Mental Math

#	Answer	Solution
1	54	9×6 = 54
2	20 [units]	5×4 = 20
3	18	2, 6, 10, 14, ? Differ by 4, so 14 + 4 = 18
4	32 [cowboys]	(8 C/3B)(12B) = (12/3)×8 = 4×8 = 32
5	20 [minutes]	15 mi/h = 15 mi/60 min = 5 mi/20 min
6	3 [dollars]	3S + S = 12 → S = 12/4 = 3
7	Monday	157 ≡ (157 – 140) (mod 7) = 17 ≡ 3 (mod 7) Friday + 3 days = Monday
8	5	20 = 4×5 = 2^2×5, so need to multiply by 5 to get 2^2×5^2

Individual Contest

#	Answer	Solution
1	69	Find smallest number
2	[$] 9.70	$4.38 + $5.32 = $9.70
3	6	54/9 = 6
4	1250	3897 – 2647 = 1250
5	21 [vitamins]	3×7 = 21
6	28.489	32.034 – 3.545 = 28.489
7	3,794,605	Write it out, translating words to numbers.
8	3/4	1 – 2/8 = 6/8 = 3/4
9	[x =] 452	Rearrange & solve: 374 – 78 = 452 = x
10	0, $\frac{1}{4}$, 2, 5	Sort into order
11	[$] 6.05	$10 – ($3.75 + $0.20) = 10 – 3.95 = $6.05

#	Answer	Solution
12	986 [Knuts]	(29 K / S)×(17 S/G)×(2 G) = 29×17×2 = 986
13	30.8 [pounds]	4.4×7 = 30.8
14	42	14 = 2·7, 21 = 3·7; LCM = union of prime factorization = 2·3·7 = 42
15	18	factors = 1, 17 → 1 + 17 = 18
16	34 [weights]	170/5 = 34
17	7,800 [polar bears]	0.3×26000 = 3×(0.1×26000) = 3× 2600 = 7800
18	24 [cubs]	10×2 + 3 + 1 = 24
19	101	2020 = 2×2×5×101 → 101 is largest
20	2/11	4 + 7 + 11 = 22 → 4/22 = 2/11
21	16	|5.5 – 17.5| + 6/3 × (9 – 7) = 12 + 2(2) = 16
22	40 [units]	12 + 12 + 8 + 8 = 40
23	19 [°C]	12 – (-7) = 19
24	1,843 [miles]	375 + 423 + 314 + 406 + 325 = 1843
25	100 [words]	25×(60/20)×10 = 750 and 17×(60/12)×10 = 850, so… 850 – 750 = 100
26	9 [feet]	(11 + 12 + 8 + 10 + x)/5 = 10 → 41 + x = 50 → x = 9
27	3/16 [kg]	(45.5 – 0.5) / (8×30) = 45/240 = 3/16
28	7 [pieces]	n(n+1)/2 + 1 = 3(3 + 1)/2 + 1 = 3×2 + 1 = 7 http://oeis.org/A000124
29	5	$2^{15} = 8^x = 2^{3x}$ so… 15 = 3x → x = 5
30	36 [ways]	stars & bars: # ways to divide 10 coins into 3 bins = # ways to choose two dividers from 9 spaces = ${}_9C_2 = C(9,2)$ = 9×8/2 = 9×4 = 36
31	280 [seconds]	In 1 minute, together they can paint 1/7 + 1/14 = 3/14 of the wall. Thus, they will need 14/3 minutes to paint the wall. (14/3)×60 = 280 seconds

#	Answer	Solution
32	$132_{[4]}$	Add a leading zero, then split into 01 11 10. Now, convert each 2-digit pair into its base 4 equivalent and write that sequence together to get 132
33	15	Solve |3@(4@2)| in pieces: |4@2 = 4*4 – 4*2| = 8 → |3@8 = 3*3 – 3*8| = 15
34	243	$(x^3 \cdot y^3)/8 = 729$ → $x^3 \cdot y^3 = 729 \cdot 8 = 9^3 \cdot 2^3 = (3^2)^3 \cdot 2^3 = 3^3 \cdot 3^3 \cdot 2^3$ $= 27 \cdot (27 \cdot 8) = 27 \cdot 216$ So, one solution is 729 + 8 = 737, and another solution is 27 + 216 = 243, which is the answer
35	634	$\Sigma\ 1 + 2 + \ldots + 40 = (1 + 40)(40)/2 = 41 \times 20 = 820$ $\Sigma\ 1 + 2 + \ldots + 11 = (1 + 11)(11)/2 = 6 \times 11 = 66$ 820 - 66 = 754 $LCM(12,40) = LCM(2^2 \cdot 3, 2^3 \cdot 5) = 8 \cdot 3 \cdot 5 = 120$ 754 – 120 = 634
36	[$] 108	$C > 0$, $M > 0$, $C + M = 11$, $8C + 13M$ = amount spent $13C + 8M = 123$ → $13C + 8(11 - C) = 13C - 8C + 88 = 123$ → $5C = 123 - 88 = 35$ → $C = 35/5 = 7$ → $11 - 7 = 4 = M$ $8C + 13M = 8(7) + 13(4) = 56 + 52 = 108$
37	4 [zeroes]	To make a 0 at the end of a number, we need pairs of 2 and 5. Factoring 20! would give many 2s (e.g., 2, 8, 16, etc.). So, 5s are the limiting factor. We have 5s in 5, 10, 15, and 20. There are no other factors of 5, so the number will end in 4 zeros. Calculationally, we can do: $floor(20/5^1) = 4$, then check the next power of 5 with $floor(20/5^2) = 0$; so, we can stop there and know that we have 4 zeros.
38	28	Because 68 is even and 2 is the only even prime number, we are looking for a prime other than 2. Trial and error leads to: 61 + 7 = 68 and 31 + 37 = 68, then: 6 + 1 + 7 + 3 + 1 + 3 + 7 = 28

#	Answer	Solution
39	60 [factors]	Get prime factorization of 5040 by dividing by small prime numbers to find that $5040 = 2^4 \cdot 3^2 \cdot 5^1 \cdot 7^1$, where all exponents are written explicitly. To calculate the number of factors, add one to each exponent in the prime factorization and multiply those numbers: $(4 + 1)(2 + 1)(1 + 1)(1 + 1) = 5 \cdot 3 \cdot 2 \cdot 2 = 15 \cdot 4 = 60$
40	182 [degrees]	Every 5-minute span on the clock equates to a 30° arc/sector. At 7:22, the hour hand is partway between 7 and 8, while the minute hand is partway between 4 and 5. So, in the smaller angle formed, there are two 5-minute sectors (from 5 to 7) plus two fractional portions. We can use the fraction of the hour and the fraction of minutes between 4 and 5 to determine the two fractional portions and add it all together: $30 + 30 + (22/60) \cdot 30 + (3/5) \cdot 30 = 60 + 11 + 18 = 89$ Now find the difference: $(360 - 89) - 89 = 360 - 180 + 2 = 182$

Multiple Choice Contest

#	Answer	Solution
1	D	in order: 12, 22, 28, 36, 50, so median = 28
2	C	36/50 = 72/100 → decrease of 28%
3	A	4×4 + 4×[(4 + 10)/2]×6 = 16 + 4×7×6 = 16 + 4×42 = 16 + 168 = 184
4	B	There are 2×3×3 = 18 possible combinations of flowers/colors. Of those, 2 combinations (Red-Pink-Pink) and (Yellow-Pink-Pink) have two flowers of the same color. So there are 18 – 2 = 16 possible arrangements.
5	E	7×7 + 1×4 + 3×5 + 4×3 = 80, which is not a listed answer
6	B	4×3 / 80 = 12/80 = 3/20 = 15/100 = 15%
7	D	permutations = 6! / (3!·2!) = 6·5·4 / 2 = 3·20 = 60
8	A	max – min = (4×7 + 3×5) – (4×3 + 3×4) = 43 – 24 = 19
9	A	7 + 5 + 4 + 3 = 19 b + s = 3+7 = 10 b + g = 3 + 5 = 8 b + p = 3 + 4 = 7 s + g = 7 + 5 = 12 s + p = 7 +4 = 11 g + p = 5 + 4 = 9 avg. = 19 + (10 + 8 + 7 + 12 + 11 + 9) / 6 = 19 + 9.5 = 28.5
10	C	60×(7 + 5 + 4 + 3) = 60×19 = 1140 60×(3 + 5) + (60/2)×7 + (60/2)(7/2) + (60×7/10)×4 +(60×3/10)(4/2) = 480 + 210 + 105 + 168 + 36 = 999 999/1140 ≈ 10/11 = 0.909090… → 90%

Team Contest

#	Answer	Solution
1	36	$12\times9/3 = 4\times9 = 36$
2	18 [years old]	$4/2 = 2$ so, difference = 2 → 20 – 2 = 18
3	3[:00] AM	$9 + 9 + 9 = 27 \equiv 3 \pmod{24}$
4	26 [m^2]	$(4 + 1 + 1)\times2 + 7\times2 = 12 + 14 = 26$
5	1 or 1:1 or 1/1	Same base and height means the unshaded areas are the same, so the total of the shaded areas in each figure are the same.
6	12 [days]	$\frac{7\ e}{3\ h \times 4\ d} \times \frac{8\ h}{56\ e} = \frac{1}{12\ d}$
7	1504	$3@7 = (3 + 7)\cdot3^2 = 90$, then… $4@90 = (4 + 90)\cdot4^2 = 94\times16 = 940 + 470 + 94 = 1504$
8	126 [times]	hundreds digit: 100 times tens digit: 10 + 3 = 13 times ones digit: 1 + 10 + 2 = 13 times → 100 + 13 + 13 = 126
9	2178	There is no carry, so A must be 1 or 2. Trying both, only 2 works for D in ABCD to give A in DCBA. Then just need to work out B and C by trial and error.
10	1/12	$1/2 \times 1/6 = 1/12$

Relay #1

#	Answer	Solution
1-1	99	**63**, 72, **81**, 90, **<u>99</u>**
1-2	54	99 → 9 + 9 = 18 and 36/12 = 3, so 3×18 = 54
1-3	5 [weeks]	54 + (-2·7 + 2)*n = 0 → 54 = 12n → n = 54/12 = 4R6 round up to 5
1-4	162 [$units^2$]	$(5+4)\times18 = 9\times18 = 180 - 18 = 162$

Relay #2

#	Answer	Solution
2-1	7	6×7 = 42 and 12 + 23 = 35 → 42 – 35 = 7
2-2	[$] 84[.00]	3×7×4 = 21×4 = 84
2-3	[$] 21[.00]	84×0.25 = 84×1/4 = 21
2-4	111 [degrees]	90 + 21 = 111

College Bowl – Round #1

#	Answer	Solution
1-1	27	9 + 9 + 9 = 27
1-2	625	Sequence is powers of five: 5, 25, 125, 625
1-3	"two-thirds" or "two over three"	A 3, 4, 5, or 6 is rolled on a six-sided die with probability of 4/6, which simplifies to 2/3.
1-4	64 [percent]	16/25 is equal to 64/100 = 64%
1-5	185 [days]	(31 - 5) + 31 + 30 +31 +30 + 31 + 6 = 5×30 + 9 + 26 = 150 + 35 = 185
1-6	560 [dollars]	800 × 0.7 = 7×80 = 560 dollars
1-7	"one-sixth" or "one over six"	18 pencils in all, 3 Statler → 3/18 = 1/6
1-8	12 [ways]	4! / 2! = 12 (divide by 2! to account for 2 identical "S"s)
1-9	2	17×25 = 425 → Tens digit = 2
1-10	"three-eighths" or "three over eight"	(1/2)·(3/4) = 3/8

College Bowl – Round #2

#	Answer	Solution
2-1	126	14 × 9 = 140 – 14 = 136
2-2	420 [minutes]	60 × 7 = 420
2-3	48 [legs]	12 × 4 = 48
2-4	4 [balloons]	7 × 60 = 420 seconds → 420/3 = 140s/balloon, so… 560/140 = 4
2-5	408 [feet]	12×17×2 = (170 + 34)×2 = 408
2-6	12 [rocks]	LCM(1,2,3,4) = 12
2-7	"two-fifths" or "two over five"	6 + 7 + 2 = 15 total → 6/15 = 2/5
2-8	24	LCM(12, 8) = 24
2-9	38 [pints]	4 × 8 = 32 and 3 × 2 = 6, so… 32 + 6 = 38
2-10	15	5 × 3 = 15

College Bowl – Round #3

#	Answer	Solution
3-1	5 [minutes]	320/64 = 320/32/2 = 10/2 = 5
3-2	206	573 – 367
3-3	512 [cu. units]	$8\times8\times8 = 2^9 = 64\times8 = 512$
3-4	7 [prime numbers]	Primes are: 23, 29, 31, 37, 41, 43, 47 → 7 numbers
3-5	729 [cents]	$(3\times3\times3)\times27 = 27\times27 = 9\times9\times9 = 729$
3-6	14 [cats]	18×2 = 36 = max # chickens, but need to reconcile with # heads 64 – 36 = 28 → 28/2 = 14
3-7	12,321 or "twelve thousand three-hundred twenty-one"	111 × 111 = 12321 (note the pattern in the palindrome)
3-8	36 pi [sq. units]	$A = \pi R^2 = 6^2\pi = 36\pi$
3-9	93 [meals]	3×31 = 93
3-10	107	321 / 3 = 107

College Bowl – Round #4

#	Answer	Solution
4-1	2880 [minutes]	60×24×2 = 1440×2 = 2880
4-2	9 [mandarin oranges]	28/3 = 9 R1
4-3	6 [nights]	(32 – 8)/4 = 6
4-4	19	Sequence difference: +1, +2, +3, +4, …, so add 5 +14 = 19
4-5	121	sum of first n odd #s = n^2 → 11^2 = 121
4-6	6 [cars]	38/7 = 5 R3 → round up to 6
4-7	4,000 [cu. cm]	20×20×10 = 4000
4-8	30 [minutes]	120/4 = 30
4-9	3	689/7 = 98 R3
4-10	8 [years]	It doesn't matter how many years pass, they will always have the same difference.

College Bowl – Round #5

#	Answer	Solution
5-1	13 [weeks]	91/7 = 13
5-2	21 [hours]	7×3 = 21
5-3	"two dollars and sixty-one cents" or "two point six one"	5 – 2.39 = 2.61
5-4	4 [units]	$\sqrt{8} \times \sqrt{2} = 2\sqrt{2} \times \sqrt{2} = 4$
5-5	9	Average = 48/4 = 12, so the four numbers are 9, 11, 13, and 15. Smallest is 9.
5-6	52	GCF = 4, LCM = 48 48 + 4 = 52
5-7	"two-fifths" or "two over five" or 40%	(8/10)×(5/10) = (4/5)×(1/2) = 2/5 (= 40%)
5-8	59 [seconds]	118/2 = 59
5-9	90 [miles]	3×30 = 90
5-10	14	(8 + 21 + 13)/3 = 42/3 = 14

College Bowl – Round #6

#	Answer	Solution
6-1	"two fifths" or "two over five"	2 out of 5 colors = 2/5
6-2	17	|-5 – 12| = 12 – (-5) = 17
6-3	25 [percent]	5/20 = 1/4 = 25%
6-4	5 [diagonals]	n(n – 3)/2 = 5(2)/2 = 5
6-5	6 [coins]	4 dimes + 2 nickels = 50 cents
6-6	5	Add digits of the number, subtracting 9s as you go to come up with 5 remaining
6-7	15 [percent]	72/480 = 9/60 = 3/20 = 15/100 = 15%
6-8	47 [petals]	5×10 – 3 = 50 – 3 = 47
6-9	9	0.3 × (60/2) = 0.3 × 30 = 3 × 3 = 9
6-10	20 [feet]	6 + 6 + 4 + 4 = 20