“Math is Cool” Championships -- 2019-20

Middle School

Mental Math Solutions

6th	Answer	Solution
1	960	80 x 12 = 960
2	24 [ways]	4! = 4x3x2x1 = 24
3	5 [inches]	95/19 = 5
4	3	1/3 x 1/9 x 54 = 3
5	[N^2 =] 36	6^2 = 36
6	400	625 – 225 = 400
7	1/4	(3 – 2)/(8 – 4) = 1/4
8	25 [minutes]	48 pens in 20 min + 12 pens in 5 min = 60 pens in 25 min

"Math is Cool" Championships -- 2019-20
Middle School
Individual Test Solutions

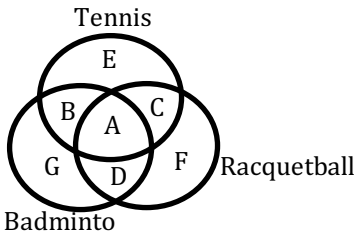
6th	Answer	Solution
1	13 [free throws]	1 x 130 = 13
2	92	7 + 17 x 5 = 7 + 85 = 92
3	5 [cups]	4/x = 16/20 → 16x = 80 → x = 5
4	441	21 x 21 = 441
5	[x =] -2	3x – 7 = 7x + 1 → -8 = 4x → -2 = x
6	25 [%]	1b + 1o + 1o + 1l makes a mixture of 4 tablespoons, one of which is balsamic vinegar, so ¼ = 25%
7	896694598	325602356 + 571092242 = 896694598
8	[D =] 119	7 x 17 = 119
9	5040	7 x 6 x 5 x 4 x 3 x 2 x 1 = 5040
10	3 [prime factors]	2^2 x 5 x 101, 3 distinct factors
11	93[°]	Corresponding angles in similar triangles are congruent so the largest angle will also be 93˚.
12	22 [inches]	506/23 = 22
13	400 [cm^2]	80/4 = 20 = side length Area = 20^2 = 400
14	13	5 + 8 = 13

15	38/7	(8 + 6 + 7 + 5 + 3 + 0 + 9)/7 = 38/7
16	5.3×10^9	$a \times 10^b$, $1 < a < 10$, so 5.3×10^9
17	49π [cm^2]	$C = 2\pi r = 14\pi$, so $r = 7$. $A = \pi r^2 = \pi \times 7^2 = 49\pi$.
18	17280	4 x 6 x 8 x 9 x 10 = 17280
19	55	16561 ÷ 131 = 126 r 55
20	187/288 [ft^2]	8.5/12 x 11/12 = 93.5/144 = 187/288
21	96	Since 9 is biggest start by writing a list of numbers meeting the first condition: 6, 15, 24. 33, 42, 51, 60, 69, 78, 87, 96, 105, 114, 123, . . . Circling the ones that also meet the 2nd condition gives 33 and 96. Of these two, only 96 also meets the third condition.
22	2112 [yards per minute]	180 mi/2.5 hr x 1 hr/60 min x 1760 yd/mi = (180 x 1760)/(2.5 x 60) yd/min = (3 x 1760)/2.5 yd/min = 5280/2.5 yd/min = 10560/5 yd/min = 2112 yd/min
23	77 [grams]	65/100 = 50/x → 65x = 5000 → 13x = 1000 → x = 1000/13 ≈ 76.9
24	[k =] 1/2	5 = k x 10 → k = 5/10 = ½
25	25 [units]	$\sqrt{(17 - -7)^2 + (4 - -3)^2} = \sqrt{24^2 + 7^2} = \sqrt{576 + 49} = \sqrt{625} = 25$
26	π/6 [cm] or 1/6 π	30/360 x 2π = 1/12 x 2π = 2π/12 = π/6
27	192 [cells]	$3 \times 2^6 = 192$
28	5/72	P[(1,1,5) or (1,5,1) or (5,1,1)] = $\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot 3$ = 3/216 P[(1,2,4) or (1,4,2) or (2,1,4) or (2,4,1) or (4,1,2) or (4,2,1)] = $\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot 6$ = 6/216 P[(1,3,3) or (3,1,3) or (3,3,1)] = $\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot 3$ = 3/216 P[(2,2,3) or (2,3,2) or (3,2,2)] = $\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot 3$ = 3/216 3/216 + 6/216 + 3/216 + 3/216 = 15/216 = 5/72
29	1490	3 x 4 x 5 = 60 → 8 x 9 x 10 = 720 2 x 5 x 6 = 60 → 7 x 10 x 11 = 770 720 + 770 = 1490
30	143 [°]	Number of degrees the minute hand has traveled – number of degrees the hour hand has traveled = (26/60) x 360 – (26/60) x 30 = 156 – 13 = 143
31	120	$T_n = n(n + 1)/2$ $T_{15} = 15(15 + 1)/2 = 15(16)/2 = 240/2 = 120$

32	1 [yd^3]	96 in = 8 ft, 81 in = 6 ft 9 in = 6.75 ft Volume = 8 x 6.75 x 0.5 = 27 ft^3 and 27 ft^3 = 1 yd^3
33	320 [minutes]	1/16 + 1/x = 1/4 → 16x(1/16 + 1/x = 1/4) → x + 16 = 4x → 16 = 3x → x = 16/3, 16/3 x 60 = 320
34	98 [in^2]	Diameter of the circle = 14 = diagonal of the square, so each side of the square = 14/rt 2, and the area of the square = $(14/\text{rt } 2)^2$ = 196/2 = 98
35	8990000 [phone numbers]	There are 900 three-digit numbers, one of which is 911 and none of which starts with 0, so there are 899 possibilities for the first three digits, then 10 possibilities each for the last four digits, so 899 x 10^4 = 6480000
36	18π [in^3]	Area of the base = πr^2, so r = 3 in Volume = $(2/3)\pi 3^3 = 18\pi$
37	329.5 [three-digit base-d numbers]	If d = 3 there are 2 possible values for a, and three possible values each for b and c, so there are 2 x 3 x 3 = 18 possible three-digit base-3 numbers. For base-4 there will be 3 x 4 x 4 = 48 possible three-digit numbers. Base-5: 4 x 5 x 5 = 100 Base-6: 5 x 6 x 6 = 180 Base-7: 6 x 7 x 7 = 294 Base-8: 7 x 8 x 8 = 448 Base-9: 8 x 9 x 9 = 648 Base-10: 9 x 10 x 10 = 900 (18+48+100+180+294+448+648+900)/8 = 329.5
38	36 [$units^2$]	Divide into 3 triangles and 1 rectangle 3 triangles: 6 x 3/2 = 9, 1 x 4/2 = 2, 2 x 5/2 = 5, 9 + 2 + 5 = 16; 1 rectangle: 4 x 5 = 20; 16 + 20 = 36 

39

2 [campers]



A	B	C	D	E	F	G	T	R	B	Tot.
0	0	38	4	0	0	38	38	42	42	80
0	…	…	…	…	…	…	38	42	42	…
0	18	20	22	0	0	2	38	42	42	62
0	19	19	23	0	0	0	38	42	42	61
0	20	18	22	0	2	0	38	42	42	62
0	…	…	…	…	…	…	38	42	42	…
0	38	0	4	0	38	0	38	42	42	80
0	0	0	42	38	0	0	38	42	42	80
0	…	…	…	…	…	…	38	42	42	…
0	18	18	24	2	0	0	38	42	42	62
0	19	19	23	0	0	0	38	42	42	61
0	20	18	22	0	2	0	38	42	42	62
0	…	…	…	…	…	…	38	42	42	…
0	38	0	4	0	38	0	38	42	42	80
0	18	20	22	0	0	2	38	42	42	62
0	0	38	4	0	0	38	38	42	42	80

This table shows that 0 is impossible, because the smallest possible total number is 61. The table was generated by first making sure that A + B + C + E = 38 and that A + C + D + F = 42. Going through all the possibilities that B + C = 38 shows that the smallest possible total occurs when B = C (rows 1 - 7). Similarly, if you start by making sure that B + D = 42, the smallest possible total occurs when B = C. So, when checking whether 1 is possible the table can be shorter, just looking for the times when B is as close to C as possible:

A	B	C	D	E	F	G	T	R	B	Tot.
1	17	20	21	0	0	3	38	42	42	62
1	18	19	22	0	0	1	38	42	42	61
1	19	18	22	0	1	0	38	42	42	61
1	20	17	21	0	3	0	38	42	42	62
1	17	17	24	3	0	0	38	42	42	62
1	18	18	23	1	0	0	38	42	42	61
1	19	18	22	0	1	0	38	42	42	61
1	20	17	21	0	3	0	38	42	42	62

Again, 61 is the smallest possible total.

When A = 2, a total of 60 is possible when B = C:

A	B	C	D	E	F	G	T	R	B	Tot.
2	18	18	22	0	0	0	38	42	42	60

So, 2 is the answer.

40	14 [ordered triples]	<table><tr><th>Regular</th><th>Broken</th></tr><tr><td>1 + 2 ^ 9</td><td>1 ^ 2 + 9 = 10</td></tr><tr><td>. . . . .</td><td>. . . . .</td></tr><tr><td>1 + 8 ^ 9</td><td>1 ^ 8 + 9 = 10</td></tr><tr><td>2 + 1 ^ 8</td><td>2 ^ 1 + 8 = 10</td></tr><tr><td>3 + 1 ^ 7</td><td>3 ^ 1 + 7 = 10</td></tr><tr><td>3 + 2 ^ 1</td><td>3 ^ 2 + 1 = 10</td></tr><tr><td>4 + 1 ^ 6</td><td>4 ^ 1 + 6 = 10</td></tr><tr><td>6 + 1 ^ 4</td><td>6 ^ 1 + 4 = 10</td></tr><tr><td>7 + 1 ^ 3</td><td>7 ^ 1 + 3 = 10</td></tr><tr><td>8 + 1 ^ 2</td><td>8 ^ 1 + 2 = 10</td></tr></table> There are 7 from 1 + 2 ^ 9 to 1 + 8 ^ 9, plus the other 7 listed so there are 14 possibilities.

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Middle School

Multiple Choice Solutions

6th	Answer	Solution
1	D	13 x 13 = 169
2	E	6 x 12 : 3 x 11 = 72:33 = 24:11
3	B	$\frac{4 \cdot 7.5}{2} \cdot 12 = 180$
4	A	Let p = side length of the pentagon and h = side length of the heptagon (ap/2 x 5)/(ah/2 x 7) = 55/182 → (5.5p/2 x 5)/(10.4h/2 x 7) = 55/182 → 27.5p/72.8h = 55/182 → p/h = (72.8/27.5)(55/182) = (2x72.8/182) = 145.6/182 = 1456/1820 = 728/910 = 364/455 = (91 x 4)/(91 x 5) = 4/5
5	B	2.5 x 4 = 10
6	E	$\frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \frac{1}{6} \cdot 3 + \frac{1}{6} \cdot 4 + \frac{1}{6} \cdot 5 + \frac{1}{6} \cdot 6 = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6} = 3.5$
7	C	$\frac{1}{36} \cdot 2 + \frac{2}{36} \cdot 3 + \frac{3}{36} \cdot 4 + \frac{4}{36} \cdot 5 + \frac{5}{36} \cdot 6 + \frac{6}{36} \cdot 7 + \frac{5}{36} \cdot 8 + \frac{4}{36} \cdot 9 + \frac{3}{36} \cdot 10 + \frac{2}{36} \cdot 11 + \frac{1}{36} \cdot 12 = \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36} = \frac{252}{36} = 7$
8	D	4 records were set in July and 4 were set in August, more than any other month
9	A	(2.7 + 0.4 + 1.4 + 3.7 + 5.4 + 1.0 + 2.5 + 2.04 + 0.39)/9 = 19.53/9 = 2.17

10	D	6/28/76 Women's WR holder: 3(60) + 55 = 235 seconds to complete 1500 meters 6/28/76 Women's WR holder: 3(60) + 56 = 236 seconds to complete 1500 meters 236 – 235 = 1 so the woman would have to continue running for 1 second Her rate in meters per second would be 1500/236 m/s so the remaining number of meters to run would be 1 x 1500/236 = 750/118 = 375/59

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Middle School

Team Test Solutions

6th	Answer	Solution
1	10.75 [feet]	129/12 = 10r9 = 10.75
2	135 [min]	2x45 = 90 90 + 45 = 135
3	2/5	There are 15 numbers and 6 of them are even, and 6/15 = 2/5.
4	36 [units^2]	6x6 = 36
5	7 [cm]	LxW = 98, 1x98, 2x49, 7x14 are the only three factor pairs of 98. The one resulting in a perimeter of 42 is 7x14, since 2x7 + 2x14 = 42, so the answer is 7.
6	16888	2000 + 2002 + 2020 + 2200 + 2022 + 2202 + 2220 + 2222 = 16888
7	2	Since a + b = 100, c = 80. The smallest value of a must be 21 so that b is still less than c. c/a would be 4 if a were 20, so that's not possible. 80 isn't divisible by 3, so it must be 2.
8	3750 [cm^3]	Let L = 2x, W = 3x, H = 5x, then 1550 = 2(6x^2) + 2(10x^2) + 2(15x^2) = 62x^2 → x^2 = 25 → x = 5 So L = 10, W = 15, H = 25 Volume equals 10 x 15 x 25 = 3750
9	45	The middle numbers must have 40 as their average. The two smallest numbers must be as large as possible, so 37, 38, 39, 41 must be the smallest four numbers. The largest number will be 21 more than the smallest number and the 2nd largest number will be 1 less than the largest number. 37,38,39,41,57,58 The mean of this set = 45

10	9 [pathways]	ACDB ACEDB ACEB ACDEB AECDB AEDB AEB AECDEB AEDCEB

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Relay Solutions

6th	Answer	Solution
P-1	12	.15 x 80 = 12
P-2	144	$12^2 = 144$
P-3	8	17 x 8 = 136 144 – 136 = 8
P-4	2	2 x 2 x 2 = 8
1-1	5	2, 4, 8, 16, 32
1-2	125	5 x 5 x 5 = 125
1-3	25	.2 x 125 = 25
1-4	13	Factors of 52 are 1 x 52, 2 x 26, and 4 x 13. The largest prime factor is 13.
2-1	2.5	50/20 = 2.5
2-2	10	2.5 x 4 = 10
2-3	50	10,20,30,40,50,60,70,80,90, the median is also the mean
2-4	8	1^2 + 7^2 = 50, so 1 + 7 = 8

"Math is Cool" Championships -- 2019-20

Middle School

College Bowl Round #1 Solutions

6th	Answer	Solution
1	3750	300 x 12.5 = 3750
2	36π [cm^2]	$\pi(6)^2 = 36\pi$
3	5 [prime numbers]	31, 37, 41, 43, 47
4	7740 [seconds]	2 hours x 3600 seconds/hour = 7200 seconds + 9 x 60 = 7740 sec
5	-1/2 or "-1 over 2" or "1 over -2"	Slope formula is $(y_2 - y_1)/(x_2 - x_1)$ (-2 - 4)/(3 - -9) = -6/12 = -1/2
6	60.75 [in^2]	13.5/3 = 4.5 13.5 x 4.5 = 60.75
7	26/51 or "26 out of 51" or "26 over 51"	$\frac{13}{18} \times \frac{12}{17} = \frac{156}{306} = \frac{26}{51}$
8	199	199 is prime
9	60	Prime factorization of 24 is $2^3 \times 3^1$; $(2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1)$ = 15*4 = 60 OR 1 + 2 + 3 + 4 + 6 + 8 + 12 + 24 = 60
10	70 [percent]	45 is 30% of 150 minutes. 100% - 30% = 70%

“Math is Cool” Championships -- 2019-20

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College Bowl Round #2 Solutions

6th	Answer	Solution
1	5 [socks]	There are 4 colors so the fifth sock will have to match one of them.
2	52 [degrees]	90 – 38 = 52
3	2	.2 x .1 x 100 = 2
4	24 [inches]	6 + 8 + 10 = 24
5	364 [times]	24/6 = 4 7 x 4 x 13 = 362
6	18 [hot dogs]	2 x 3 x 3 = 18
7	6 [factors]	48 – 1, 2, 3, 4, 6, 8, 12, 16, 24, 48 60 – 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 common factors – 1, 2, 3, 4, 6, 12
8	24 [ways]	4! = 24
9	32 [cups]	4 C = 1 Qt, 4Qt = 1G, 16 C = 1 G, 32 C = 2G
10	1088	$33^2 = 1089$ 1089 – 1 = 1088

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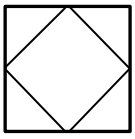
College Bowl Round #3 Solutions

6th	Answer	Solution
1	8	The median is also the mean, which is 8
2	20 [lawns]	35/7 = 5, 4 x 5 = 20
3	8 [hours]	12 x 14 = 168 168/21 = 8
4	7 [threes]	$3^6 = 729$, $3^7 > 1000$
5	23 [memes]	They will both like 1 (0 x 35 + 1), 36 (1 x 35 + 1), . . . , 736 (21 x 35 + 1), 771 (22 x 35 + 1) 0 through 22 makes 23
6	[x =] 900	915 + x = 2x + 15 x = 900
7	32 [in^2]	The inside square has half the area of the outside square. 64/2 = 32 
8	1131.2	.56 x 2020 = 1131.2
9	27 [diagonals]	n(n – 3)/2 9(9 – 3)/2 = 27
10	120 [penguins]	2 12-hr periods per day 30 x 2 = 60, 60 x 2 = 120

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College Bowl Round #4 Solutions

6th	Answer	Solution
1	7	1, 3, 4, 7, 8, 15, 22 The median is 7
2	2.2 [cups]	1/3 = x/6.6 x = 2.2
3	20	8/(2/5) = 8 x 5/2 = 20
4	8800	5 x 4 x 440 = 8800
5	10 [cm]	The only perfect square, other than 1, that 90 is evenly divisible by is 9, so the area of the base would be 9. Then the height would be 10, since 9 x 10 = 90.
6	12 [inches]	The formula for the area of a trapezoid is (a + b)/2 x h 132 = (8 + 14)/2 x h 132 = 11h h=12
7	360 [ways]	6!/2! = 720/2 = 360
8	14	9 x 5 = 45, 9 – 5 = 4, 9 + 5 = 14
9	4	18 x 1/3 x 2/3 = 4
10	12	$276 = 2^2 \times 3 \times 23$ $84 = 2^2 \times 3 \times 7$ $GCF = 2^2 \times 3 = 12$

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College Bowl Round #5 Solutions

6th	Answer	Solution
1	3235	3600 - 365 = 3235
2	36 [songs]	3 songs/month x 12 months/year = 36 songs/year
3	7/13 or “7 over 13” or “7 out of 13”	Half the deck is black, including the eights of spades and clubs, and there are two red eights for (26+2)/52 = 28/52 = 7/13
4	[x=] -7	17x + 45 = 4x – 46 13x = -91 x = -7
5	40 [%]	D-bound train goes 180 miles in 2hrs. C-bound train goes 120 miles in 2hrs. So they meet after 2 hrs. It will take the C-bound train 300/60 = 5 hrs to make the trip. 2 is 2/5 of 5 or 40%.
6	693	$53^2 – 46^2 = (53 + 46)(53 – 46) = 99 \times 7 = 693$
7	2 [books]	5! = 120, 5!/2! = 60 so there must be two identical books
8	5 [centuries]	$20^3 = 8000$ 8000/16 = 500 = 5 centuries
9	[$] 46000	81000 – 35000 = 46000
10	567 [followers]	7, 21, 63, 189, 567 (x3 each time)

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College Bowl Round #6 Solutions

6th	Answer	Solution
1	42 [faces]	7 x 6 = 42
2	11	2, 3, 5, 7, 11 – 5th smallest
3	5/36 or "5 out of 36" or "5 over 36"	(2, 6), (3, 5), (4, 4), (5, 3), (6, 2) = 5 ways out of 36 total
4	12 [miles]	63360/5280 = 12
5	4 [shirts]	1sh = 3h, 2sh = 6h, 2h = 5so , 6h = 15so, 2sh = 15 so, 4sh = 30so
6	1/15 or "1 out of 15" or "1 over 15"	.6 x 30 = 18, 12 left 2/3 of 12 = 8, 4 left 4 – 2 = 2 left 2/30 = 1/15
7	3645 [iPhones]	5000 x .9 = 4500 4500 x .9 = 4050 4050 x .9 = 3645
8	3/5 or "3 out of 5" or "3 over 5"	Since the events are independent, just convert the percent to a fraction, 60% = 3/5
9	71	8 + 7 x 9 = 71
10	156 [hours]	It's a non-leap year so there will be 28 days, which means 20 weekdays and 8 weekend days. 20 x 5 + 8 x 7 = 156

“Math is Cool” Championships -- 2019-20

Middle School

College Bowl Round (Extra) Solutions

6th	Answer	Solution
1	9 [days]	14 chapters in 2 days = 7 chapters per day 77/7 = 11 days total 11 – 2 = 9 days to go
2	[$] 4.50 or “four fifty” or “four dollars and fifty cents”	5 x 1.2 = 6 6 x .75 = 4.50
3	39	1 + 2 + 3 + 6 + 9 + 18 = 39
4	200 [in^2]	10 x 40/2 = 200
5	7/150 or “7 out of 150” or “7 over 150”	97500/325 = 300 = total number of pages 21 x 14 = 294 so there are 14 page numbers that are multiples of 21. The answer is 14/300 = 7/150
6	20 [feet]	$20^2 + 21^2 = 29^2$ so the answer is 20.