“Math is Cool” Championships -- 2019-20

High School

Mental Math Solutions

	Answer	**Solution**
1	20 [cm]	
2	3	Slope formula
3	9	Do the cubed root first then squared
4	$4\sqrt{3}$ [cm]	30-60-90
5	256	
6	1000	Difference of squares
7	180	Factorial, then divide by the amount of repeat letters
8	(4, -18)	-b/2a for x and insert to find the y

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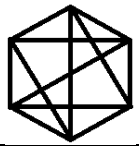
Individual Test Solutions

	Answer	Solution
1	9/19	
2	155	18^2-13^2=(18+13)(18-13)=31*5=155 Or just subtract.
3	$16\sqrt{3}$	
4	32π [units^2]	Half the area of a circle with radius 8
5	2160 [double stitches]	210 yd = 630 ft = 7560 in. 7560 / (7/2) = 2*1080 = 2160 It is faster to divide by 7/2 before converting yards to inches, in which case you have 60 * 3 * 12 = 60 * 36 = 2160
6	-9	(-3)(x^4) + (x)(-6x^3)
7	672	n + (n + 1) + (n + 2) = 2019. 3n + 3 = 2019. n = 672.
8	4/21	
9	17	$7 = 3x - y$ $8 = 5x - 2y$ Subtract twice the first equation from the second to get: $6 = 1x \rightarrow y = 11$
10	1849	One trick for numbers between 25 and 50 is that x^2 = 100(x-25) + (50-x)^2. In this case, 1800+7^2 = 1849.
11	(-7, -5)	5x-14x-7y+7y=0+63 -> -9x=63 -> x=-7, y=-5
12	5	X cancels out of the lower equation; solve it for a and use that to solve the upper equation.

13	2/51	P(both red) = 4/18 * 3/17 = 2/51.
14	[$] 32.50	25 * $1.30 = 32.50
15	2	Area and height are linearly related.
16	11/15 [hours]	1/3 hr + 2/5 hr.
17	-7/6	c/a, or factor into $2(6x-7)(x+1)=0$
18	256/625	(4/5)^4=256/625
19	14 [polygons]	There is only one configuration of diagonals that satisfies both conditions, and it divides the interior into 10 triangles and 4 kites. 
20	-11	-3, 1, -2, -1, -3, -4, -7, -11
21	20/9 [hours]	(Child A * Child B)/(Child A + Child B) = (4 * 5)/(4 + 5).
22	5	215 is a product of the primes 43 and 5. Neither 215 or 1 have a digit sum of 7, while 43 does, which requires A to be 43 and B to be 5.
23	-16	-1, 2, -4, 8, -16, 32.
24	8 [units]	Complete the square in x: $(x+6)^2+y^2=28+36=64$
25	5y + 40	xy + 5y – xy - 5x - 5 + 5x - [8x - 40 – 13x -5 + 5x] = xy – xy + 5y - 5x + 5x - 5 - 8x - 5x + 13x + 40 + 5 = 5y + 40
26	1188	12+16+...+92+96= (1/2)22*(12+96)= 11*108=1188
27	Grande [pizza]	Area for tall is 9pi, grande is 16π, venti is 25π. $10/9π = $0.35, $15/16π = $0.30, $25/25π = $0.32.
28	13,608	Mean = 8, Mode = 9, Median = 9, Range = 21.

29	1 [factors]	Only happens in numbers of the form p^x, for prime p and positive integer x.
30	16 [quarts]	32 ounces equals 1 quart.
31	140/27 [days]	Knowing that pump A drains 1/10 in a day, we can set up a system of two equations in two variables: $$\frac{1}{10}+\frac{1}{b}=\frac{1}{7}, \frac{1}{10}+\frac{1}{c}=\frac{1}{4}$$ Add the two equations to each other to get $$\frac{1}{5}+\frac{1}{b}+\frac{1}{c}=\frac{1}{7}+\frac{1}{4}$$ $$\frac{1}{b}+\frac{1}{c}=\frac{1}{7}+\frac{1}{4}-\frac{1}{5}=\frac{27}{140}$$ This is the equation that describes B and C alone draining the reservoir. Each day they drain 27/140, therefore the whole job takes 140/27 days, and we never had to solve for b or c.
32	925	$P(n) = (3n^2-n)/2$
33	48	Divide both sides by $\log_2 x \log_4 x \log_6 x$ first to make it easier. Then use the log identity $$\frac{1}{\log_a b}=\log_b a$$
34	11/64	7, 8, 9, and 10 mL are all success states. Adding their probabilities together goes like this. $$\frac{\frac{10!}{3!\,7!}+\frac{10!}{2!\,8!}+\frac{10!}{1!\,9!}+\frac{10!}{0!\,10!}}{2^{10}}=\frac{120+45+10+1}{2^{10}}=\frac{176}{1024}$$ $$=\frac{88}{512}=\frac{44}{256}=\frac{22}{128}=\frac{11}{64}$$
35	8	a+b=16, b+c=17, a+c=9. If you add all three equations together $$2a+2b+2c=42$$ $$a+b+c=21$$ We straightforwardly acquire from there that a,b,c=12,4,5.
36	$\frac{1}{4}$	There are either 2 or 3 tails, so our sample space is TTH, THT, HTT, and TTT. One result with more tails out of four possible equally-likely states.
37	6 [factors]	Equivalent to asking for the positive integer factors of 300/15=20, which are 1, 2, 4, 5, 10, and 20.

38	5/12	The boundary case is if the polynomial has a double root, which happens if c is such a value as to allow us to say $y = 3(x-r)^2 = 3x^2 - 8x + c$. Then $$3(x^2 - 2rx + r^2) = 3x^2 - 8x + c$$ $$3x^2 - 6rx + 3r^2 = 3x^2 - 8x + c$$ From which we get 6r=8, or r=4/3, and then c=16/3. Therefore, any c below 16/3 results in real roots.
39	106/31	Consider the sequence as a … sequence … of chunks, divided by the repetition in the pattern of numerators. By factoring out the first five terms from the entire sequence, you can express the series like so $$\left(1 + \frac{1}{32} + \frac{1}{1024} + \cdots\right)\left(\frac{1}{1} + \frac{2}{2} + \frac{4}{4} + \frac{2}{8} + \frac{1}{16}\right)$$ And then use the infinite geometric series formula to get a solvable expression. $$= \frac{1}{\left(1 - \frac{1}{32}\right)} * \frac{53}{16} = \frac{32}{31} * \frac{53}{16} = \frac{106}{31}$$
40	4 [boxes]	The hammer's torque is 3 lb * 4 ft = 12 ft-lbs = 144 in-lbs. The first box of nails exerts 1 * 48 in-lbs., and each successive box applies 8 fewer in-lbs. 48+40+32+24 = 144.

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Multiple Choice Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	C	The units digits of 7 to any power are the same 4 numbers repeated: 7, 9, 3, and 1. Divide 247 by 4 and the depending on the remainder is your answer
2	**99**	A	Quadratic formula
3	**3**	A	Follow order of operations and exponent properties
4	**4**	D	When making a triangle the two smaller sides have to add up to be larger than the largest side.
5	**5**	B	Slope formula first and then either plug in one of the coordinate points into y=mx+b or into point-slope
6	**99**	A	SA = 64pi, V = 256pi/3 SA-V = (192pi-256pi)/3
7	**7**	B	The area of a regular hexagon is $\frac{1}{2}Pa$
8	**8**	C	First find the term of 642 using the arithmetic formula and work backwards. After finding the n, plug into the arithmetic sum formula
9	**99**	D	Volume of a cone is BH/3 = 365. Convert 365 to base 8. 365 is clearly less that 64*8, so start by dividing 365/8. The dividend is the sixty-four's digit. Divide the remainder by 8 again, and the dividend is the 8's digit. The remainder is the ones digit.
10	**10**	C	Since we only ever talk about 6 rooms, treat it as a single unit, such as 1 space. If Pam cleans 1 space in 70 minutes, she cleans 1/70 of a space in 1 minute. Similarly, Angela cleans 1/55 of a space in 1 minute. Together, in 1 minute they clean $\frac{1}{70}+\frac{1}{55}=\frac{125}{3850}$ of a space. Therefore they clean the whole space in $\frac{3850}{125}$ minutes, which is 30.8.

99	**2**	D	Change 27 to a power of 3
99	**6**	B	N objects in a line is N! N objects in a circle equals (N-1)!. This particular circle can be flipped around in three-dimensional space, which halves the number of possible combinations compared to a fixed circle. Answer is 7! / 2 = 2520
99	**9**	B	There is a trick that only works for 3x3 matrices where $$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$ $$= aei + bfg + cdh$$ $$-ceg - bdi - afh$$ Notice that each term is a diagonal of the matrix, accounting for wraparound. Positive terms are negative slope and vice-versa. Most of the time, it's not much slower to decompose the 3x3 into a trio of 2x2 matrices and calculate the determinant traditionally.

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Team Test Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	1	Subtracting 4 from both sides may marginally speed up the arithmetic.
2	**99**	IV or 4	The midpoint is (0.5, -0.5)
3	**3**	15 [mph]	Assume 60 miles from home to the beach. 120 / (1 hour + x hour) = 24 mph. 120 = 24 + 24x. x = 4 hours. 60 miles / 4 hours = 15 mph
4	**4**	72 [cm^2]	12^2 /2
5	**5**	11/20	4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20
6	**99**	$6\sqrt{2}$ [units]	The formula for the distance form a point (x,y) to a line Ax+By+C=0 is: $\frac{\lvert Ax+By+C\rvert}{\sqrt{A^2+B^2}}$ $= \frac{\lvert 4-4-12\rvert}{\sqrt{1+1}} = \frac{12}{\sqrt{2}} = 6\sqrt{2}$
7	**7**	24	Substitute 2a=c for a possibly swifter calculation. $40 = (c+b)^3 = c^2 + 2cb + b^2$ $40 - 16 = c^2 + b^2 = 24$
8	**8**	13	You want y to be as large as possible as it multiplied by 18. y = 10, x = 3 works, but y = 11 has no integer solution. 10 + 3 = 13.
9	**99**	360 [ways]	There are 6!=720 ways of arranging the pages, and page four is after page 3 in exactly half of them.

10	**10**	6	Do the arithmetic-geometric sequence trick twice; the first iteration reduces it from a quadratic-geometric sequence to an arithmetic-geometric sequence. $$2S = \frac{1}{1} + \frac{4}{2} + \frac{9}{4} + \cdots$$ $$2S - S = S = 1 + \frac{3}{2} + \frac{5}{4} + \frac{7}{8} + \cdots$$ And again: $$2S = 2 + 3 + \frac{5}{2} + \frac{7}{4} + \frac{9}{8} + \cdots$$ $$2S - S = S = 4 + \frac{2}{2} + \frac{2}{4} + \frac{2}{8} + \cdots$$ $$= 4 + 2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\right)$$ $$= 4 + 2\left(\frac{\frac{1}{2}}{1 - \frac{1}{2}}\right) = 4 + 2 = 6$$
99	**2**	3/4	$$\log_{16} 8 = \frac{\log_2 8}{\log_2 16} = \frac{3}{4}$$
99	**6**	625/26	g(-5)=-25. f(-25)=625/26 Be careful handling the negatives.
99	**9**	$\frac{9\pi}{8}$	Sin and cos are equal (and thus have a difference of 0) at an argument of $\frac{\pi}{4} + \pi k, k \epsilon \mathbb{Z}$. The third positive zero will then be at $2t = \frac{\pi}{4} + 2\pi = \frac{9\pi}{4}$ $$t = \frac{9\pi}{8}$$

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Pressure Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	10	Represent the product as $(a-1)a(a+1)=990$ $a(a^2-1)=990$ $a^3-a=990$ From there notice that 990=1000-10.
2	**2**	10	The sequence is alternating signs of triangular numbers.
3	**99**	$3\frac{15}{28}$	$\frac{11}{4}*\frac{9}{7}=\frac{99}{28}$
4	**4**	$8+8\sqrt{2}$	Divide the shaded area into 2 triangles and a rectangle. Since the octagon is regular, we know the exterior angle is 45 degrees, which makes the triangles 45-45-90 right triangles with side length $\frac{4}{\sqrt{2}}=2\sqrt{2}$. The rectangle then has side lengths $2\sqrt{2}$ and 4.
5	**99**	14	The turtles in each pond could be 4-0-0: 1 way 3-1-0: 4 ways 2-2-0: 3 ways 2-1-1: 6 ways Since the turtles are distinguishable but the ponds are not, 2-2-0 is the same as 2-0-2, and we only have to worry about ordering the turtles within these configurations.
99	**3**	13	$-2+15$
99	**5**	$\sqrt{3}-1$	Though the form is slightly weirder than the normal infinite expression templates, the same strategy applies. Let the whole expression = X. Then, since the expression is infinite, the first unit of repetition is also equal to X. $2-X\sqrt{3}=X$ $X=\frac{2}{1+\sqrt{3}}\frac{1-\sqrt{3}}{1-\sqrt{3}}=\frac{2-2\sqrt{3}}{-2}=\sqrt{3}-1$

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College Bowl Round #1 Solutions

	Answer	**Solution**
1	12 [factors]	$2^2 * 5 * 101$
2	1.143	24 / 21 = 1.142857
3	[$]19.80	$20 * 0.9 * 1.1 = $19.80
4	188	1 1 2 3 5 8 13 21 34 55 89 144. 2+8+34+144= 188.
5	780 [handshakes]	40C2 = 40 * 39 / 2
6	2 [squares]	1 4 9 16 25 36 49 64 81 100
7	3 [hours]	The net change is 4-3.8 = 0.2 gallons per minute. The time is: 36/0.2 = 180 minutes, which is 3 hours
8	30	$5^5 = 3125$. 3 * 1 * 2 * 5 = 30
9	64 [sq inches]	64 inch square has a diagonal measuring 8 root 2
10	100	2 3 5 7 11 13 17 19 23

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College Bowl Round #2 Solutions

	Answer	Solution
1	35 [ways]	7C3 = 7! / (4! * 3!)
2	90	12 + 23 + 34 = 1 + 8 + 81
3	0	The remainder of 100/25 is 0.
4	3	The harmonic mean is the reciprocal of the average of the reciprocals. HM=3/(1/2+1/3+1/6)
5	9,600	The b^2 term is: $(4C2)\ (5a)^2\ (8b)^2=6(25)(64)\ a^2\ b^2=9600\ a^2\ b^2$
6	7 [numbers]	The numbers 1, 2, 3, 4, 5, 6 and 10 appear.
7	Sunday	The day after tomorrow would be Wednesday. 7 full weeks plus 4 days later would be Sunday.
8	42 [degrees]	Look at the difference in exterior angles instead: (360 / 5) - (360 / 12)= 72 – 30 = 42 degrees
9	97	97 is the only prime number
10	1	x = 5, y = 0

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College Bowl Round #3 Solutions

	Answer	Solution
1	7/8	1 – P(no tails) = 1 – 1/8 = 7/8
2	9050	It may be faster to instead view the sequence as 1 + … + 100 +100(40). =5050+4000=9050.
3	162	$3^5 = 243$, $3^4 = 81$
4	1	$x^2 - 5x + 6 > 4x - 12$ $Let\ x = 1: 1 - 5 + 6 > 4 - 12$ $2 > -8$ As the statement is true for x=1, and 1 is the smallest positive integer, the answer is one. Note that if you factorize the left side and divide by (x-3), you are losing information by dividing by a term that contains the variable.
5	36 [sq units]	This is an isosceles triangle base 19 – 7 = 12, the height is 8 – 2 = 6. The area is: A=1/2 (12)(6)= 36.
6	2332 [base 5]	6667 = 34210. 34210 = 23325.
7	$6\pi\sqrt{73}$ [units^2]	Equivalent to asking for twice the slant area of one of the cones. The slant height can be derived as the hypotenuse of a 3-8-x right triangle by the Pythagorean theorem, $2\pi rl = 2 * 3 * \sqrt{73} * \pi = 6\pi\sqrt{73}$
8	$\frac{43}{54}$	$\frac{43}{12} * \frac{2}{9} = \frac{43}{54}$
9	24 [inches]	Triangle inequality 17 + 8 > 24.
10	4 [factors]	1, 3, 673, 2019 are the factors

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College Bowl Round #4 Solutions

	Answer	Solution
1	66 [units]	6* (7+15)/2=6*11=66
2	1,010	The numbers are: n,n+1,n+2,n+3. The middle two terms add to the same as the outer two so the sum is 2,020 / 2=1,010.
3	105	Cubic root is 3, 5 and 7, respectively
4	13/10	½ + 1/3 + 1/6 = 1. 1/5 + 1/10 = 3/10
5	38	Twenty terms (4^{th} to 24^{th}) adds 20 * 1.5 = 30. The 24^{th} term is 8 + 30 = 38.
6	$150\sqrt{3}$	Radius 10, which is the side length of the six equilateral triangles that form the hexagon.
7	34,650 [ways]	11! / (4! 4! 2!)
8	76	The range is 81 – 1 = 80, the mode is 4.
9	420	LCM(2*2*3*5,2*5*7)=2*2*3*5*7=420
10	205 [cents]	1 + 5 + 10 + 25 = 41 cents. 5 sets of 41 cents equals 205 cents.

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College Bowl Round #5 Solutions

	Answer	Solution
1	24 [cm]	V = Base * height / 3
2	15 [cows]	2 * chickens + 4 * cows = 70 legs. -2 * (Chickens + cows = 20).
3	45	The square root of 2019 is 44.9.
4	August 5 or 8-5	Start with 30 * 7 equals 210. Then, +1, -2, +1, 0, +1, 0 +1 for the first 7 months from 30, which equals 212.
5	517	97 + 101 + 103 + 107 + 109 = 517
6	$\frac{63}{8}$	Common factor is $\frac{4}{3}$. $14 * \frac{3}{4} * \frac{3}{4} = \frac{63}{8}$
7	$48\sqrt{3}$ [square feet]	The hexagon is 6 equilateral triangles with side length the radius of a circle, 8 feet. Area of 1 triangle is $8^2\sqrt{3}/4$. Half the hexagon is 3 triangles.
8	10080 [ways]	8! / 2*2
9	61	Sequence is 01, 11, 21, 31, etc.
10	7	571428 are the repeating digits for 4/7. The 20^{th} digit is 7

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College Bowl Round #6 Solutions

	Answer	Solution
1	9600	
2	9 [zeros]	There are 8 factors of 5 and one of 25 and lots of factors of 2 – 9 zeroes.
3	[x =] -12 & 10	(x + 12) (x – 10) = 0
4	38	The sum of the first 50 numbers is 50 * 51 / 2 = 1275. The sum of the numbers skipped is 75. n + (n + 1) =75, n =38, n + 1 = 39.
5	39	35! through 39! results in 8 zeroes.
6	20	1 + 4 + 9 + 16 + 25 + 36 + 49 = 140.
7	17/25	The probability that the next phone sold is an iPhone is 4/5 and a Galaxy is 1/5. P(same) = (4/5) (4/5) + (1/5) (1/5) = 17/25
8	20 [feet]	One side of the rhombus is 5 feet.
9	420	2 * 2 * 3 * 5 * 7 = 420
10	125/4	0.25 * 0.125 * 1000 = 31.25

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College Bowl Round (Extra) Solutions

	Answer	Solution
1	21	-9 through 9 are 19 consecutive integers, leaving 10 and 11.
2	7	$6^1 = 0$, then follows the pattern of 3, 1, 9, 7, 5. The 30^{th} power is 7.
3	-40	-256 * 250 = 64000
4	9 [times]	5! = 120, 6! = 720, 7! = 5,040, 8! = 40,320, 9! = 362880, 10! = 3628800.
5	30	
6	77	Possible lengths of the 3^{rd} side is from 8 and 14.