Use global search and replace to change the following variable

Year: #year → 2019-20

Champs: #champs → Championships or Masters

For the individual, delete 41-45 on answer sheet and problems.

“Math is Cool” Masters -- 2019-20

High School

Mental Math Solutions

	Answer	**Solution**
1	14	Cancel first: 7*2 = 14
2	6.3×10^4	
3	120 [degrees]	(6 – 2) * 180 / 6
4	360	6! / 2
5	1	The units digit is cyclic: 7, 9, 3, 1. 28 mod 4 = 1.
6	8	$2^8 = 256$.
7	24	16 / (1 – 1/3)
8	3, −5 [any order]	Factor and solver for x: 2x^2 + 4x – 30 = 0 2(x^2 + 2x – 15) = 0 2(x – 3)(x + 5) = 0

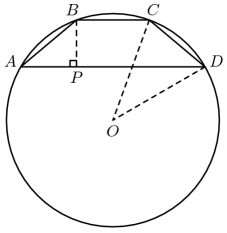
"Math is Cool" Masters -- 2019-20
High School
Individual Test Solutions

	Answer	**Solution**
1	-5	
2	1631	
3	9π [sq. inches]	
4	19	
5	75 [degrees]	$180 - 61 - 44 = 75$
6	-1	Since $m < 0$, we only need to solve $6m - 3 = -9$.
7	17	$18^2 = 324, 17^2 = 289$
8	4 [games]	Five games costs $2 + 5 * 3 = 17$, too much. Four games costs $2 + 4 * 3 = 14$, so \$15 is enough.
9	1320	$(13 - 1)(13 - 2)(13 - 3) = 1320$
10	17 [paper clips]	
11	-2	
12	2112	
13	675	
14	$\frac{5}{18}$	
15	$\frac{1}{2}$	

16	$60\sqrt{14}$	
17	$(2, 1)$	
18	18	18 comes up twice.
19	12π [cubic meters]	
20	1155	Arithmetic series has $\frac{75-35}{2} + 1 = 21$ terms, so $\left(\frac{21}{2}\right)(75 + 35) = 1155$.
21	9	There are five numbers, so $\sqrt[5]{1 \times 3 \times 9 \times 27 \times 81} = 9$.
22	$3120_{[4]}$	Convert to base 10 first, then use repeated division to convert to base 4.
23	8	Cross multiply and solve the resulting equation: $3(x - 3) = x + 7$.
24	90 [points]	12(80) = 8(75) + 4(x) or realize that 8 students is twice 4 students, so the 4 students must average twice as far from 80% as the first 8. x must be 90%.
25	20 [units]	Solve $5 + (x - 5) + (x - 4) = 10 + (x - 10) + 16$.
26	3	Factor out the 3^x in the numerator to make solving easier: $$\frac{3^x}{39}(3 + 9 + 27) = 27$$
27	$14\sqrt{2}$ [units]	Get the points of intersection by solving the system of equations. Substitute into the circle equation and solve $x^2 + (x + 2)^2 = 100$. Then use the distance formula.
28	51	We have $f(-1) = -7$ and $g(-7) = 51$.
29	56	There are C(8,5) = 56 ways to place the consonants; there is only one alphabetical ordering for them. Place vowels in remaining spots, also only one ordering.
30	36 [meters]	The altitude creates two 5-12-13 right triangles.

31	66 [outcomes]	"Stars-in-bars" problem, with $w + l + t = 10$.
32	15	Inverse is $\begin{bmatrix} 5 & -13/3 & 23/3 \\ 1 & -2/3 & 4/3 \\ 9 & -23/3 & 40/3 \end{bmatrix}$
33	9	Solve $x = \sqrt[3]{24 + x}$ by guess-and-check. We find that $x = 3$, so answer is $3 \times 3 = 9$.
34	998	Experiment with values of n just below 1000 to discover that for $n < 1000$, $f(n) = 997$ if n is even and $f(n) = 998$ if n is odd.
35	354	Rewrite equation as $(2^y - x)(2^y + x) = 615$. Guess-and-check by setting factors of 615 equal to factors of LHS of equation. $x = 59$ and $y = 6$.
36	14	Integers from 1 to 19 add up to 190. Go through values of $\frac{7k}{15k}$, add up the numerator and denominator, and get to within 190. Alternatively, if Jack's sum is $7x$, Jill's sum is $15x$, and n is the one neither picked, then $7x + 15x + n = 1 + 2 + \cdots + 19 = 190$. Solve Diophantine Equation, for $n \leq 19$.
37	$\frac{175}{256}$	Say we have two randomly made pizzas. For any one topping, there is a $\frac{3}{4}$ probability it is not common to both pizzas. Since there are four toppings per pizza, the probability of no toppings in common is $\left(\frac{3}{4}\right)^4$, so answer is $1 - \left(\frac{3}{4}\right)^4 = \frac{175}{256}$.

38	$\frac{29}{3}$	By drawing the picture, we see that the internal tangents have slopes of 2 and m, and the external tangents have a slope of 3 and 4. Let a equal the slope of the line through the centers of the two circles. By symmetric, we have the relationships $\arctan a - \arctan 3 = \arctan 4 - \arctan a$ and $\arctan a - \arctan 2 = \arctan m - \arctan a$. Use sum-and-difference formulae for tangent and reduce the equations to $$\frac{a-3}{1+3a} = \frac{4-a}{1+4a}$$ and $$\frac{a-2}{1+2a} = \frac{m-a}{1+ma}$$ Solve the system for m.
39	192π [sq. units]	Radius of circle is $\sqrt{(c-13)^2+(c-7)^2}$, which cleverly works out to be $\sqrt{2(c^2-20c+13)+192} = \sqrt{2(0)+192}$.
40	2058	The numerator of question factors to $(a+b+c)^2$.
41	84 [units]	Break up quadrilateral into two right triangles, apply Pythagorean Theorem twice.
42	32400	Number needs to be of the form $2^a 3^b 5^c$, where $(a+1)(b+1)(c+1) = 75$. Guess-and-check to find that $(a,b,c) = (4,4,2)$.
43	$\frac{25}{162}$	Choose the digit of the three-of-a-kind; choose distinct, different kind digits for the other results; then order the results: $C(6,1) * C(5,2) * \left(\frac{5!}{3!}\right) = 1200$ is the numerator, 6^5 is the denominator.

44	72	Notice that the expression is equivalent to $(12-1)^{2019}+(12+1)^{2019}$, and when expanding via the Binomial Theorem, we get something of the form $2(\text{STUFF}+2019*12)$, where "STUFF" has factors of 12^2 and above. Thus, that part of the expression has remainder of 0 upon division by 144. The remaining portion, $2*2019*12$ has remainder of 72 when divided by 144.
45	50 [units]	Without loss of generality, let quadrilateral be $ABCD$, and let AB, BC, and CD be the congruent sides. Refer to diagram below:  Since the angles hit the same arc $\widehat{CD}$, $m\angle BAP = m\angle COD$. Use the Law of Cosines on triangle COD to obtain $\cos\angle COD = \frac{3}{4}$. Use SOH-CAH-TOA on triangle ABP to obtain $AP = AB\cos\angle BAP = 20\left(\frac{3}{4}\right) = 15$. By symmetry, $AD = 15+20+15 = 50$.

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Multiple Choice Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	C	Solve to get $x = k = 6$. Substitute this value into $3k^2/4$.
2		E	Simplify fraction from the lowest level, carefully working your way up.
3	**3**	D	We have $\frac{\Delta y}{\Delta x} = \frac{15-6}{3-k} = 3$. Solve the equation.
4	**4**	B	A heptagon has $n = 7$ sides. The formula for sum of all interior angles is $180(n-2)$.
5	**5**	B	In an arithmetic sequence, any term is the average of its neighbors. Thus, we need to solve $2(2x+11) = (x) + (4x-3)$. Solve to get $x = 25$, obtain two adjacent terms, and subtract to find the common difference.
6		C	Look at units digits first and you can get 10 for each block of 100. That gives you 40 4s. If you look at tens digits you get an additional 5 for each block of 100. That gives you 60 total, plus 1 for 400 yields a grand total of 61 appearances.
7	**7**	E	Draw a picture, break up the trapezoid into two triangles, and apply the Triangle Inequality to the sides. The possible integer values of the other leg are 2, 3, 4, … 14, or 13 possible values.
8	**8**	B	First, we write b as $b = \sqrt[20]{2}$. Raising b, c, and d to the 300th power yields 2^{15}, 5^6, and 6^5. So among those three numbers the largest is b. To compare a and b, we raise them both to the 140th power to obtain 3^4 and 2^7. Therefore, b is larger, making it the largest overall.

9		A	The number of multiples of 2, 3, and 5 are $\left\lfloor \frac{1000}{LCM(2,3,5)} \right\rfloor = 33$. The number of multiples of 2, 3, 5, and 8 are $\left\lfloor \frac{1000}{LCM(2,3,5,8)} \right\rfloor = 8$. The positive difference of these two numbers is the desired count.
10	**10**	D	a+b*c=20 → a = 20 – b*c c*d-a=19 → c*d – 20+b*c c*(d-b)=39 = 3*13 = 1*39
	2	B	Same as finding the units digit of 8^8. Go through units digits of powers of 8 and find a pattern.
	6	C	Use the change-of-base formula and log rules to convert the equation into $\log_6 \sqrt{(x-7)(x-2)} = 1$, or $(x-7)(x-2) = 36$. Solve this quadratic, keeping in mind that $x > 7$.
	9	A	If $LCM(x, y) = 72$, then either 8 divides x, 9 divides x, or both. However, since $LCM(x, z) = 600$ and 9 does not divide 600, we know that 9 does not divide x. Thus, x is either 8 or 24. By similar reasoning, we find that y can be 9, 18, or 36. Neither x nor y have a power of 5; therefore z must be the source of all powers of 5. The possible values of z are 25, 50, 75, 100, 150, 300. Experiment with various powers of 2 and 3 to get valid values for z.

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Team Test Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	122	Convert to base 10, add, and then write in base 9.
2		80 [mph]	240/60 = 4 hours. David must cover that distance in 3 hours. Thus, answer is 240/3 = 80 mph.
3	**3**	30 [sq. units]	Use the Shoelace Method, or draw a picture.
4	**4**	1	Factor $(2^x - 2)(2^x - 6) = 0$. The sub-equation $2^x - 2 = 0$ yields the smallest solution.
5	**5**	-1594	Use your favorite method to evaluate determinant.
6		248	Let the number in base 7 equal abc, so that in base 9, the number is cba. These are both the same number, just different bases, so convert to a common base 10. This yields the equations $49a + 7b + c = 81c + 9b + a$, or $48a = 80c + 2b$. Guess and check—factoring in the constraints on the values of the digits—we have $a = 5$, $b = 0$, and $c = 3$. We want the number in base 10, and $503_7 = 248_{10}$.

7	**7**	480 [meters]	Let x be the distance Jacob runs before the first meeting. Since the two start at diametrically opposite points, note that $x + 100$ is half the circumference of the track. By the second meeting, Jacob has run a total of $100 + (x + 100) - 60 = x + 140$. Jillian has run a total of $x + 60$. We have the proportion $\frac{x}{100} = \frac{x+140}{x+60}$, with solution $x = 140$, so the circumference of the track is $2(140 + 100) = 480$.
8	**8**	$\frac{25}{72}$	
9		180,001	Sum of 1 through 9 is 45. Each digit appears 4 times in each position. From 1 through 9,999 we have: 45 * 4 * 10000 = 180,000.
10	**10**	187,902	Principle of Inclusion-Exclusion
	2	$2\sqrt{2}e^{\frac{3\pi}{4}i}$	In the complex plane, the magnitude is $\sqrt{a^2 + b^2}$ for $a + bi$. The angle is 135° or 3π/4.
	6	2	Keep taking reciprocals of both sides of the equation rather than simplifying the complex fraction.

	9	120	Let $M = 2020$ and $N = 800$. We have $(a^2 + b^2)(c^2 + d^2) = M^2$. Since $ac = bd$, this implies that $(ac)^2 + (bd)^2 = 2abcd$. Use this along with the previous equation to determine that $ad + bc = M$. Notice that $(a^2 + b^2) + (c^2 + d^2) - 2(ad + bc) = M + M - 2M = 0$. But this factors to $(a - d)^2 + (b - c)^2 = 0$, which means that $a = d$ and $b = c$. Consequently, $a + b = c + d$. Therefore, $(a + b)(c + d) = ac + bd + ad + bc = N + N + M = 2N + M$, or $a + b = c + d = \sqrt{2N + M}$. Finally, we have $a + b + c + d = 2\sqrt{2N + M}$ $= 2\sqrt{1600 + 2020} = 2\sqrt{3620}$ $\approx 2(60) = 120.$

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Pressure Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	64π	Rearrange to the standard form and calculate the area as: πab. Standard form is: $\frac{(x-h)^2}{a^2}+\frac{(x-k)^2}{b^2}=\frac{(x-8)^2}{128}+\frac{(y+4)^2}{32}=1$ $A=\pi\sqrt{32*128}=64\pi$
2		4:25 PM	Volume of the cone is $\frac{\pi}{3}*h*r^2$. 80% of the volume is 0.8*(30 π) = 24π and 37.5% of the volume is 11.25π. Take the difference and divide by 1.5π to calculate how long it takes Balderdash to drink all that water. Then add the hours to 7:55 AM.
3	**3**	$1800-18000i$	$(5-5i)^3(6+6i)^2$ $=5^3 6^2[(1-i)(1+i)]^2(1-i)$ $=125(36)(4)(1-i)$ $=18000-18000i$
4	**4**	25,401,600	
5		$\frac{5}{2}$	Plotting the graphs, you get one right triangle that is 2x2 and another that is 1x1. Total area is 5/2.
	2	$\frac{1}{8}$	Move the ¼ exponent out and take the derivative, using the chain rule.
	5	73	Formula for 3x3 matrix. Determinants are 81 and -8.

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College Bowl Round #1 Solutions

	Answer	Solution
1	$8\sqrt{15}$ [units]	The length of the leg is $\sqrt{34^2 - 14^2} = \sqrt{960}$.
2	22/51 [twenty-two over fifty-one]	(41 - - 3) / (123 – 21)
3	4/13 [Four-thirteenths]	(4 + 12) / 52
4	864	330=2*3*5*1, Sum of factors = (1+2)(1+3)(1+5)(1+11)=3*4*6*12=864
5	6	To find the rational root, plug in values until you get a solution. That is 2. Divide out (x – 2) to get the remaining 2nd degree polynomial of x^2 – 4x + 2. Use the quadratic equation to find the remaining two roots of $2 \pm \sqrt{2}$. Shortcut: - b / a = - (- 6) / 1
6	50 [squares]	3+8+15+24
7	3	Expand and combine like terms.
8	2 [pieces]	600 – 300 – 150 = 150. 150 / 4 = 37.5 so he gives away 4*37. 150 – 4*37 = 2.
9	468,000 [license plates]	26*25*10*9*8
10	13	Set up system of equations to find that there are 14 cats and 27 birds. Then take the difference.

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College Bowl Round #2 Solutions

	Answer	Solution
1	499,500	This is just $\frac{n(n+1)}{2}$.
2	(-11, 7)	Work into vertex form by completing the square.
3	150 [minutes]	Find the rates of weeds pulled per hour for each. Then divide 175 by the combined rate.
4	(-1/3, 6) Negative one-third [comma] six	Find the equation for the line that goes through (1, 7) and (5, 10). Then use whatever method to find where it intersects with the line 6x + 2y = 10.
5	8	Convert to the standard ellipse form by completing the square for the x and y parts of the equation.
6	128 [square inches]	A sphere with surface area 64π has a radius of 4, so the space diagonal of the cube is 8 and the side length is $8/\sqrt{3}$. Square it and multiply by 6 to get the surface area of the cube.
7	$\frac{4095}{4096}$	1 – Pr(all bulbs light up) = 1-$\binom{12}{12}(\frac{1}{2})^{12}(\frac{1}{2})^{0}$
8	$\frac{1}{12}$	The possibilities are {5, 5}, {4, 6}, and {6, 4}. There are (1/6)^2 combinations. So the probability is 3/36.
9	160 [degrees]	The formula is 180 * (S – 2) / S.
10	5	Basic log rules.

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College Bowl Round #3 Solutions

	Answer	**Solution**
1	15 [centimeters]	If you draw it out, it is obvious that the Pythagorean theorem can be used to find the length of the segment.
2	8/11	$\Pr(A\|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$ where A is the event that a red card is drawn and B is the event that a face card or heart is drawn. The denominator is $22/52$ and the numerator is $16/52$.
3	-3/2	$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$, $\csc\left(\frac{\pi}{6}\right) = 2$
4	2	Volume of a sphere = $\frac{4}{3}\pi r^3$ Surface area of a sphere = $4\pi r^2$
5	-576	$\binom{6}{1}(3)(-2)^5$
6	3,780 [ways]	There are $\frac{8!}{2!2!2!}$ ways to rearrange the letters in CALCULUS in any order. There are $\frac{7!}{2!2!}$ ways to rearrange the letters in CALCULUS so that the Us are next to each other. The difference is the answer.
7	32 [factors]	1080 = 2^3 * 3^3 * 5^1 (1+3)*(1+3)*(1+1) = 32
8	-3/2	Slope = $\frac{5-11}{-3-6}$ Invert it to get the slope of the perpendicular line.
9	20	
10	165π [units squared]	$(13^2 - 2^2)\pi$

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College Bowl Round #4 Solutions

	Answer	Solution
1	-3	Plug in 1 to get one solution. Divide it out and then use the quadratic formula.
2	40,320	
3	1/221	(4/52)*(3/51)
4	$54\sqrt{3}$ [units squared]	The length of a side of the hexagon is the same as the circle's radius.
5	100	Even digits are 0, 2, 4, 6, 8. There are 5 choices for each place in the 3-digit number, except that 0 can't be in the hundred's spot. 4*5*5=100.
6	8	Harmonic mean is the inverse of the average of the inverses.
7	22	The 1^{st} is 6 and the 2^{nd} is 28. $1 + 2 + 3 = 6$ and $1 + 2 + 4 + 7 + 14 = 28$
8	156 [units]	Use the triangle inequality theorem to figure out the largest and smallest possible lengths.
9	6,552	Factor each number completely (until primes). Then multiply each factor the greatest number of times it occurs in either number.
10	-7	PEMDAS

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College Bowl Round #5 Solutions

	Answer	Solution
1	1071	Difference of squares
2	81	
3	1/2	-b/2a=x, then plug in x into your parabola to find the value of y.
4	9 [raised] to the 48th [power]	Make the bases common.
5	150 [hotdogs]	System of equations 2n+1.5h=525 and n+h=300. Solve the second equation for n, then substitute into the first equation.
6	-1	Unit circle
7	2/11	There are 11 different combinations that can be made where at least one die is a 3. Out of those 11, {3, 4} and {4, 3} sum up to 7.
8	720 radical 3 or 720 square root 3 [units cubed]	To find the area of a hexagon it is 1/2Pa then multiply by the height to get volume.
9	1 [solution]	You could either find the discriminant of the equation or factor out and solve for x.
10	32 [units squared]	The diagonal of the square is 8, and you will get a 45-45-90 triangle. The side of the square= 4 radical 2, then square that answer and you should get 16 radical 4, which give you 32

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College Bowl Round #6 Solutions

<table>
<tr><th></th><th>Answer</th><th>Solution</th></tr>
<tr><td>1</td><td>$\sqrt{93}$ [inches]</td><td>$5^2 + 8^2 + 2^2 = 93$</td></tr>
<tr><td>2</td><td>7</td><td>$\ln(x^2 - 36) - \ln(x + 6) = \ln(x - 6)$</td></tr>
<tr><td>3</td><td>-1</td><td>$i^{10} + i^{17} + i^{-29} = i^2 + i + i^{-1} = -1 + i - i = -1.$</td></tr>
<tr><td>4</td><td>0, 1 and 6</td><td>Test a few cubes…
<table><tr><th>#</th><th>(#)^3</th><th>Remainder</th></tr><tr><td>1</td><td>1</td><td>1</td></tr><tr><td>2</td><td>8</td><td>1</td></tr><tr><td>3</td><td>27</td><td>6</td></tr><tr><td>4</td><td>64</td><td>1</td></tr><tr><td>5</td><td>125</td><td>6</td></tr><tr><td>6</td><td>216</td><td>6</td></tr><tr><td>7</td><td>343</td><td>0</td></tr><tr><td>8</td><td>512</td><td>1</td></tr><tr><td>9</td><td>729</td><td>1</td></tr><tr><td>10</td><td>1000</td><td>6</td></tr></table></td></tr>
<tr><td>5</td><td>639</td><td>Find the first 20 primes and add them.
2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 + 29 + 31 + 37 + 41 + 43 + 47 + 53 + 59 + 61 + 67 + 71 = 639</td></tr>
<tr><td>6</td><td>12 [males]</td><td>3M=2F and F=2(M-3).</td></tr>
<tr><td>7</td><td>12</td><td>You can add the squares until 650, or use the equation for the sum of the first n numbers:
$\frac{n(n+1)(2n+1)}{6} = 650$</td></tr>
<tr><td>8</td><td>25 [sq inches]</td><td>One diagonal measures $5\sqrt{2}$ inches.</td></tr>
<tr><td>9</td><td>1080 [diagonals]</td><td>d = n (n – 3) / 2. 48 * 45 / 2 = 1080.</td></tr>
</table>

10	[x =] 3	$x^2 - 6x + 9 = 0$. $(x - 3)^2 = 0$

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College Bowl Round (Extra) Solutions

	Answer	**Solution**
1	2	$log_{2019}4076351 = x$ 2019 squared is 4076351
2	14641	Students could quickly use Pascal's triangle to solve
3	41/333	Any repeating decimal could be placed on the numerator and the same amount of digits in the denominator as 9s so 123/999 and reduce
4	$2\sqrt{2}$ [meters]	Call the radius of the smaller circle r_1 and the radius of the larger circle r_2. The area between the circles is $\pi({r_2}^2 - {r_1}^2)$. So ${r_2}^2 - {r_1}^2 = 2$. Use the Pythagorean theorem to calculate half of the length of the chord – this is just $\sqrt{2}$. Double it to get the answer.
5	49.95	Solve by adding 1.11 + 2.22 + 3.33 + … + 9.99. Or factor and solve: 1.11 * (1 + 2 + 3 + … + 9) = 1.11 * 45
6	7920	11! / 7! = 11 * 10 * 9 * 8