqrt{3}}{3}$ and area $\frac{16\pi}{3}$
7	91	n(n+1)(2n+1)/6
8	$56°15'$ 56 degrees 15 minutes	$\frac{5\pi}{16} \cdot \frac{180}{\pi} = \frac{225}{4} = 56\frac{1}{4} = 56°15'$