"Math is Cool" Championships -- 2019-20
Middle School
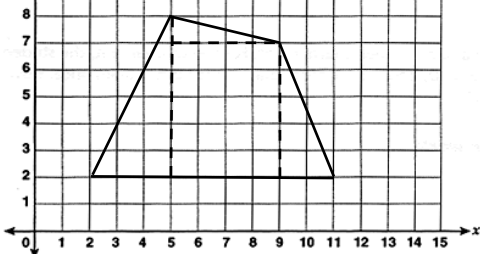## <u>Mental Math Solutions</u>

7th/8th	Answer	Solution
1	780	13 x 60 = 780
2	24 [ways]	4! = 4x3x2x1 = 24
3	5 [inches]	95/19 = 5
4	3	1/3 x 1/9 x 54 = 3
5	9 [edges]	3 on each base and 3 lateral edges = 9
6	20 [minutes]	(2/3) x 30 = 20
7	1/4	(3 − 2)/(8 − 4) = 1/4
8	2244	(62-40)(62+40)=2244

"Math is Cool" Championships -- 2019-20
Middle School
Individual Test Solutions

7th/8th	Answer	Solution
1	13 [free throws]	1 x 130 = 13
2	92	7 + 17 x 5 = 7 + 85 = 92
3	5 [cups]	4/x = 16/20 → 16x = 80 → x = 5
4	441	21 x 21 = 441
5	[x =] -2	3x – 7 = 7x + 1 → -8 = 4x → -2 = x
6	25 [%]	1b + 1o + 1o + 1l makes a mixture of 4 tablespoons, one of which is balsamic vinegar, so ¼ = 25%
7	896694598	325602356 + 571092242 = 896694598
8	[D =] 119	7 x 17 = 119
9	5040	7 x 6 x 5 x 4 x 3 x 2 x 1 = 5040
10	3 [prime factors]	2^2 x 5 x 101, 3 distinct factors
11	8 [in^2]	4x4/2 = 8
12	300 [in^2]	5 x 60 = 300
13	76540 [ft^2]	(7 + 13)7654/2 = (20 x 7654)/2 = 10 x 7654 = 76540
14	5	5 x 5 x 5 = 125

15	90 [°]	Hour hand points at 12 and minute hand points at 3. There are 360/12 or 30 degrees between each consecutive number. 3 x 30 = 90
16	197	$98 = 2 \times 7^2$ and $99 = 3^2 \times 11$, so they are relatively prime 98 + 99 = 197
17	15 [inches]	$\sqrt{17^2 - 8^2} = \sqrt{289 - 64} = \sqrt{225} = 15$
18	17280	4 x 6 x 8 x 9 x 10 = 17280
19	55	16561 ÷ 131 = 126 r 55
20	187/288 [ft²]	8.5/12 x 11/12 = 93.5/144 = 187/288
21	96	Since 9 is biggest start by writing a list of numbers meeting the first condition: 6, 15, 24. 33, 42, 51, 60, 69, 78, 87, 96, 105, 114, 123, . . . Circling the ones that also meet the 2nd condition gives 33 and 96. Of these two, only 96 also meets the third condition.
22	2112 [yards per minute]	180 mi/2.5 hr x 1 hr/60 min x 1760 yd/mi = (180 x 1760)/(2.5 x 60) yd/min = (3 x 1760)/2.5 yd/min = 5280/2.5 yd/min = 10560/5 yd/min = 2112 yd/min
23	77 [grams]	65/100 = 50/x → 65x = 5000 → 13x = 1000 → x = 1000/13 ≈ 76.9
24	[y =] 75	5 x 300 = 20y → 1500 = 20y → 1500/20 = 75 = y
25	2.4 [feet]	$10^2 - 6^2 = 8^2$, so h = 8 $12^2 - 6^2 = \sqrt{108}^2$ $H = \sqrt{108} = 6\sqrt{3} \approx 6 \times 1.732 \approx 10.392 \approx 10.4$ 10.4 − 8 = 2.4
26	5π/6 [cm] or 5/6 π	C = πd, so C = 10π, so the arc length is 1/12 x 10π = 5π/6
27	6.25 [revs per second]	In 8 minutes it will be cut in half four times 100, 50, 25, 12.5, 6.25
28	-15	7(-3) − 3(-2) = -15
29	1	2x − 1 = 3 → 2x = 4 → x = 2 and 2x − 1 = -3 → 2x = -2 → x = -1; so -1 + 2 = 1
30	2048π/3 [in³] or 2048/3 π	$S = 4\pi r^2 = 256\pi \rightarrow r^2 = 64 \rightarrow r = 8$ $V = 4/3\pi r^3 = 4/3\pi 8^3 = 4\pi \times 512/3 = 2048\pi/3$

31	1/4	P(Amy gets red on her 1st draw) = 1/16 P(Amy gets red on her 2nd draw) = 15/16 x 14/15 x 13/14 x 12/13 x 1/12 = 1/16 P(Amy gets red on her 3rd draw) = 15/16 x 14/15 x 13/14 x 12/13 x 11/12 x 10/11 x 9/10 x 8/9 x 1/8 = 1/16 P(Amy gets red on her 4th draw) = 15/16 x 14/15 x 13/14 x 12/13 x 11/12 x 10/11 x 9/10 x 8/9 x 7/8 x 6/7 x 5/6 x 4/5 x 1/4 = 1/16 P(Amy gets red on her 1st, 2nd, 3rd, or 4th draw) = 1/16 + 1/16 + 1/16 + 1/16 = 4/16 =1/4
32	240π [cm^3]	$5^2 + r^2 = 13^2 \rightarrow r^2 = 144 \rightarrow r = 12$ $V = \pi r^2 h/3 = 144\pi(5)/3 = 240\pi$
33	320 [minutes]	$1/16 + 1/x = 1/4 \rightarrow 16x(1/16 + 1/x = 1/4) \rightarrow x + 16 = 4x \rightarrow 16 = 3x \rightarrow x = 16/3$, 16/3 x 60 = 320
34	216 [cm^2]	L = 3W, H = 2L, so H = 6W W x 3W x 6W = $18W^3 \rightarrow 18W^3 = 144 \rightarrow W^3 = 8 \rightarrow W = 2$, L = 6, H = 12, so surface area = 2(2)(6) + 2(2)(12) + 2(6)(12) = 24 + 48 + 144 = 216
35	512 [ways]	9 squares and two possibilities for each square so $2^9 = 512$
36	$\frac{75\sqrt{3}}{2}$ [in^2]	There are 6 equilateral triangles, each with side lengths of 5 inches. The area of one triangle is 5^2(rt3)/4, so 6 x 25rt3/4 = 75rt3/2
37	329.5 [three-digit base-d numbers]	If d = 3 there are 2 possible values for a, and three possible values each for b and c, so there are 2 x 3 x 3 = 18 possible three-digit base-3 numbers. For base-4 there will be 3 x 4 x 4 = 48 possible three-digit numbers. Base-5: 4 x 5 x 5 = 100 Base-6: 5 x 6 x 6 = 180 Base-7: 6 x 7 x 7 = 294 Base-8: 7 x 8 x 8 = 448 Base-9: 8 x 9 x 9 = 648 Base-10: 9 x 10 x 10 = 900 (18+48+100+180+294+448+648+900)/8 = 329.5
38	36 [units^2]	Divide into 3 triangles and 1 rectangle 3 triangles: 6 x 3/2 = 9, 1 x 4/2 = 2, 2 x 5/2 = 5, 9 + 2 + 5 = 16; 1 rectangle: 4 x 5 = 20; 16 + 20 = 36

39

2 [campers]

Tennis, E, B, C, A, G, F, Racquetball, D, Badminton
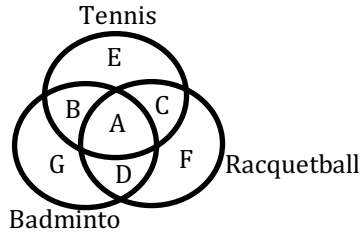
A	B	C	D	E	F	G	T	R	B	Tot.
0	0	38	4	0	0	38	38	42	42	80
0	...	...	...	...	...	...	38	42	42	...
0	18	20	22	0	0	2	38	42	42	62
0	19	19	23	0	0	0	38	42	42	61
0	20	18	22	0	2	0	38	42	42	62
0	...	...	...	...	...	...	38	42	42	...
0	38	0	4	0	38	0	38	42	42	80
0	0	0	42	38	0	0	38	42	42	80
0	...	...	...	...	...	...	38	42	42	...
0	18	18	24	2	0	0	38	42	42	62
0	19	19	23	0	0	0	38	42	42	61
0	20	18	22	0	2	0	38	42	42	62
0	...	...	...	...	...	...	38	42	42	...
0	38	0	4	0	38	0	38	42	42	80
0	18	20	22	0	0	2	38	42	42	62
0	0	38	4	0	0	38	38	42	42	80

This table shows that 0 is impossible, because the smallest possible total number is 61. The table was generated by first making sure that A + B + C + E = 38 and that A + C + D + F = 42. Going through all the possibilities that B + C = 38 shows that the smallest possible total occurs when B = C (rows 1 - 7). Similarly, if you start by making sure that B + D = 42, the smallest possible total occurs when B = C. So, when checking whether 1 is possible the table can be shorter, just looking for the times when B is as close to C as possible:

A	B	C	D	E	F	G	T	R	B	Tot.
1	17	20	21	0	0	3	38	42	42	62
1	18	19	22	0	0	1	38	42	42	61
1	19	18	22	0	1	0	38	42	42	61
1	20	17	21	0	3	0	38	42	42	62
1	17	17	24	3	0	0	38	42	42	62
1	18	18	23	1	0	0	38	42	42	61
1	19	18	22	0	1	0	38	42	42	61
1	20	17	21	0	3	0	38	42	42	62

Again, 61 is the smallest possible total.

When A = 2, a total of 60 is possible when B = C:

A	B	C	D	E	F	G	T	R	B	Tot.
2	18	18	22	0	0	0	38	42	42	60

So, 2 is the answer.

| 40 | $64\sqrt{2} - 64$ [m] | $\frac{x}{\sqrt{2}} + x + \frac{x}{\sqrt{2}} = 8 \rightarrow x\sqrt{2} + x = 8 \rightarrow x(\sqrt{2} + 1) = 8 \rightarrow x = 8/(\sqrt{2} + 1) \times (\sqrt{2} - 1)/ (\sqrt{2} - 1) = 8(\sqrt{2} - 1)$ = one side length
Perimeter = $8 \times 8(\sqrt{2} - 1) = 64(\sqrt{2} - 1) = 64\sqrt{2} - 64$

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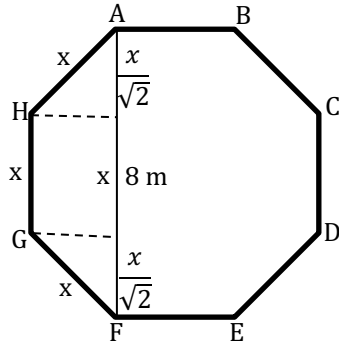
"Math is Cool" Championships -- 2019-20
Middle School
<u>Multiple Choice Solutions</u>

7th/8th	Answer	Solution
1	D	13 x 13 = 169
2	E	12 x 12 : 6 x 11 = 144:66 = 24:11
3	B	$\frac{6 \cdot 9.2}{2} \cdot 10$ = 276
4	A	Let p = side length of the pentagon and h = side length of the heptagon (ap/2 x 5)/(ah/2 x 7) = 55/182 → (5.5p/2 x 5)/(10.4h/2 x 7) = 55/182 → 27.5p/72.8h = 55/182 → p/h = (72.8/27.5)(55/182) = (2x72.8/182) = 145.6/182 = 1456/1820 = 728/910 = 364/455 = (91 x 4)/(91 x 5) = 4/5
5	B	2.5 x 4 = 10
6	E	$\frac{1}{8} \cdot 1 + \frac{1}{8} \cdot 2 + \frac{1}{8} \cdot 3 + \frac{1}{8} \cdot 5 + \frac{1}{8} \cdot 3 + \frac{1}{8} \cdot 4 + \frac{1}{8} \cdot 3 + \frac{1}{8} \cdot 2$ = $\frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{5}{8} + \frac{3}{8} + \frac{4}{8} + \frac{3}{8} + \frac{2}{8}$ = $\frac{23}{8}$ = 2.875
7	C	$\frac{1}{36} \cdot 2 + \frac{2}{36} \cdot 3 + \frac{3}{36} \cdot 4 + \frac{4}{36} \cdot 5 + \frac{5}{36} \cdot 6 + \frac{6}{36} \cdot 7 + \frac{5}{36} \cdot 8 + \frac{4}{36} \cdot 9 + \frac{3}{36} \cdot 10 + \frac{2}{36} \cdot 11 + \frac{1}{36} \cdot 12$ = $\frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36}$ = $\frac{252}{36}$ = 7
8	C	4 records were set in September, more than any other month
9	A	(2.7 + 0.4 + 1.4 + 3.7 + 5.4 + 1.0 + 2.5 + 2.04 + 0.39)/9 = 19.53/9 = 2.17

| 10 | D | Current Men's WR holder: 3(60) + 26 = 206 seconds to complete 1500 meters
6/28/76 Women's WR holder: 3(60) + 56 = 236 seconds to complete 1500 meters
236 – 206 = 30 so the slower runner would have to continue running for 30 seconds
Her rate in meters per second would be 1500/236 m/s so the remaining number of meters to run would be 30 x 1500/236 = 30 x 750/118 = 30 x 375/59 = 11250/59 |

"Math is Cool" Championships -- 2019-20
Middle School
<u>Team Test Solutions</u>

7th/8th	Answer	Solution
1	10.75 [feet]	129/12 = 10r9 = 10.75
2	276 [students]	72/6 = 12, so the total is 12x12 + 6x12 + 5x12 = 276
3	2/5	There are 15 numbers and 6 of them are even, and 6/15 = 2/5.
4	-9/2	0 = 2/3x + 3 $\Rightarrow$ -3 = 2/3x $\Rightarrow$ -9/2 = x
5	7 [cm]	LxW = 98, 1x98, 2x49, 7x14 are the only three factor pairs of 98. The one resulting in a perimeter of 42 is 7x14, since 2x7 + 2x14 = 42, so the answer is 7.
6	25524	3135 + 3136 + 3145 + 3146 + 3235 + 3236 + 3245 + 3246 = 25524
7	2	Since a + b = 100, c = 80. The smallest value of a must be 21 so that b is still less than c. c/a would be 4 if a were 20, so that's not possible. 80 isn't divisible by 3, so it must be 2.
8	3750 [cm^3]	Let L = 2x, W = 3x, H = 5x, then 1550 = 2(6x^2) + 2(10x^2) + 2(15x^2) = 62x^2 → x^2 = 25 → x = 5 So L = 10, W = 15, H = 25 Volume equals 10 x 15 x 25 = 3750

9	64	The 2nd smallest number is 50, so the smallest number is 49 and the largest number is 75. Therefore the 2nd largest number is 74 and the 3rd largest number is 73. These 5 numbers add up to 321, so their average is 64.2. Try making the median 64 and the 3rd smallest number 63 and the mean equals the median. 49,50,63,64,73,74,75 The mean and the median of this set = 64 If the median were 65, the total of all the numbers would have to increase by 7 in order for the mean to also be 65, but this is not possible since the only other number that can be increased is the 3rd smallest number, but it can only be increased by 1.
10	9 [pathways]	ACDB ACEDB ACEB ACDEB AECDB AEDB AEB AECDEB AEDCEB

7th/8th	Answer	Solution
P-1	12	.15 x 80 = 12
P-2	144	12^2 = 144
P-3	8	17 x 8 = 136 144 – 136 = 8
P-4	2	2 x 2 x 2 = 8
1-1	2.5	50/20 = 2.5
1-2	10	2.5 x 4 = 10
1-3	50	10,20,30,40,50,60,70,80,90, the median is also the mean
1-4	8	1^2 + 7^2 = 50, so 1 + 7 = 8
2-1	4	8 – 4 = 4
2-2	3	143/4 = 35r3 The remainder is 3.
2-3	28	3 x 47 = 141 13^2 = 169 169 – 141 = 28
2-4	12.25π [cm^2]	28/4 = 7 so the diameter of the circle is 7 and the radius is 3.5. 3.5^2π = 12.25π

"Math is Cool" Championships -- 2019-20
Middle School
College Bowl Round #1 Solutions

7th/8th	Answer	Solution
1	3750	300 x 12.5 = 3750
2	36π [cm^2]	$\pi(6)^2 = 36\pi$
3	147 [minutes]	7 cups x 3 minutes x 7 days = 147 minutes
4	7740 [seconds]	2 hours x 3600 seconds/hour = 7200 seconds + 9 x 60 = 7740 sec
5	-1/2 or "-1 over 2" or "1 over -2"	Slope formula is $(y_2 - y_1)/(x_2 - x_1)$ (-2 - 4)/(3 - -9) = -6/12 = -1/2
6	12 [inches]	The diagonal of a square is n root 2 which means the sides are all 3. 3x4 = 12 OR 3 + 3 + 3 + 3 = 12
7	26/51 or "26 out of 51" or "26 over 51"	$\frac{13}{18} \times \frac{12}{17} = \frac{156}{306} = \frac{26}{51}$
8	78	.3 x 260 = 78
9	60	Prime factorization of 24 is $2^3 \times 3^1$; $(2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1) =$ 15*4 = 60 OR 1 + 2 + 3 + 4 + 6 + 8 + 12 + 24 = 60
10	540 [degrees]	180(n - 2) where n is the number of sides; 180 x 3 = 540 degrees

"Math is Cool" Championships -- 2019-20
Middle School
College Bowl Round #2 Solutions

7th/8th	Answer	Solution
1	5 [socks]	There are 4 colors so the fifth sock will have to match one of them.
2	28.75 [dollars] or "twenty-eight seventy-five"	1.15 x 25 = 28.75
3	-9/5 or "-9 over 5" or "9 over -5"	a/b → -b/a so 5/9 → -9/5
4	24 [inches]	6 + 8 + 10 = 24
5	364 [times]	24/6 = 4 7 x 4 x 13 = 362
6	3 [videos]	1000 x 1.4 = 1400 1400 x 1.4 = 1960 1960 x 1.4 > 2000
7	2 [factors]	221 – 1, 13, 17, 221 850 – 1, 2, 5, 10, 17, 25, 34, 50, 85, 170, 425, 850 common factors – 1, 17
8	24 [ways]	4! = 24
9	32 [cups]	4 C = 1 Qt, 4Qt = 1G, 16 C = 1 G, 32 C = 2G
10	1088	33^2 = 1089 1089 – 1 = 1088

"Math is Cool" Championships -- 2019-20
Middle School
College Bowl Round #3 Solutions

7th/8th	Answer	Solution
1	8	The median is also the mean, which is 8
2	192π [in^3]	r = 4, $4^2\pi$ x 12 = 192π
3	120	5! = 120
4	7 [threes]	3^6 = 729, 3^7 > 1000
5	23 [memes]	They will both like 1 (0 x 35 + 1), 36 (1 x 35 + 1), . . . , 736 (21 x 35 + 1), 771 (22 x 35 + 1) 0 through 22 makes 23
6	-7	x^2 + 7x – 120 = 0 (x + 15)(x – 8) = 0 x = -15 and x = 8 -15 + 8 = -7
7	7/25 or "7 out of 25" or "7 over 25"	Total area of the board = $5^2\pi = 25\pi$ $4^2\pi - 3^2\pi = 7\pi$ P(inside 4, outside 3) = $7\pi/25\pi$ = 7/25
8	1131.2	.56 x 2020 = 1131.2
9	27 [diagonals]	n(n – 3)/2 9(9 – 3)/2 = 27
10	120 [penguins]	2 12-hr periods per day 30 x 2 = 60, 60 x 2 = 120

"Math is Cool" Championships -- 2019-20
Middle School
<u>College Bowl Round #4 Solutions</u>

7th/8th	Answer	Solution
1	7	1, 3, 4, 7, 8, 15, 22 The median is 7
2	2.2 [cups]	1/3 = x/6.6 x = 2.2
3	[x =] 7	3x + 2 = 23 3x = 21 x = 7
4	8800	5 x 4 x 440 = 8800
5	2 [cm]	The largest perfect square that 800 is divisible by is 400, so the base would be 400. Then the height would be 2, since 400 x 2 = 800.
6	Friday	9 x 5 = 45 and 2 x 11 = 22, so the first whole week takes 67 hours. 110 − 67 = 43 43/9 = $4\frac{7}{9}$, so it will be on the fifth day of the week, Friday.
7	127 [billion dollars] or [$] 127,000,000	1 + 2 + 4 + 8 + 16 + 32 + 64 = 127 billion
8	14	9 x 5 = 45, 9 − 5 = 4, 9 + 5 = 14
9	4	18 x 1/3 x 2/3 = 4
10	12	276 = 2^2 x 3 x 23 84 = 2^2 x 3 x 7 GCF = 2^2 x 3 = 12

College Bowl Round #5 Solutions

7th/8th	Answer	Solution
1	10	3600/360 = 10
2	1155	3 x 5 x 7 x 11 = 1155
3	7/13 or "7 over 13" or "7 out of 13"	Half the deck is black, including the eights of spades and clubs, and there are two red eights for (26+2)/52 = 28/52 = 7/13
4	21 [in^2]	If the diagonal is 5 rt 2, then the sides are all 5 and the area would be 25, so 25 – 4 = 21
5	40 [%]	D-bound train goes 180 miles in 2hrs. C-bound train goes 120 miles in 2hrs. So they meet after 2 hrs. It will take the C-bound train 300/60 = 5 hrs to make the trip. 2 is 2/5 of 5 or 40%.
6	693	$53^2 – 46^2 = (53 + 46)(53 – 46) = 99 \times 7 = 693$
7	September	3/13 date 1 The 100th date will be 2(99) = 198 days later 3/14 - 3/31 = 18 days April = 30 days May = 31 days June = 30 days July = 31 days August = 31 days 18 + 30 + 31 + 30 + 31 + 31 = 170, 198 – 170 = 28 and September has more than 28 days, so the answer is September.
8	5 [centuries]	20^3 = 8000 8000/16 = 500 = 5 centuries
9	[$] 46000	81000 – 35000 = 46000
10	567 [followers]	7, 21, 63, 189, 567 (x3 each time)

"Math is Cool" Championships -- 2019-20
Middle School
College Bowl Round #6 Solutions

7th/8th	Answer	Solution
1	42 [faces]	7 x 6 = 42
2	[$] 159	265 x 3 = 795 795/5 = 159
3	5/36 or "5 out of 36" or "5 over 36"	(2, 6), (3, 5), (4, 4), (5, 3), (6, 2) = 5 ways out of 36 total
4	12 [miles]	63360/5280 = 12
5	84 [square inches]	$7^2 + 24^2 = 25^2$ so it's a right triangle 7 x 24/2 = 84
6	1/15 or "1 out of 15" or "1 over 15"	.6 x 30 = 18, 12 left 2/3 of 12 = 8, 4 left 4 − 2 = 2 left 2/30 = 1/15
7	3645 [iPhones]	5000 x .9 = 4500 4500 x .9 = 4050 4050 x .9 = 3645
8	9 [numbers]	209 = 11 x 19 297 = 11 x 27 27 − 18 = 9 multiples of 11
9	71	8 + 7 x 9 = 71
10	[x =] [0].15	2x/3 + 1/5 = 3/10 20x + 6 = 9 20x = 3 x = 3/20 = 0.15

"Math is Cool" Championships -- 2019-20
Middle School
College Bowl Round (Extra) Solutions

7th/8th	Answer	Solution
1	9 [days]	14 chapters in 2 days = 7 chapters per day 77/7 = 11 days total 11 – 2 = 9 days to go
2	[$] 4.50 or "four fifty" or "four dollars and fifty cents"	5 x 1.2 = 6 6 x .75 = 4.50
3	39	1 + 2 + 3 + 6 + 9 + 18 = 39
4	200 [in^2]	10 x 40/2 = 200
5	7/150 or "7 out of 150" or "7 over 150"	97500/325 = 300 = total number of pages 21 x 14 = 294 so there are 14 page numbers that are multiples of 21. The answer is 14/300 = 7/150
6	20 [feet]	$20^2 + 21^2 = 29^2$ so the answer is 20.