"Math is Cool" Masters -- 2018-19

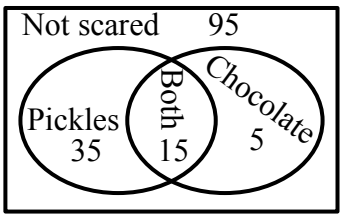
Middle School

Mental Math Solutions

6th	Answer	Solution
1	21 [miles]	7 x 3 = 21
2	12 [days]	.8 x 15 = 12
3	7	$\sqrt{49} = 7$
4	1/2	26 black cards in a standard deck, 26/52 = 1/2
5	96 [ft^2]	16x12/2 = 96
6	11	(14 + 7 + 21 + 2)/4 = 11
7	[x =] 2/13	91x + 6 = 20 91x = 14, x = 14/91 = 2/13
8	4 [sets]	${}_4C_3 = 4$

"Math is Cool" Masters -- 2018-19
Middle School
Individual Test Solutions

6th	Answer	Solution
1	4408	76 x 58 = 4408
2	885833	8735207 – 7849374 = 885833
3	2	$\sqrt{36}$ - $\sqrt{16}$ = 6 – 4 = 2
4	4.369 x 10^7	a x 10^b where a is a number between 1 and 10 and b is the number of places needed to move the decimal to the left to make a such a number
5	66 [cm^2]	A = bh/2, so (12)(11)/2 = 66
6	1/26	There are two cards out of 52 that could match the card on the third draw, so 2/52 = 1/26
7	120 [ways]	5! = 120
8	9 [days]	If it doubles every day then it must have been half the size the day before. The day before day 10 is day 9.
9	6 [ways]	\$1 x 18 \$1 x 13 and \$5 x 1 \$1 x 8 and \$5 x 2 \$1 x 8 and \$10 x 1 \$1 x 3 and \$5 x 3 \$1 x 3 and \$5 x 1 and \$10 x 1,
10	73/105	3/5 + 2/3 – 4/7 = (63 +70 – 60)/105 = 73/105
11	105 [blips]	35 blips = 15 blaps, 15 blaps = 27 blops, so 35 blips = 27 blops, and 105 blips = 81 blops
12	-2/3	(7 – 1)/(-8 – 1) = 6/-9 = -2/3

13	25	$\|3 - 4 \cdot 7\| = \|3 - 28\| = \|-25\| = 25$
14	24 [inches]	$7^2 + b^2 = 25^2 \rightarrow 49 + b^2 = 625 \rightarrow b^2 = 576 \rightarrow b = 24$
15	[x =] 5.5	6x + 21 – 12x + 20 = 8 -6x + 41 = 8 -6x = -33 x = 33/6 = 5.5
16	53 [pencils]	21 + 3 + 3 – 2 + 3 + 2 + 3 – 2 + 3 + 3 – 2 + 2 + 3 + 3 – 2 + 3 + 2 + 3 – 2 + 3 + 3 – 2 + 2 = 21 + 36 – 12 + 8 = 53
17	270 [degrees]	360/12 = 30, 9x30 = 270
18	ABDC	5683 +7462 13145, so A = 8, B = 4, C = 2, D = 3, so ABDC is greatest to least.
19	27 [triangles]	Let the length of the side of the smallest triangle = 1 unit and P = perimeter, assuming the triangles are equilateral. Then, P = 3: 16 triangles P = 6: 7 triangles (incl. upside down) P = 9: 3 triangles P = 12: 1 trangle 16 + 7 + 3 + 1 = 27
20	95 [people]	 All four numbers add up to 150, the numbers in the Pickles circle add up to 50, and the numbers in the Chocolate circle add up to 20. So the answer is 95.
21	40 [minutes]	Focus on Eho. Ten minutes to go, come back and get backpack. Then thirty more to walk a mile, for a total of 40.
22	38 [children]	$c + 2c + 4c = 266 \rightarrow 7c = 266 \rightarrow c = 38$

23	44 [cubes]	Only cubes on the edges and the corners will have two or three faces painted. The rest have only one face or zero faces painted. There are 8 vertices and 12 edges. Subtract 2 (for the corners) from 5 to get the number of cubes on the edges. So the answer will be 12 x 3 + 8 = 44.
24	1 [hole]	3 holes with 1 putt and 2 putts on the other 15 holes makes a total of 33 putts. 47 – 33 = 14 If he has one extra putt on 14 of the 15 holes with at least 2 putts, he could conceivably have just one hole with 2 putts.
25	22/105	P(gg) = 3/15x2/14 = 6/210 P(bb) = 4/15x3/14 =12/210 P(br,br) = 3/15x2/14 = 6/210 P(bl,bl) = 5/15x4/14 = 20/210 (6+12+6+20)/210 = 22/105
26	126 [ways]	${}_9C_4$ = (9x8x7x6)/(4x3x2x1) = 9 x 2 x 7 = 126
27	[y =] -21/5	-4(9x + 4y = 21) → -36x – 16y = -84 9(4x + 9y = -21) → 36x + 81y = -189 Add the new equations: 65y = -273 → y = -168/65 = -21/5
28	63/16 [hours]	1/9 + 1/7 = 1/x → 7x + 9x = 63 → 16x = 63 → x = 63/16
29	$486_{[9]}$	16 + 8 + 3 = 27 = 123_4 320 + 48 + 7 = 375 = 567_8 27 + 375 = 402 = 4 x 9^2 + 8 x 9^1 + 6 x 9^0 = 486_9
30	3 [cm]	V = $\pi r^2 h$ so $108\pi = \pi r^2(12)$ → 9 = r^2 → 3 = r
31	66 [seconds]	r_1 = d/t and r_2 = d/60, $r_2 = 1.1r_1$ = 1.1d/t → d/60 = 1.1d/t → 1/60 = 1.1/t → t = 66 seconds
32	24	x = 5 and y = 9 or vice versa z = either 4 or 10 5 + 9 + 10 is the larger sum

33	83 [bills]	20-1s - 20 15-1s & 1-5 - 16 10-1s & 2-5s - 12 10-1s & 1-10 - 11 5-1s & 3-5s - 8 5-1s & 1-5 & 1-10 - 7 4-5s - 4 2-5s & 1-10 - 3 2-10s - 2 20+16+12+11+8+7+4+3+2=83
34	$5\sqrt{2}$ [units]	D((3 + 7)/2, (2 + 8)/2) = (5, 5) $AD^2 = (-2 - 5)^2 + (6 - 5)^2 = 50$ $AD = \sqrt{50} = 5\sqrt{2}$
35	[q =] 4800	Even multiples of 13 include 26, 52, 78, 104, etc. Even multiples of 17 include 34, 68, 102, etc. The only pair from these two lists that are close enough together to be part of a set of three consecutive even integers are 102 and 104. Then the smallest possible third of the three consecutive even integers would be 100. The prime factorization of 100 x 102 x 104 is 2 x 2 x 5 x 5 x 2 x 3 x 17 x 2 x 2 x 2 x 13. If you multiply this out without the 13 or 17 included, you get 4800.
36	[x =] 3/2	$\frac{x}{\frac{x}{\frac{x}{4}-\frac{x}{10}}+\frac{x}{\frac{x}{4}-\frac{x}{10}}} = \frac{9}{80} \Rightarrow$ $\frac{x}{\frac{x}{\frac{6x}{40}}+\frac{x}{\frac{6x}{40}}} = \frac{9}{80} \Rightarrow \frac{x}{\frac{2x}{\frac{6x}{40}}} = \frac{9}{80} \Rightarrow$ $\frac{x}{\frac{80x}{6x}} = \frac{9}{80} \Rightarrow \frac{x}{\frac{80}{6}} = \frac{9}{80} \Rightarrow$ $\frac{6x}{80} = \frac{9}{80} \Rightarrow 6x = 9 \Rightarrow x = 3/2$
37	3/6622	44 cards total, three 6s, two 12s, one 18 (3C1 x 2C1 x 1C1)/44C3 or 1/44 x 2/43 x 3/42 x 6
38	[C =] 46	When S = 6, there are three terms, $15x^4y^2$, $16x^3y^3$, and $15x^2y^4$. 15 + 16 + 15 = 46

39	37/90	All sums created this way are multiples of 11. Any that are 2-digit are palindromes. The only 3-digit multiple of 11 that is a palindrome and that is less than 99 + 99 is 121. All the rest of the 3-digit sums are not palindromes. 1 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 37, so 37/90.
40	[n =] 10	b must be 4, 5, 6, 7, 8, or 9 Check them all $123_4 = 27$, $123_5 = 38$, $123_6 = 51$, $123_7 = 66$, $123_8 = 83$, and $123_9 = 102$, so all of them meet the first condition. Then check $321_4 = 57$, $321_5 = 86$, $321_6 = 121$, $321_7 = 162$, $321_8 = 209$, and $321_9 = 262$. It only meets the second condition when b = 9 and $(10 + 6)^2 + 6 = 262$. So, n = 10.
41	13	$-9x+9y+8z=155$ and $-9(-x-2y+5z=83)$ added together is $27y-37z=-592$ $2(-x-2y+5z=83)$ and $2x-5y+5z=74$ added together is $-9y+15z=240$ $-3(9y+15z=240)$ and $27y-37z=-592$ added together is $8z = 128$, so $z = 16$ $9y+15(16)=240$ means that $y=0$ $-x-2(0)+5(16)=83$ means that $x=-3$ $-3 + 0 + 16 = 13$
42	[d =] 117	From the first equation you can see that b x c must equal 8 and by solving the second equation you get that b + c = a. So, when b = 1 and c = 8, then a = 9. When b = 2 and c = 4, then a = 6. Swapping the values of b and c in each of these cases gives the same value of a. Since $a^2 = d$, there are two possible values of d, 6^2 and 9^2 and 36 + 81 = 117.
43	36/7 [ft]	Let ? = x and the right and left segments that add up to 6 be y and 6 – y Then x/12 = y/6 and x/9 = (6 - y)/6 So 6x = 12y and 6x = 54 - 9y 12y = 54 - 9y y = 18/7 x/12 = 18/42 x = 216/42 = 36/7

44	13 [cm]	Solve for the radius of the sphere $(4/3)\pi r^3 = 2304\pi$ $r = 12$ If x is the side length of the cube, then $x\sqrt{3} = 24$ and $x = 8\sqrt{3}$ 8 x 1.7 = 13.6 So, the largest integer less than 13.6 is 13
45	$\sqrt{3}/3$ or $\frac{1}{\sqrt{3}}$	Points chosen on the circle result in right triangle, points outside the circle result in acute triangles, points inside the circle result in obtuse triangles. The height of the triangle is 1 so half of a side is 1/rt 3, meaning the perimeter is 6/rt 3. The parts of $\overline{AO}$ and $\overline{BO}$ that are inside the semicircle are each of length 1. To get the answer simplify 2/(6/rt 3). Points O, C, and the other two intersections of the triangle and the semicircle are dimensionless, so they do not effect the probability

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Multiple Choice Solutions

6th	Answer	Solution
1	E	A(-6, 8) and A'(-6, -8) From 8 to -8 is 16 units
2	D	C(-4, 4), C'(4, 4), C"(4, -2) CC" = $\sqrt{(-4-4)^2+(4--2)^2}$ = $\sqrt{64+36}$ = $\sqrt{100}$ = 10
3	C	B(-9, 6), D(-5, 7), B"'(-2, -4), and D"'(2, -3) are the vertices of the quadrilateral. It can be surrounded by an 11x11 square and there are 2-7x10 triangles and 2-1x4 triangles that are subtracted from this square to get the area of the quadrilateral, 121 – 70 – 4 = 47.
4	C	6,929,725,043 in scientific notation is 6.929725043×10^9 and if the decimal number is rounded to the nearest hundredth, it would be 6.93×10^9
5	A	Only the number of internet users in the year 2016 has digits that add to a multiple of 3: 3 + 4 + 2 + 4 + 9 + 7 + 1 + 2 + 3 + 7 = 42, a multiple of 3 2001: 31, 2004: 40, 2007: 49, 2010: 31, 2013: 41, not multiples of 3
6	B	According to the equations, the number of internet users is increasing annually by 13.2% and the world population is increasing annually by 1.2% 13.2/1.2 = 11
7	B	(0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), and (6, 6) are all of the doubles. There are 7 out of 28 total tiles. 7/28 = 1/4
8	C	There are a total of 8 of each number from 0 to 6, so 8(0 + 1 + 2 + 3 + 4 + 5 + 6) = 8(21) = 168

9	B	To calculate the number of tiles in a double-nine set: there are 10 tiles in the list (0, 0), (0, 1), (0, 2), . . . , (0, 9), 9 tiles in the list (1, 1), (1, 2), (1, 3), . . . , (1, 9), 8 tiles in the list (2, 2), (2, 3), (2, 4), . . . , (2, 9), and so on until the last list is the one tile (9, 9). So the total is 1 + 2 + 3 + . . . + 8 + 9 + 10 = 55
10	A	To calculate the number of tiles in a double-twelve set: there are 13 tiles in the list (0, 0), (0, 1), (0, 2), . . . , (0, 12), 12 tiles in the list (1, 1), (1, 2), (1, 3), . . . , (1, 12), 11 tiles in the list (2, 2), (2, 3), (2, 4), . . . , (2, 12), and so on until the last list is the one tile (12, 12). So the total is 1 + 2 + 3 + . . . + 11 + 12 + 13 = 91 The probability of drawing these two tiles is 1/91 x 1/90 x 2 = 1/4095

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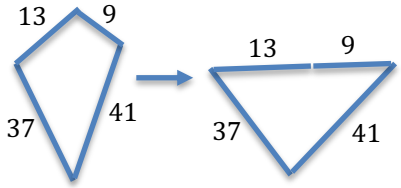
Middle School

Team Test Solutions

6th	Answer	Solution
1	5.5 [hours]	165/55 + 175/70 = 5.5
2	11 [prime numbers]	23,29,31,37,41,43,47,53,59,61,67
3	17/28	(4/7 + 9/14)/2 = 17/28
4	38 [Ruby-Throated Hummingbirds]	5/12 of 228 is 95 Ruby-Throated Hummingbirds, so there are 133 Bee Hummingbirds. 133 – 95 = 38
5	441	21^2 = sum of 1st 21 odd integers = 441, or just add them.
6	22 [feet]	If the segments of 13 and 9 are collinear, then the greatest distance would be 9 + 13 = 22 
7	3003 [dogs]	1% = 115.5, 2% = 231, 10% = 1155, 50% = 5775, 52% = 6006, half of 6006 is 3003
8	$1499.80 or 1499 dollars and 80 cents	65/100 x orig. price = 974.87 Orig. price = 100 x 974.87/65 = 1499.80
9	$14234_{[6]}$	21_3 = 7, 32_4 = 14, 43_5 = 23 7x14x23 = 2254 = 1 x 1296 + 4 x 216 + 2 x 36 + 3 x 6 + 4 x 1 = 14234_6

10	72 [paths]	ABCDE, ABEDC, ACBDE, ACBED, ACDBE, ACDEB, ADCBE, ADEBC, AEBCD, AEBDC, AEDBC, AEDCB – 12 that start with A There are also 12 that start with B CABDE, CABED, CADBE, CADEB, CAEBD, CAEDB, CBADE, CBAED, CBDAE, CBDEA, CBEAD, CBEDA, CDABE, CDAEB, CDBAE, CDBEA, CDEAB, CDEBA – 18 that start with C There are also 18 that start E DACBE, DAEBC, DBCAE, DBEAC, DCABE, DCAEB, DCBAE, DCBEA, DEABC, DEACB, DEBAC, DEBCA – 12 that start with D

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Middle School

Relay Solutions

6th	Answer	Solution
1-1	100	20 x 5 = 100
1-2	10000	$100^2 = 10000$
1-3	100001	100001
1-4	33333.7 [inches]	$100001/3 \approx 33333.7$
2-1	46 [centimeters]	2(15 + 8) = 46
2-2	45	1,2,23.46 46 – 1 = 45
2-3	[x =] 21	3.5/7.5 = x/45 x = 21
2-4	3 [values]	1+2+3+4=5+6 → X = 6 6+7+8 → X = 3 10+11 → X = 2 3 values

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College Bowl Round #1 Solutions

6th	Answer	Solution
1	[x=] 23	3x+16=85 3x=69 x=23
2	4/25 or "4 over 25" or "4 out of 25"	2/5x2/5=4/25
3	196	7x28=196
4	42 [cents]	9D – (3P+3N+3D) 90 – (3+15+30)=42
5	60 [in^2]	$8^2 + b^2 = 17^2$, b = 15 8x15/2 = 60
6	32768	$8^5 = 2^{15} = 2^{10}x2^5 = 1024x32 = 32768$
7	3	# of vertices of a nonagon is 9 and a tetrahedron has 6 edges, so 9 – 6 = 3
8	9 [days]	18 – 3 + 1 – 3 + 1 – 3 + 1 – 3 + 1 – 3 + 1 – 3 + 1 – 3 + 1 – 3 + 1 – 3 + 1 = 18 – 2(9) = 0, 9 days
9	40 [cups]	4C=1Qt, 4Qt=1Gal, 16C=1Gal, 40C=2.5Gal
10	20 [lbs]	35A = 14D, 5A = 2D, 20A = 8D, 20 lbs

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College Bowl Round #2 Solutions

6th	Answer	Solution
1	3/10 or "3 over 10" or "3 out of 10"	P(3 blue in a bag with 10 total) = 3/10
2	East	90x7=630=360+270 Starting from north, 270 CCW = East
3	146 [cents]	6+25+40+75=146
4	20 [diagonals]	n(n-3)/2 8(5)/2=20
5	18 [slices]	d = 2ft, r = 12 in, A = $12^2\pi = 144\pi$, $144\pi/8\pi = 18$ slices
6	[x=] 16	x + y = 50, 2x + 2= y, x + 2x + 2 = 50, 3x = 48, x = 16
7	5:20 am	60x33=1980 → 2000 min = 33 hrs, 20 min → 33 hrs, 20 min – 24 hrs = 9 hrs, 20 min → 8 PM + 9 hrs, 20 min = 5:20 am
8	4/11 or "4 over 11" or "4 to 11"	10-39 is 30 numbers 8 primes: 11, 13, 17, 19, 23, 29, 31, 37 The rest (= 22) are composite 8/22 = 4/11
9	8 [multiples]	22, 44, 66, 88, 110, 132, 154, 176 = 8 multiples
10	5 [%]	7/12x6/25x5/14 = (7x6x5)/(12x25x14) = (1x1x1)/(2x5x2) = 1/20 = 5%

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College Bowl Round #3 Solutions

6th	Answer	Solution
1	4.2 [times]	21/5 = 4.2
2	95	97 – 2 = 95
3	18	3^6 = 729, 7+2+9=18
4	11 [people]	C = 100π ≈ 314 11 < 314/28 < 12 So 11 people
5	14	ab = 147, a = 3b → $3b^2$ =147 → b^2 = 49 → b = 7 and a = 21 21 – 7 = 14
6	102 [pages]	85 = 20%, 8.5 = 2%, 17 = 4%, 170 = 40%, 187 = 44% 187 – 85 = 102
7	19/26 or "19 out of 26" or "19 over 26"	6 red face cards, 4 aces, 4 twos = 14 winning cards and 38 losing cards 38/52 = 19/26
8	330	LCM(60, 150) = 300 GCF(60, 150) = 30 300 + 30 = 330
9	[0].45	11/5 – 7/4 = (44 – 35)/20 = 9/20 = 0.45
10	672/673 or "672 over 673"	(2016/3)/(2019/3) = 672/673

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College Bowl Round #4 Solutions

6th	Answer	Solution
1	10	6 x 99 = 594, the closest multiple of 11 less than 604 604 – 594 = 10
2	228	19(3 + 21)/2 = 228
3	[x =] 11	15x – 11 = 154 15x = 165 x = 11
4	115 [%]	p/100(7000)=8050 p=8050(100)/7000 = 115
5	10	5^1, 5^2, 5^3, and $5^4 < 1000 \rightarrow N = 1 + 2 + 3 + 4 = 10$
6	20 [factors]	$648 = 2^3 \times 3^4$ Total number of factors = (3 + 1)(4 + 1) = 20
7	1 [triangle]	The only possible triangle has sides 2, 3, and 3. The sum of the two shorter sides must be greater than the third side.
8	7 [dates]	Possible Thursdays include: 22nd thru 28th, so 7 dates. The 29th could never be the 4th Thursday, but could be the 5th Thursday.
9	119	7/12 + 2/5 = (35 + 24)/60 = 59/60 59 + 60 = 119
10	20 [palindromes]	22, 33, 44, 55, 66, 77, 88, 99, 101, 111, 121 131, 141, 151, 161, 171, 181, 191, 202, 212 – 20 palindromes

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College Bowl Round #5 Solutions

6th	Answer	Solution
1	6 [years old]	¼ of 24 = 6 He'll be 30 in 24 years, which is 5 times 6.
2	11/15 [beats per second] or "11 over 15"	44 beats in 60 sec = 44/60 beats per sec = 11/15
3	50	A/B = C/D, B = 5A, C is a 2-digit number → $A/5A = 1/5 = C/D$ The smallest 2-digit number C can be is 10, so D = 50
4	$38.04 or 38 dollars and 4 cents or thirty-eight-oh-four	4 x 2.99 = 11.96 50 – 11.96 = 38.04
5	42 [inches]	3-4-5 Pythagorean triple 56 = 4(14), 70 = 5(14), so the other leg is 3(14) = 42
6	11/40 or "11 out of 40" or "11 over 40"	3 x 13 = 39, so 13 multiples of 3. Multiples of 3 that are also multiples of 5 are multiples of 15. There are 2 multiples of 15 less than 40. 13 – 2 = 11 → $11/40$
7	14 [prime factors]	Just count the prime numbers less than 46: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 → 14 prime factors
8	16 [oranges]	6B = 10A and 5A = 12O → $10A = 24O \rightarrow 6B = 24O \rightarrow 4B = 16O$
9	9 [ways]	20P, 15P 1N, 10P 2N, 10P 1D, 5P 1D 1N, 5P 3N, 4N, 2N 1D, 2D = 9
10	23	11 → 2, 13 → 4, 17 → 8, 19 → 10, 23 → 5, so 23 is the answer

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College Bowl Round #6 Solutions

6th	Answer	Solution
1	[x =] 7	400x – 600 = 2200 400x = 2800 x = 7
2	16/81	(2/3)^4 = 16/81
3	35 [M&Ms]	Minimum = .15(140) = 21 5 x 21 = 105 140 – 105 = 35
4	5.6	(2 + 3 + 5 + 7 + 11)/5 = 5.6
5	$25_{[9]}$	$23_{10} = 2(9^1) + 5(9^0) = 25_9$
6	2 [hours]	3G = 5H = 18000A 2G = 5H = 12000A 2G = 1H = 2400A 2G = 2H = 4800A So, 2 hrs
7	784π [cm^3]	$14^2\pi(4) = 784\pi$
8	4 [minutes]	.9(150) = 135 540/135 = 4 min
9	2/13 or "2 out of 13" or "2 over 13"	4 Jacks + 4 Queens = 8 cards 8/52 = 2/13
10	12 [centimeters]	A = bh/2 and b = 2h → 2h(h)/2 = 36 → $h^2 = 36$ → h = 6 → b = 12

"Math is Cool" Masters -- 2018-19

Middle School

College Bowl Round (Extra) Solutions

6th	Answer	Solution
1	[x =] 24	6x – 11 = 4x + 37 2x = 48 x = 24
2	56 [centimeters]	$\sqrt{196}$ = 14 14 x 4 = 56
3	24 [ordered pairs]	(26, 24), (27, 23), . . . , (48, 2), (49, 1) 49 – 25 = 24 pairs
4	11/12	(6 – -5)/(3 - -9) = 11/12
5	39601	(200 – 1)(200 – 1) = 40000 – 2(200)(1) + 1 = 40000 – 400 + 1 = 39601
6	11/45	12 to 96 = 4(3) to 4(24), so 22 multiples of 4 out of 90, 22/90 = 11/45
7		
8		
9		
10		