"Math is Cool" Championships -- 2020-21

6th Grade

Mental Math Solutions

	Answer	Solution
1	113	What is seventy-seven plus thirty-six? 77 + 36 = 113
2	32	A composite number is a number with at least one factor in addition to one and the number itself. What is the smallest composite number greater than thirty? 31 is prime, 32 is divisible by 1, 32, and several other numbers
3	45 [mph]	A train travels at an average speed of thirty miles per hour. A car travels at an average speed that is one and a half times as fast as the train. How many miles per hour is the car's average speed? 30 * 1.5 = 45
4	[A + B =] 37	When rolling two standard dice the probability of rolling two ones as a reduced common fraction is A over B. What is the value of A plus B? 1/6*1/6 = 1/36, so 1 + 36 = 37
5	2 [inches]	A rectangle is four inches wide and fifteen inches long. What is the number of inches in the radius of the largest circle that can completely fit inside the rectangle without going outside the rectangle? The shorter dimension of 4 inches limits the diameter of the circle to 4 inches, so the radius would be 2 inches.
6	144	What is one hundred multiplied by one point two and then multiplied by one point two again? 100*1.2 = 120 120*1.2 = 144

7	[A + B =] 5	Abe, Ben, Cara, and Dina are randomly arranged in a line. The probability that Dina stands at the front of the line as a reduced common fraction is A over B. What is the value of A plus B? There are 4! ways to arrange the four people and 3! ways that Dina could be at the front, so the probability is 3!/4! = 1/4, so 1 + 4 = 5
8	5 [rectangles]	How many distinct rectangles with whole number side lengths are possible if the perimeter is twenty inches and a one by nine rectangle counts as the same as a nine by one rectangle? Possible values for (L, W) include (1, 9), (2, 8), (3, 7), (4, 6), (5, 5)

"Math is Cool" Championships -- 2020-21

6th Grade

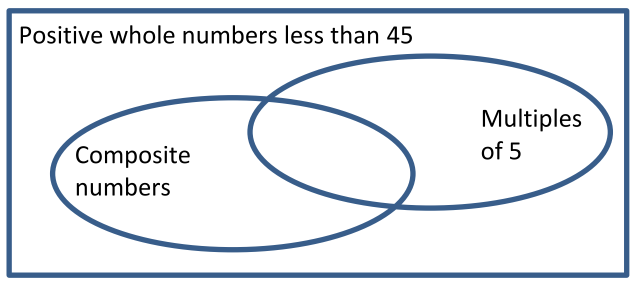
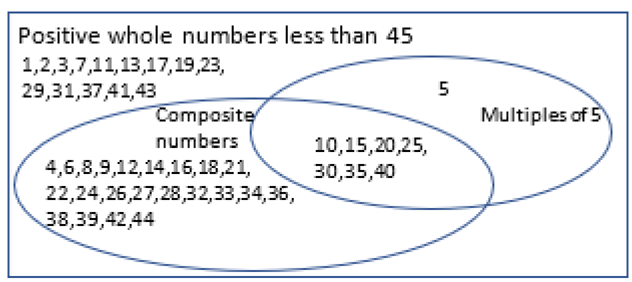
Individual Test Solutions

	Answer	**Solution**
1	0	What is 10 + 20 – 30? 10 + 20 – 30 = 0
2	[A + B =] 10	The probability that a number chosen randomly from the set {1, 2, 3, 4, 5, 6, 7} is even as a reduced common fraction is A/B. What is the value of A + B? 3 out of the 7 numbers are even, so 3/7 is the probability 3 + 7 = 10
3	90 [degrees]	What is the number of degrees in a right angle? 90 degrees in a right angle
4	20 [dimes]	How many dimes are equal in value to 2 dollars? 200/10 = 20
5	560	What is the product of 7 and 80? 7 x 80 = 560
6	230 [pages]	Rashid has read 46 pages of *Tears of the Giraffe*, from the Number One Ladies Detective Agency book series, which is 20 percent of the number of pages in the book. How many pages are in *Tears of the Giraffe*? 46 = 20%, so 23 = 10%, so 230 = 100%
7	5	Olga multiplies her favorite number by 8 and then adds 3. The result is 43. What is Olga's favorite number? 8x + 3 = 43, so 8x = 40, so x = 5
8	8 [vertices]	How many vertices (corners) are there on a cube? A cube has 8 vertices
9	10	What is the square root of 100? $\sqrt{100}$ = 10
10	5	What is the remainder when 77 is divided by 12? 77/12 = 6 r5

11	[x =] 14	Solve the equation for x: 10x – 25 = 115 10x – 25 = 115, so 10x = 140, so x = 14
12	6 [days]	A vine grows 6 inches per day. In how many days will the vine be 3 feet longer than it is now? 3 ft = 36 in, 36/6 = 6
13	[A + B =] 9	A jar contains 32 red marbles, 24 green marbles, and 8 blue marbles. The ratio of the total marbles in the jar that are blue is A/B, as a reduced common fraction. What is the value of A + B? 8/64 = 1/8 1 + 8 = 9
14	221 [in^2]	A rectangle has a length of 17 inches and a width of 13 inches. What is the number of square inches in the area of the rectangle? 13 * 17 = 221
15	[A + B =] 106	As a decimal, the mean (average) of the four smallest three-digit whole numbers is A.B, where A is a three-digit number and B is a single digit. What is the value of A + B? 100 + 101 + 102 + 103 = 406, 406/4 = 101.5, and 101 + 5 = 106
16	18 [bananas]	One nectarine costs the same as two oranges. One orange costs the same as three bananas. How many bananas cost the same as three nectarines? 1N = 2O, 1O = 3B, 1N = 6B, 3N = 18B
17	5 [miles]	A car is driving at an average rate of 30 miles per hour. How many miles does the car travel in 10 minutes? 10 min is 1/6 of an hour, so 1/6 of 30 = 5
18	216	A set of numbers consists of the numbers shown: {2, 3, 4, 5, 6, 7, 8, 9, 10} A new set of numbers is created by multiplying all the numbers in the first set by 4. What is the sum of all the numbers in the new set? 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 54 4 * 54 = 216
19	6 [elbow bumps]	In a doubles tennis match there are two two-person teams. After a certain match, each player gives every other player in the match an elbow bump. How many total elbow bumps are exchanged? n(n – 1)/2 4(3)/2 = 6

20	854917632	Garfield writes the words representing the numbers 1 through 9 in alphabetical order. Then he replaces each word with the number it represents to form a 9-digit number. What is this 9-digit number? eight, five, four, nine, one, seven, six, three, two becomes 854917632
21	75 [inches]	The perimeter of a parallelogram is 200 inches, and the length of the longer side is three times the length of the shorter side. What is the number of inches in the length of the longer side? L = 3W and 2W + 2L = 200. Substituting 3W for L results in 8W = 200, so W = 25 and L = 75
22	20 [routes]	On the lower half of a hill there are four paths to choose from. All four paths lead to the same spot half-way up the hill. From this spot to the top there are five paths to choose from. How many total routes are there to hike up the hill from bottom to top? 4 * 5 = 20
23	1 [multiple]	How many positive two-digit multiples of 7 are also multiples of 10? {14, 21, 28, 35, 42, 49, 56, 70, 77, 84, 91, 98} ∩ {10, 20, 30, 40, 50, 60, 70, 80, 90} = {70}
24	81 [palindromes]	A palindrome is a number that reads the same forwards as backwards, like 232. How many positive three-digit palindromes are there in which the three digits are not all the same? In each group, 100 – 199, 200 – 299, . . . , 900 - 999, there are 10, for example, from 100 – 199 the palindromes are 101, 111, 121, 131, 141, 151, 161, 171, 181, 191. However, one in each group has all three digits the same, in the case of 100 – 199, it is 111. So, there are 9 groups and 9 in each, so 9 * 9 = 81.
25	[A + B =] 47	In jar 1 there are 6 green skittles and 2 red skittles. In jar 2 there are 8 green skittles and 3 red skittles. The probability of drawing a red skittle from jar 1 and then a red skittle from jar 2, as a reduced common fraction, is A/B. What is the value of A + B? P(R,R) = 2/8 * 3/11 = 3/44 3 + 44 = 47
26	1270	Let S_1 be the infinite arithmetic series whose first three terms are 10, 19, 28, . . . , let S_2 be the infinite geometric series whose first three terms are 5, 15, 45, . . . , and let S_3 be the sum of S_1 and S_2. This means that the 1st term of S_3 equals the sum of the 1st terms of S_1 and S_2, the 2nd term of S_3 equals the sum of the 2nd terms of S_1 and S_2, and so on. What is the 6th term in S_3? The first 6 terms of S_1 are: 10, 19, 28, 37, 46, 55 The first 6 terms of S_2 are: 5, 15, 45, 135, 405, 1215 55 + 1215 = 1270

27	[C =] 400	Circle A has an area of 16π square centimeters. Circle B has a radius that is five times the radius of Circle A. In terms of π, the number of square centimeters in the area of Circle B is $C\pi$. What is the value of C? For Circle A, Area = 16π, so r = 4 For Circle B, r = 5 * 4 = 20, so Area = 400π and A = 400
28	13 [donkeys]	In a field there are donkeys and geese. There are a total of 72 feet and 23 heads among them. How many donkeys are in the field? $4d + 2g = 72$ → $4d + 2g = 72$ → $-2g = -20$ → $g = 10$ $d + g = 23$ $-4d - 4g = -92$ $d + 10 = 23$ → $d = 13$
29	[A + B =] 36	In a survey of 600 households, exactly 17% of the households have at least one dog and at least one cat as pets, exactly 12% have at least one cat and no dogs as pets, and exactly 27% have at least one dog and no cats as pets. The remaining households have neither dogs nor cats as pets. The probability, as a reduced common fraction, that a household chosen at random from the 600 households has at least one dog as a pet is A/B. What is the value of A + B? 17% have at least 1 dog and at least 1 cat and 27% have at least 1 dog and no cats, so 44% have at least one dog = 44/100 = 11/25 11 + 25 = 36
30	11 [matches]	Susan and Lisa are playing a series of tennis matches against each other. They use the board shown to keep track of wins and losses. The small oval around the 0 is moveable and when Susan wins a match, she moves it one number to the right. When Lisa wins a match, she moves is one number to the left. If Susan wins three matches and there have been no ties, how many total matches will they have played when the oval is moved to the left-hand 5 for the first time? 5 4 3 2 1 0 1 2 3 4 5 The order of who wins each match does not matter in solving this problem. So, if you assume Lisa wins the first 3 matches, then Susan will win the next 8 so that the oval ends up on the left-hand 5. 3 + 8 = 11

31	[A + B =] 33	Junior has one gallon of red paint and one gallon of blue paint. He takes one cup of red paint and adds it to the gallon of blue paint. After mixing it thoroughly, he takes out one cup of the mixed paint. Inside this cup, the ratio of paint that is blue is A/B, as a reduced common fraction. What is the value of A + B? After 1 cup of red paint is added to the blue paint, 16/17 of the mixture is blue paint and 1/17 of the mixture is red paint. Any amount of this mixture that is taken out will have this same ratio, which will be 1/17th red and 16/17 blue, and 16 + 17 = 33
32	14 [numbers]	In the Venn diagram shown, how many positive whole numbers less than 45 are inside the rectangle but outside both ovals?  
33	26 [cm]	The height of a trapezoid is 8 centimeters and the trapezoid has an area of 108 cm^2. Both bases (the two parallel edges) have lengths that are whole numbers of centimeters. What is the number of centimeters in the longest possible length of one of the bases? Area = (a + b)h/2 → 108 = (a + b)8/2 → a + b = 27 Since both are whole numbers and must be positive, one of them is 1 and the other is 26.
34	[N =] 375	The base-8 number 567_8 equals the base-10 number N. What is the value of N? $5 * 8^2 + 6 * 8^1 + 7 * 8^0 = 5 * 64 + 6 * 8 + 7 * 1 = 320 + 48 + 7 = 375$

35	30 [mph]	Bettina drives at an average rate of 40 miles per hour during the 4-mile drive from home to work in the morning. On the way back from work to home along the same route, she averages 24 miles per hour. What is her overall average rate in miles per hour for the two trips combined? 40 miles in 60 min means she goes 4 miles in 6 min on the way to work 24 miles in 60 min means she goes 4 miles in 10 min on the way home 8 miles in 16 min for the combined trips means 32 miles in 64 min 32(15/16) miles in 64(15/16) min equals 30 miles in 60 min
36	8 [series]	A certain infinite series of numbers is in the form abcabcabc . . . where a, b, and c, each represent a distinct positive single digit number. For example, the series . . . 123123123 . . . matches the given description, whereas . . . 445445445 . . . does not. How many infinite series that match this description exist in which any three adjacent digits add up to more than 21? Note, the series . . . 123123123 . . . and . . . 231231231 . . . count as the same series, since they are infinite series and it therefore cannot be said what digit each series begins or ends with. 589589589 . . . → sum of any 3 adjacent digits = 22 679679679 . . . → sum of any 3 adjacent digits = 22 689689689 . . . → sum of any 3 adjacent digits = 23 789789789 . . . → sum of any 3 adjacent digits = 24 For each series the orders . . . abcabc . . . , . . . bcabca . . . , and . . . cabcab . . . count as the same and the orders . . . acbacb . . . , . . . cbacba . . . , and . . . bacbac . . . , count as the same, so there are two versions of each of the 4 series written above for a total of 8 distinct series.
37	[T – S =] 360	An increasing arithmetic sequence begins with 10 as its first term, has 9 more terms, and the common difference is d. If d = 2, then the sum of the 10 terms is S. If d = 10, then the sum of the 10 terms is T. What is the value of T – S? If d = 2 then term number ten equals 10 + 9(2) = 28, and the sum of the ten numbers, S, would be (10 + 28)*10/2 = 38*5 If d = 10 then term number ten equals 10 + 9(10) = 100, and the sum of the ten numbers, T, would be (10 + 100)*10/2 = 110*5 T – S = 110x5 – 38x5 = 550 – 190 = 360

38	91	Consider the equation 15 = *k*x – 21. What is the sum of all possible whole number values of *k* for which the equation has a positive whole number as a solution for x? 15 = x – 21 → x = 36 and *k* = 1 15 = 2x – 21 → x = 18 and *k* = 2 15 = 3x – 21 → x = 12 and *k* = 3 15 = 4x – 21 → x = 9 and *k* = 4 15 = 6x – 21 → x = 6 and *k* = 6 15 = 9x – 21 → x = 4 and *k* = 9 15 = 12x – 21 → x = 3 and *k* = 12 15 = 18x – 21 → x = 2 and *k* = 18 15 = 36x – 21 → x = 1 and *k* = 36 1 + 2 + 3 + 4 + 6 + 9 + 12 + 18 + 36 = 91
39	12 [values]	A data set has four different positive whole numbers. The mean of the data set is 14. How many possible values of the smallest number in the set are there? If the mean is 14 and there are 4 numbers, then the sum of the numbers in the set must be 56. If the smallest number is 13, then the smallest set of four different numbers would be 13, 14, 15, 16, and these add up to 58. So, 13 is not a possible value of the smallest number and anything larger than 13 is also not possible. Therefore, the smallest number in the set can be anything from 1 to 12, which means there are 12 possible values for the smallest number.
40	[$] 2000 [dollars]	A local club plans to spend $10,000 to cover the costs of hosting a baseball game. They expect to sell tickets that will generate a total of $15,000 in income. If it rains on the day of the game, they won't sell any tickets and the club will lose all the money invested. The weather forecast for the day of game is a 20% probability of rain. On average and in dollars, how much profit can the club expect to earn? 80% of the time they will earn $5000 and 20% of the time they will lose $10000. .8(5000) + .2(-10000) = 4000 – 2000 = 2000

"Math is Cool" Championships -- 2020-21

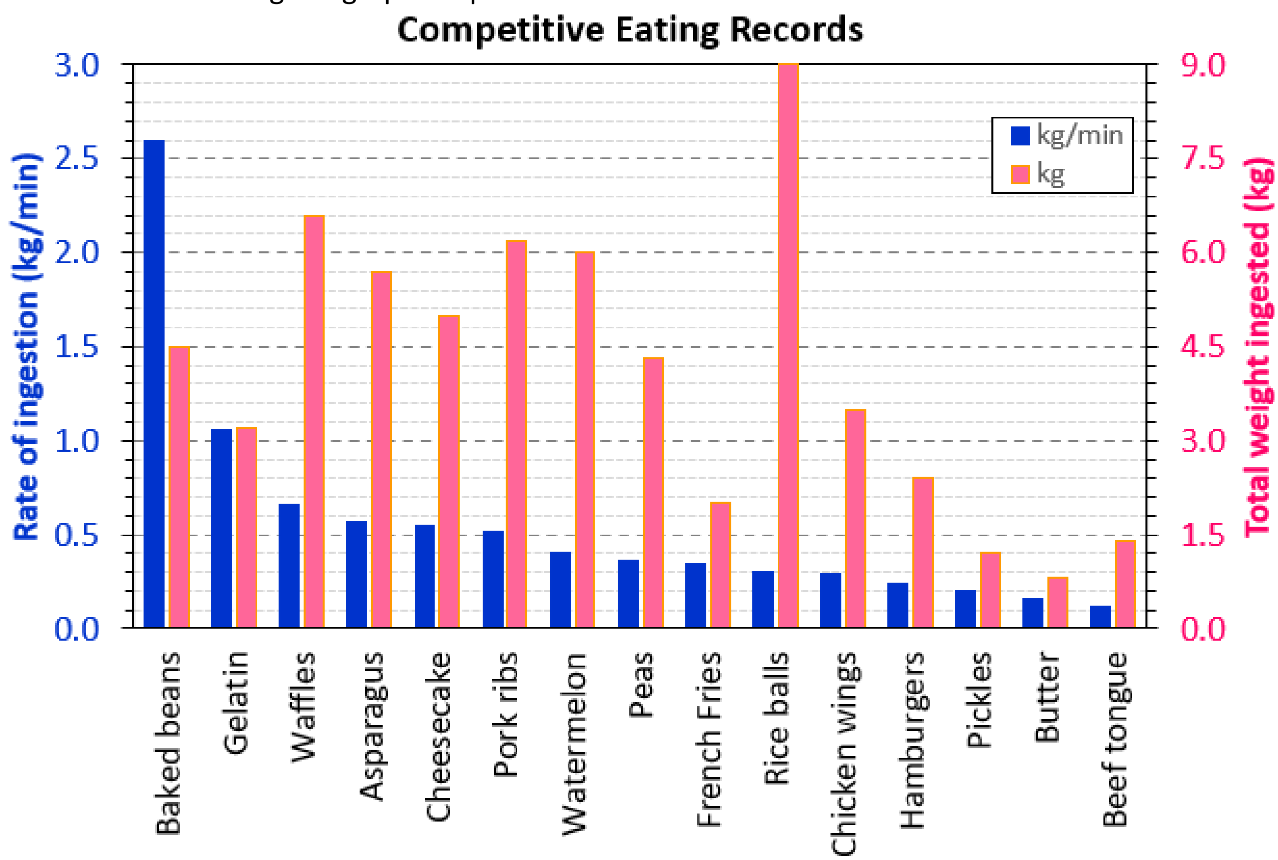
6th Grade

Multiple Choice Solutions

	Answer	Solution
USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #1 THROUGH #4. The equation y = 6x – 10 has an infinite number of solutions in the form (x, y) where x stands for one number and y stands for the resulting number when the number that x represents is multiplied by 6 and then 10 is subtracted. For example (2, 2) is a solution, because if x is replaced with two, the resulting value of y is 6(2) – 10, which equals 2.		
1	C	Which of the ordered pairs is also a solution to the equation? A) (-1, 4) B) (-1, -4) C) (3, 8) D) (3, 14) E) (4, 16) 6(3) – 10 = 8, so (3, 8) is a solution
2	D	When x equals 100, what does y equal? A) 50/3 B) 30 C) 300 D) 590 E) 610 y = 6(100) – 10 = 590
3	B	What is the value of x in the ordered pair (x, 50)? A) $6.\overline{6}$ B) 10 C) 30 D) 32 E) 290 50 = 6x – 10, 60 = 6x, x = 10

4	E	What is the sum of all the x- and y-values in the four solutions in which x is a single digit prime number? A) 37 B) 72 C) 75 D) 78 E) 79 The four solutions are (2, 2), (3, 8), (5, 20), and (7, 32) so 2 + 2 + 3 + 8 + 5 + 20 + 7 + 32 = 79

Refer to the following bar graph for problems 5 – 7.

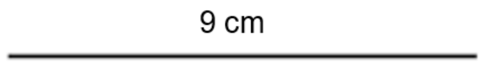


5	B	Which competitive eating record has the largest number of kilograms ingested? A) Baked Beans B) Rice Balls C) Waffles D) Gelatin E) Watermelon Rice Balls has the tallest pink bar.

6	E	Each food record in the table can be ranked based on how it compares to the other foods' rate of consumption and weight ingested. For example, we would rank Butter as 15th (last) for total weight ingested, and 14th for rate of ingestion. With rankings, the lower the number the better the ranking. In other words, a rank of 1 is better than a rank of 10. What is the lowest sum when the rank of rate of ingestion is added to the rank of total weight ingested, for any of the 15 foods listed? A) 35 B) 18 C) 7 D) 6 E) 5 Waffles has rank 3 for rate of ingestion and rank 2 for total weight ingested, so 3 + 2 = 5.
7	B	The ranks for rate of ingestion go from best to worst as you read from left to right on the bar graph. If the graph were to be redrawn so that the ranks for total weight ingested were to go from best to worst as you read left to right, how many records would not move from their current spot on the graph? A) 0 B) 1 C) 2 D) 4 E) 7 The only one that would not move is the 'Peas' category, which is in the 8th position for rate of ingestion, and is also in the 8th position for total weight ingested.

USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #8 THROUGH #10.

Consider the segment shown with length 9 cm as Shape 0.

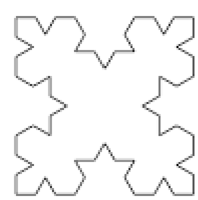
Shape 0 

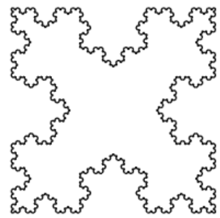
Imagine the middle three centimeters were removed and replaced with two 3-cm segments to form the new shape shown, which is Shape 1.

Shape 1 

Then the process is repeated where all four of the segments have their middle third replaced by two segments of the same length to create Shape 2.

Shape 2 

8	C	What is the sum of the lengths of the four segments in Shape 1? A) 9 cm B) 10 cm C) 12 cm D) 27 cm E) 36 cm 4 segments that are 3 cm each has a total length of 12 cm
9	D	A figure made of four Shape 2s, each of which has its bumps pointing toward the middle is shown here.  What is the sum of the lengths of all the segments in the perimeter of this figure? A) 48 cm B) 54 cm C) 60 cm D) 64 cm E) 72 cm Each Shape 2 has sixteen 1-cm long segments, so 4 * 16 = 64 cm.

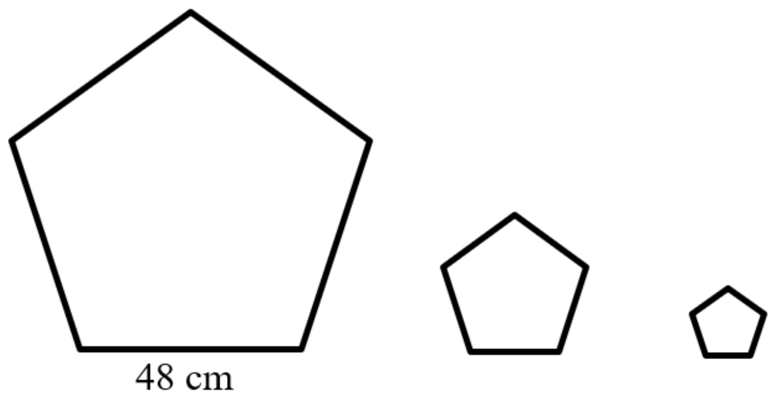
10	A	The process of replacing the middle third of every segment with two segments of the same length is repeated two more times to make Shape 4. In the same way that the figure in problem 9 is made of four shape 2s with the bumps pointing toward the middle, the figure shown here is made of four Shape 4s.  What is the sum of the lengths of all the segments in the perimeter of this figure? A) 1024/9 cm B) $85\frac{1}{3}$ cm C) 108 cm D) 256/9 cm E) Answer not given To go from Stage 0 to Stage 1 you multiply by 4/3. To go from Stage 1 to Stage 2 you multiply by 4/3. So to get to Stage 4 you multiply by 4/3 two more times. So the answer is 64 * 4/3 * 4/3 = 1024/9

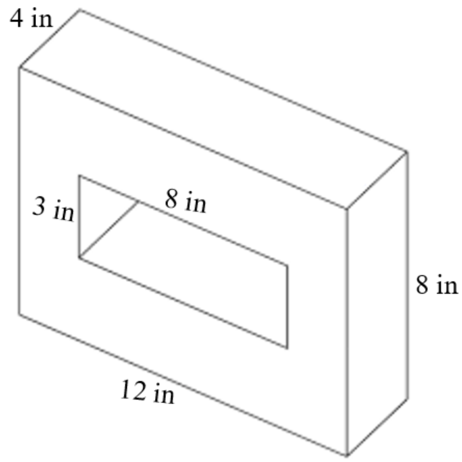
“Math is Cool” Championships -- 2020-21

6th Grade

Team Test Solutions

	Answer	**Solution**
1	53	Evaluate the expression: 11(2 + 5) – 24 11(2 + 5) – 24 = 11(7) – 24 = 77 – 24 = 53
2	6 [colors]	All the socks in a drawer are mixed up and there is at least one pair of each color. If socks are pulled randomly from the drawer, the minimum number needed to be pulled in order to get at least one matching pair is 7 socks. How many different sock colors are in the drawer? It’s possible to draw one of each of the six colors and not have a match. When you draw the 7^{th} one, there must be a match.
3	50 [%]	A chessboard is a square grid with 8 rows and 8 columns of small squares. When a game is set up pieces are placed on all squares in two rows on one side of the board and on all squares in two rows on the other side of the board. At the start of the game, what percent of the squares on the board have pieces on them? There are a total of 8*8 = 64 squares on the board and there are 8 squares in each row. Since four rows have pieces on all their squares, then there are 4*8 = 32 pieces. 32/64 = 1/2 or 50%.
4	2	While Romy is out walking, she takes three 90 degree right turns and ends up facing north. What direction was she facing before the three turns? Answer 1 for North, 2 for East, 3 for South, and 4 for West. When going east, turning 90 degrees right results in going south. When going south, turning 90 degrees right results in going west. When going west, turning 90 degrees right results in going north. So she was going east when she started.
5	[A + B =] 5	The product of $\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}$ as a reduced common fraction is A/B. What is the value of A + B? 1/2*2/3*3/4 = 1/4, so 1 + 4 = 5

6	144 [degrees]	Two of the angles in a certain triangle have measures of 36 degrees and 76 degrees. What is the number of degrees in the sum of the measures of the largest two angles in the triangle? 36 + 76 = 112, 180 – 112 = 68, so 68 is the third angle and 68 + 76 = 144
7	60 [cm]	In a regular polygon all sides have the same length. In the series of regular pentagons shown here, going left to right each successive pentagon has side lengths that are one-half the length of the sides of the previous pentagon. If the lengths of the sides on the largest pentagon are 48 cm, then what is the number of centimeters in the perimeter of the smallest pentagon?  1/2*48 = 24 and 1/2*24 = 12 and 5*12 = 60
8	2 [inches]	A basketball team of 5 players has an average height of 74 inches. A new player joins the team who is 7 ft 2 in tall. Reminder: there are 12 inches in 1 foot. By how many inches does the average height of the team increase when the new player joins? 5 * 74 = 370 7 ft 2 in = 86 in 370 + 86 = 456 456/6 = 76 76 – 74 = 2

<table>
<tr>
<td>9</td>
<td>288 [cm^3]</td>
<td>A rectangular prism is 4 inches by 8 inches by 12 inches and has hole in it in the shape of a different rectangular prism as shown. The lengths of the two sides of the rectangular prism-shaped hole that you can see in the figure are 3 inches and 8 inches. The 3rd dimension of the hole is the same as the corresponding dimension of the larger prism. What is the number of cubic inches in the volume of this solid?


The volume of the larger prism is 12*8*4 = 384
The volume of the prism-shaped hole is 3*8*4 = 96
The volume of the solid is 384 – 96 = 288</td>
</tr>
<tr>
<td>10</td>
<td>[A + B =] 38</td>
<td>Aaron and Bonita are playing a game. Each chooses two numbers randomly from the list {1,2,3} and multiplies them together. Individual numbers can be chosen multiple times. For example, it's possible for Aaron to choose two 1s and for Bonita to also choose two 1s. A person only wins if they have the bigger product. As a reduced common fraction, the probability that Bonita wins is A/B. What is the value of A + B?

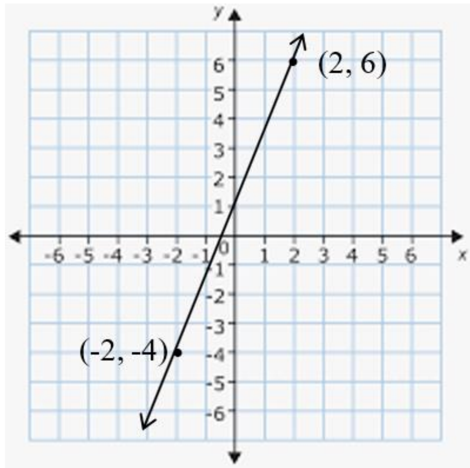
The list of possible outcomes for each person is: (1,1)=1, (1,2)=2, (1,3)=3, (2,1)=2, (2,2)=4, (2,3)=6, (3,1)=3, (3,2)=6, (3,3)=9
There are 9*9 = 81 total outcomes in which one person's outcome is compared to the other person's outcome. Each of these 81 outcomes is equally likely. Winning outcomes for Bonita include: 9v6 – 2 outcomes, 9v4 – 1 outcome, 9v3 – 2 outcomes, 9v2 – 2 outcomes, 9v1 – 1 outcome, 6v4 – 2 outcomes, 6v3 – 4 outcomes, 6v2 – 4 outcomes, 6v1 – 2 outcomes, 4v3 – 2 outcomes, 4v2 – 2 outcomes, 4v1 – 1 outcome, 3v2 – 4 outcomes, 3v1 – 2 outcomes, 2v1 – 2 outcomes
So, 2 + 1 + 2 + 2 + 1 + 2 + 4 + 4 + 2 + 2 + 2 + 1 + 4 + 2 + 2 = 33 and the probability is 33/81 = 11/27 and 11 + 27 = 38</td>
</tr>
</table>

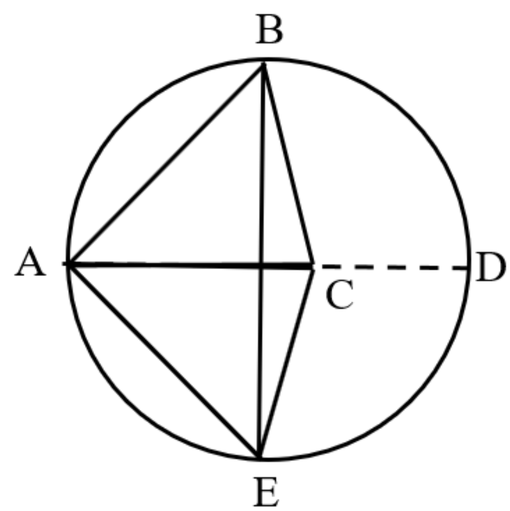
"Math is Cool" Championships -- 2020-21

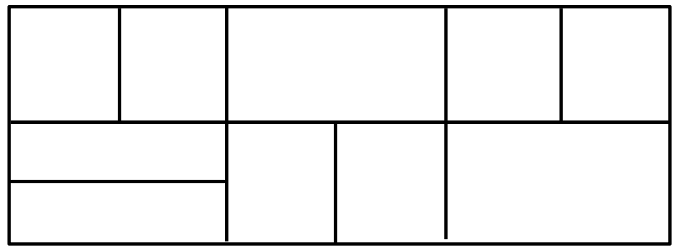
6th Grade

Linda Moore Triple Jump Solutions

	Answer	Solution
1	[A + B =] 38	In the sentence, 'The quick brown fox jumps over the lazy dog', the ratio of e's to total letters as a reduced common fraction is A/B. What is the value of A + B? There are 35 total letters and 3 e's, so the ratio is 3/35 and 3 + 35 = 38
2	-50	If x = 10 and y = -5, what is the value of 10x + 2xy + 10y? 10(10) + 2(10)(-5) + 10(-5) = 100 – 100 – 50 = -50
3	36 [minutes]	Thomas the Tank Engine travels at an average rate of 10 miles per hour. How many minutes does it take him to travel 6 miles? 6/10*60 = 36
4	[$] 5 [dollars]	The currency system in the USA has $1 bills, $2 bills, $5 bills, $10 bills, $20 bills, $50 bills and some other bills worth more than $50. Malik has 7 bills in his wallet worth 11 dollars and none of them are $2 bills. What is the number of dollars in the value of the largest bill? When no $2 bills are present, $11 dollars with 7 bills is only possible if there are six $1 bills, and one $5 bill. The largest bill is the $5 bill.

5	[A + B =] 7	The slope of the line shown as a reduced common fraction is A/B. What is the value of A + B?  (6 - -4)/(2 - -2) = 10/4 = 5/2, so 5 + 2 = 7
6	[E + F =] 29	In the expression A/B + C/D, each letter is replaced with a distinct digit from the set {2,3,4,5}. The largest possible value that can result from this process is the reduced common fraction E/F. What is the value of E + F? Check 5/3 + 4/2 and 5/2 + 4/3. The first one is 22/6 and the second one is 23/6, so 23 + 6 = 29
7	6 [mixtures]	A jar has a total of 7 beads in it. The beads are all either red or blue, and there is at least one of each color. How many different sets of 7 beads are possible that match this description? For example, one set would be 1 red and 6 blue. (R, B) → (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

8	135 [cm^2]	In the figure shown, $\overline{AD}$ and $\overline{BE}$ are both diameters of the circle, AD = BE = 30 cm, and AC = 0.6AD. $\overline{AC}$ lies on $\overline{AD}$. What is the number of square centimeters in the area of ΔABC?  AC = 0.6(30) = 18 The height of ΔABC = 0.5(10) = 15 Area = 18(15)/2 = 135
9	144	What is the smallest positive three-digit whole number that has exactly six distinct perfect square factors? The number must have a prime factorization in the form $a^2*a^2*b^2$, because this number would have 1, a^2, a^4, b^2, $(ab)^2$, and $(a^2b)^2$ as its 6 perfect square factors. Or it could be a multiple of a number with this prime factorization. Since we're looking for the smallest number, we don't need to consider multiples. The smallest such number is $2^2*2^2*3^2$ = 144

10	34 [rectangles]	How many rectangles of any size are in the drawing? Assume all angles are right angles.  2-flat horizontal ones 6-vertical stubby ones 6-taller horizontal ones 4-1.5 x as wide taller horizontal ones 3-double tall horizontal ones 5-double wide taller horizontal ones 2-double tall double wide horizontal ones 2-2.5 x as wide taller horizontal ones 2-triple wide taller horizontal ones 1-overall rectangle 1-rectangle made of two vertical stubby ones and one flat horizontal one 2 + 6 + 6 + 4 + 3 + 5 + 2 + 2 + 2 + 1 + 1 = 34

“Math is Cool” Championships -- 2020-21

6th Grade

College Bowl Round #1 Solutions

	Answer	Solution
1	4 [weeks]	Selina plays soccer three days a week for two hours each day. How many weeks will it take her to play twenty-four hours of soccer? 6 hours per week 24/6 = 4
2	44	What is the product of twenty-four, one sixth, and eleven? 24*1/6*11 = 44
3	115 [bad guys]	Jack Frost can freeze three bad guys in two minutes. Elsa can freeze seven bad guys in three minutes. Working together, how many bad guys could they freeze in thirty minutes? 3 guys in 2 min = 9 guys in 6 min 7 guys in 3 min = 14 guys in 6 min 9 + 14 = 23, 23 *5 = 115
4	80 [inches]	What is the number of inches in the perimeter of a right triangle with legs of length sixteen and thirty inches? (8, 15, 17) is a Pythagorean triple, so (16, 30, 34) is too. 16 + 30 + 34 = 80
5	92	Austin’s scores for three math tests are eighty-nine, ninety-seven, and ninety. What score would he need on his fourth test to have an average of ninety-two? 89 + 97 + 90 + x = 4(92) 276 + x = 368, x = 92

6	135 [minutes]	An airplane cruises at an average rate of five hundred miles per hour. A bullet train travels at an average rate of two hundred miles per hour. How many minutes longer does a bullet train take to make the same seven-hundred-and-fifty-mile trip? 750/500 = 1.5 hrs = 90 min 750/200 = 3.75 hrs = 225 min 225 – 90 = 135
7	28 [in^2]	A square has side lengths of six inches. If the sides are all increased by thirty-three and one third percent, what is the number of square inches in the positive difference between the area of the new square and the area of the original square? 6*1.3333333… = 8 $8^2 - 6^2 = 64 - 36 = 28$
8	37 [rounds]	A group of kids are playing a certain card game where six cards are played every round. How many rounds will it take them to play two hundred and twenty-two cards? 222/6 = 37
9	127 [cents]	Riley has three quarters, two dimes, and thirty-two pennies. What is the total number of cents in Riley's money? 3(.25) + 2(.10) + .32 = 1.27 = 127 cents
10	8 [hours]	How many hours would it take Nat to drive two hundred and thirty-two miles if she drives at an average rate of twenty-nine miles per hour? 232/29 = 8

“Math is Cool” Championships -- 2020-21

High School

College Bowl Round #2 Solutions

	Answer	Solution
1	114 [cents]	Together, a water bottle and a bag of chips cost two dollars and thirty-eight cents. In cents, how much does the bag of chips cost if it is ten cents cheaper than the water? 238 = w + w – 10 2w = 248, w = 124, c = 114 cents
2	399	What is the product of twenty-one and nineteen? 21*19 = 399
3	81	What is the positive difference between forty squared and forty-one squared? $41^2 - 40^2 = (41 + 40)(41 - 40) = 81*1 = 81$
4	196 [inches]	An equilateral triangle has a perimeter of one hundred and forty-seven inches. What is the perimeter, in inches, of a square with the same side length as the triangle? 147/3 = 49 49*4 = 196
5	4 [factors]	What is the number of distinct prime factors in thirty thousand three hundred? $30300 = 300*101 + 3*2^2*5^2*101$, So 2, 3, 5, and 101
6	96 [ways]	How many ways can the letters A-B-C-D-E be put in order, not counting any arrangements that begin with E? 5! – 4! = 120 – 24 = 96

7	47	The first two Fibonacci numbers are one and one. What is the sum of the seventh and ninth Fibonacci numbers? 1, 1, 2, 3, 5, 8, 13, 21, 34 13 + 34 = 47
8	18 [inches]	What is the number of inches in the radius of a circle with a circumference of thirty-six pi inches? d = 36, r = 18
9	62	What is the next term in the sequence that begins two, six, fourteen, thirty, and so on? 2, 6, 14, 30, . . . is increasing by powers of 2, +4, +8, +16, . . ., so 30 + 32 = 62 will be the next number
10	[A + B =] 115	Two cards are randomly drawn from a standard deck without replacement. As a reduced common fraction, the probability that the first card is a diamond and the second card is black is A over B, where A is a two-digit whole number and B is a three-digit whole number. What is the value of A plus B? 1/4*26/51 = 13/102, so 13 + 102 = 115

"Math is Cool" Championships -- 2020-21

High School

College Bowl Round #3 Solutions

	Answer	Solution
1	394	What is the sum of thirteen squared and fifteen squared? $13^2 + 15^2 = 169 + 225 = 394$
2	23	What is fifty percent of ten percent of four hundred and sixty? .5*.1*460 = .5*46 = 23
3	10 [shots]	Miriam makes seventy-five percent of her free throws. If she shoots forty free throws, how many shots will she miss? .25*40 = 10
4	64	Find the product of the mean, median, and mode of the following set of data: two, four, one, three, four, six, and eight 2, 4, 1, 3, 4, 6, 8 1 + 2 + 3 + 4 + 4 + 6 + 8 = 28, 28/7 = 4 Median, Median and Mode = 4 $4^3 = 64$
5	24 [cups]	How many cups are in one and a half gallons? 4 C = 1 Q, 4 Q = 1 G, 16 C = 1 G, 24 C = 1.5 G
6	40,320	What is the product of all the numbers from one to eight inclusive? 8! = 40320
7	[A + B =] 157	Jeff plays Tetris for twenty-five minutes on all twenty-nine days during the month of February in a leap year. The number of hours Jeff plays Tetris in February is the reduced common fraction A over B. What is the value of A plus B? 25/60*29 = 5/12*29 = 145/12, so 145 + 12 = 157

8	33 [days]	Thanos is trying to find six stones to complete his collection. If it takes him five and a half days on average to find each stone, how many total days will he need to find all six stones? 5.5 * 6 = 33
9	7	Rita thinks of a number in her head, then adds seventy to the number, then divides the outcome by eleven. Her final number is seven what number did Rita start with? (x + 70)/11 = 7, x + 70 = 77, x = 7
10	2,431	What is the product of the prime numbers between ten and eighteen? 11*13*17 = 2431