“Math is Cool” Masters -- 2020-21

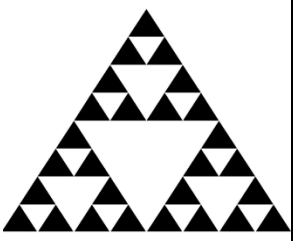
6th Grade

Mental Math Solutions

6th	Answer	Solution
1	756	What is the positive difference between one thousand and six and two hundred and fifty? 1006 – 250 = 756
2	6	What is the greatest common factor of eighteen and twenty-four? 18 = 6*3, 24 = 6*4, so 6 is the GCF
3	2100	What is the product of the numbers five, six, seven, and ten? 5*6*7*10 = 2100
4	[A + B =] 5	As a reduced common fraction, the probability of flipping no heads when you flip a fair coin twice is A over B. What is the value of A plus B? P(T,T) = 1/2*1/2 = 1/4, so 1 + 4 = 5
5	[x =] 12	Solve for X in the equation two X plus four equals twenty-eight. 2x + 4 = 28, 2x = 24, x = 12
6	7 [letters]	Sunday is one day after Saturday. How many letters are in the name of the day that is seventeen days after Saturday? Since 14 is a multiple of 7, 14 days from now will be Saturday, 15-Sun, 16-Mon, 17-Tue and Tuesday has 7 letters
7	[A + B =] 7	As a reduced common fraction, the slope of a line with points eight comma three and twelve comma six is A over B. What is the value of A + B? (8, 3) and (12, 6) Slope = (6 – 3)/(12 – 8) = 3/4, and 3 + 4 = 7

8	[A + B =] 45	As a decimal to the nearest hundredth, the number of square inches in the area of a square whose perimeter is eighteen inches is A point B, where A and B each represent two-digit integers. What is the value of A plus B? 18/4 = 4.5, 4.5^2 = 20.25 and 20 + 25 = 45

"Math is Cool" Masters -- 2020-21
6th Grade
Individual Test Solutions

6th	Answer	Solution
1	18	What is the sum of the numbers 3 through 6? 3 + 4 + 5 + 6 =18
2	[A + B =] 3	As a reduced common fraction, the probability of drawing a red card from a standard deck of cards is A/B. What is the value of A + B? 26 red out of 52 total = 1/2 and 1 + 2 = 3
3	27 [triangles]	How many small black triangles are in the figure shown?  3 * 9 = 27
4	45 [in^2]	What is the number of square inches in the area of a rectangle with a length of 3 inches and a width of 15 inches? 3 x 15 = 45
5	315	A proper factor of a number is a factor that is not the number itself. The largest proper factor of 210 is 105. What is the first number greater than 210 that also has 105 as its largest proper factor? 315 = 3 x 105 Since 315 can't be divided evenly by 2, 105 must be the largest proper factor
6	84 [skittles]	Marshawn has eaten 1/7 of the skittles in a bag. There are 72 skittles left in the bag. How many were in the bag to begin with? 72 = 6/7 of the bag, 72/6 = 12 = 1/7 of the bag, 7 * 12 = 84
7	1	Evaluate and express as a whole number: $\frac{12}{37} \cdot \frac{37}{12}$ The product of reciprocals is 1

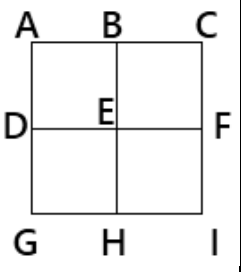
8	37 [cents]	Cory buys lunch for $4.63 and pays with a 5-dollar bill. How many cents does he get in change? 5 – 4.63 = 0.37
9	984 [miles]	The side length of a regular octagon is 123 miles. What is the number of miles in the perimeter of the octagon? 8 x 123 = 984
10	5	What is the median of the following list of numbers? {6, 2, -1, 12, 8, 4, -3, 9} -3, -1, 2, 4, 6, 8, 9, 12 (4 + 6)/2 = 5
11	-15	Evaluate and express as an integer: $(2 - 6)^3 + (15 - 22)^2$ $(2 – 6)^3 + (15 – 22)^2 = (-4)^3 + (-7)^2 = -64 + 49 = -15$
12	2 [hours]	Zhiyuan's pet ant colony doubles in size every two hours. If at one point in time there are 100,000 ants in his colony, how many hours before that time were there 50,000 ants? 50000 x 2 = 100000 so it had 50000 ants 2 hours earlier
13	3 [ft]	An 18-foot rope is cut 5 times at regular intervals so that each of the resulting pieces are the same length. What is the number of feet in the length of each piece? 5 cuts result in 6 pieces 18/6 equals 3
14	[x =] 18	Solve for x: 30 – 7.5x = -105 30 – 7.5x = -105 -7.5x = -135 x = 18
15	15 [pounds of oranges]	Twelve pounds of oranges cost $24. Eight pounds of bananas cost $4. How many pounds of oranges have the same cost as sixty pounds of bananas? Oranges cost $2 per pound, bananas cost $.50 per pound, so you can get ¼ the amount of oranges for the same cost as bananas. ¼ of 60 is 15.
16	601	The first four terms of an arithmetic sequence are 7, 13, 19, and 25. What is the 100^{th} term in the sequence? 7 + 99(6) = 601

<table>
<tr><td>17</td><td>[A + B =] 9</td><td>Ahaan flips a nickel, a dime, and a quarter. As a reduced common fraction, the probability that the nickel is heads, the dime is heads, and the quarter is tails is A/B. What is the value of A + B?

1/2 x 1/2 x 1/2 = 1/8 and 1 + 8 = 9</td></tr>
<tr><td>18</td><td>96 [minutes]</td><td>John can write two paragraphs in 24 minutes. Oliver can write three paragraphs in 48 minutes. How long, in minutes, would it take for them to write fourteen paragraphs together?

John: 2 in 24 min = 4 in 48 min
Oliver: 3 in 48 min
Together: 7 in 48 min is the same as 14 in 96 min</td></tr>
<tr><td>19</td><td>13 [cards]</td><td>Allison is holding 13 cards, Bella is holding 12 cards and Cam is holding 11 cards. Allison gives Bella and Cam 2 cards each. Bella gives Allison 1 card and Cam 3 cards. Cam gives Allison 3 cards and Bella 2 cards. After these trades, what is the number of cards held by the person holding the largest number of cards?

<table>
<tr><td>A</td><td>B</td><td>C</td></tr>
<tr><td>13</td><td>12</td><td>11</td></tr>
<tr><td>9</td><td>14</td><td>13</td></tr>
<tr><td>10</td><td>10</td><td>16</td></tr>
<tr><td>13</td><td>12</td><td>11</td></tr>
</table>
</td></tr>
<tr><td>20</td><td>15</td><td>All the prime numbers are removed from the set of integers from 10 to 20, inclusive. What is the mean of the remaining numbers?

(10 + 12 + 14 + 15 + 16 + 18 + 20)/7 = 15</td></tr>
<tr><td>21</td><td>18
[segments]</td><td>What is the number of segments in the figure shown that have endpoints A, B, C, D, E, F, G, H, or I? Note: segments that aren't shown don't count. For instance, you could connect A and I to create a segment $\overline{AI}$, but since it is not shown, it doesn't count. Only count a segment one time. For example, $\overline{AB}$ is the same as $\overline{BA}$.
 

12 short ones: $\overline{AB}$, $\overline{BC}$, $\overline{DE}$, $\overline{EF}$, $\overline{GH}$, $\overline{HI}$, $\overline{AD}$, $\overline{DG}$, $\overline{BE}$, $\overline{EH}$, $\overline{CF}$, $\overline{FI}$
6 long ones: $\overline{AC}$, $\overline{DF}$, $\overline{GI}$, $\overline{AG}$, $\overline{BH}$, $\overline{CI}$
12 + 6 = 18</td></tr>
</table>

22	1	Let A and C be two consecutive odd numbers and let B be the mean of A and C. What is $B^2 - AC$? If the first odd number, A, is 2N + 1, where N is an integer, then the second odd number, C, is 2N + 3, then B would be 2N + 2. B^2 would be $4N^2 + 8n + 4$ and AC would be $4N^2 + 8N + 3$ and $B^2 - AC = 4N^2 + 8n + 4 - (4N^2 + 8N + 3) = 1$
23	15	On a coordinate plane, point A (-10, 21) is translated 3 units to the right and down 13 units resulting in point A'. Then point A' is reflected over the y-axis resulting in point A''. What is the sum of the coordinates of A''? A(-10, 21) → A'(-7, 8) → A''(7, 8) 7 + 8 = 15
24	98 [inches]	What is the smallest possible perimeter in inches of a rectangle with sides of integer length in inches and an area of 600 square inches? Closest two numbers that multiply to 600 are 24*25, so 2*(24 + 25) = 98
25	[A + B =] 11	As a reduced common fraction, the mean of the following list of fractions is A/B. What is the value of A + B? $\frac{7}{50}, \frac{11}{50}, \frac{13}{50}, \frac{17}{50}, \frac{19}{50}, \frac{23}{50}, \frac{29}{50}, \frac{31}{50}$ (7 + 11 + 13 + 17 + 19 + 23 + 29 + 31)/50 = 150/50 = 3 3/8 and 3 + 8 = 11
26	224 [%]	If the circumference of a circle is increased by 80%, by what percent does its area increase? $1.8^2 = 3.24$ 3.24 - 1 = 2.24, hence 224%
27	$1256_{[7]}$	Express the base 5 number 3412_5 as a base 7 number. Do not include the base 7 in your answer. $3 \times 5^3 + 4 \times 5^2 + 1 \times 5^1 + 2 \times 5^0 = 482_{10}$ $482 = 343 + 98 + 35 + 6 = 1 \times 7^3 + 2 \times 7^2 + 5 \times 7^1 + 6 \times 7^0 = 1256_7$

28	6 [ways]	How many ways are there to make a sum of 10 by adding together the digits 1, 3 and 7, or any combination of these digits? 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 1 + 1 + 1 + 1 + 1 + 1 + 1 + 3 1 + 1 + 1 + 1 + 3 + 3 1 + 1 + 1 + 7 1 + 3 + 3 + 3 3 + 7
29	90 [minutes]	An 80-gallon bathtub has a faucet and a drain. When the faucet is on and the drain is closed, it takes 15 minutes to fill the bathtub. When the bathtub is full, the faucet is off, and the drain is open, it takes 18 minutes to empty the bathtub. If the drain is open while the faucet is on, how long will it take to fill the bathtub? 80/15 – 80/18 = 16/3 – 40/9 = 48/9 – 40/9 = 8/9 gallons per minute 80/(8/9) = 80*9/8 = 90 minutes
30	[A + B =] 37	As a reduced common fraction, the sum of $\frac{\frac{3}{5}}{7} + \frac{3}{\frac{5}{7}}$ is A/B. What is the value of A + B? (3/5)/7 = 3/35 and 3/(5/7) = 21/5 3/35 + 21/5 = 150/35 = 30/7 and 30 + 7 = 37
31	15325	What is the sum of the first 25 terms of an arithmetic sequence with first term 37 and common difference (the amount added from one term to the next) of 48? 37 + 24(48) = 1189 so this is the 25th term The sum of the first 25 terms would be 25(37 + 1189)/2 = 25(1226)/2 = 25(613) = 12260 + 3065 = 15325

<table>
<tr><td>32</td><td>[A + B =] 338</td><td>A jar has four red marbles and some other marbles in it. When drawing two marbles out of the jar without replacement, the probability of getting one red and one blue marble is determined using the following calculation: $\left(\frac{4}{17}\right)\left(\frac{7}{16}\right)(2)$
Once the marbles are put back in the jar, as a reduced common fraction, the probability of drawing two blue marbles out of the jar with replacement is A/B, where A is a two-digit whole number and B is a three-digit whole number. What is the value of A + B?

7/17*7/17 = 49/289 and 49 + 289 = 338</td></tr>
<tr><td>33</td><td>[A + B =] 84</td><td>A data set has ten distinct positive whole numbers and a mean of 50. As a decimal to the nearest tenth, the largest possible median of the set is A.B, where A is a two-digit whole number and B is a single digit. What is the value of A + B?

The sum of the ten numbers is 10 * 50 = 500
To make the median as large as possible, make the lowest 4 numbers, 1, 2, 3, and 4 so the other 6 numbers add up to 490. For the median to be as large as possible, the remaining 6 numbers need to be consecutive, or nearly consecutive and they need to add up to 490. 490/6 = 81 and 4/6, so the last 6 numbers would be 79, 80, 81, 82, 83, 85, because 79 + 80 + 81 + 82 + 83 + 85 = 490. The median of the set would be (79 + 80)/2 = 79.5 and 79 + 5 = 84</td></tr>
<tr><td>34</td><td>54</td><td>Let A = m/n, let B = p/q, and let m, n, p, and q be distinct single-digit positive whole numbers. If A + B = 12, what is the product of m and p?

<table>
<tr><td>9/1 + 8/2</td><td>13</td><td>9/2 + 8/1</td><td>12.5</td></tr>
<tr><td>9/1 + 7/2</td><td>12.5</td><td>9/2 + 7/1</td><td>11.5</td></tr>
<tr><td>9/1 + 6/2</td><td>12</td><td>9/2 + 6/1</td><td>10.5</td></tr>
<tr><td>9/1 + 5/2</td><td>11.5</td><td>8/1 + 7/2</td><td>11.5</td></tr>
</table>
The 1st column shows the decreasing pattern in the sums that are made when 9 is over 1. The 2nd column shows the decreasing pattern in the sums that are made when 9 is over 2, and the largest possible sum that does not include 9. From this sample, it is apparent that there is only one way for a sum of 12 to occur, namely if m = 9 and p = 6, or if m = 6 and p = 9. Either way, m * p = 9 * 6 = 54.</td></tr>
</table>

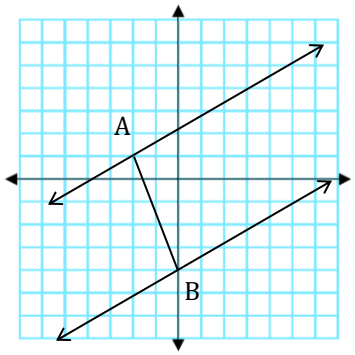
<table>
<tr><td>35</td><td>128
[scenarios]</td><td>A math tournament sudden-death round is played as 8th position playing 7th position, and the winner of that round playing 6th position, and so on until the tournament is over. Once you lose a round, you can no longer play. How many distinct paths leading to one winner can this sudden-death round have?

For each round, there are two possible winners. The rounds are independent. For 8 participants, there would be 7 total rounds. 2^7 = 128.</td></tr>
<tr><td>36</td><td>11</td><td>What is the sum of all integer solutions to the following inequality?
$|3x - 2| \leq 16$

$3x - 2 \leq 16$ and $3x - 2 \geq -16$
$3x \leq 18$ $3x \geq -14$
$x \leq 6$ $x \geq -14/3$
$-14/3 \leq x \leq 6$
The integer solutions are -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6 and their sum is 11.</td></tr>
<tr><td>37</td><td>98 [lineups]</td><td>A certain sport has three positions: forward, middle, and back, and during a game each team has five players on the field at one time. If the Redtown Raptors team has two forwards, four middles, and three backs on their roster, how many different five-player lineups can they use if there must always be at least one of each position on the field?

<table>
<tr><td colspan="5">Combos of Forwards, Middles & Backs</td></tr>
<tr><td>F</td><td>M</td><td>B</td><td></td><td></td></tr>
<tr><td>1</td><td>1</td><td>3</td><td>2*4*1</td><td>8</td></tr>
<tr><td>1</td><td>2</td><td>2</td><td>2*6*3</td><td>36</td></tr>
<tr><td>1</td><td>3</td><td>1</td><td>2*4*3</td><td>24</td></tr>
<tr><td>2</td><td>1</td><td>2</td><td>1*4*3</td><td>12</td></tr>
<tr><td>2</td><td>2</td><td>1</td><td>1*6*3</td><td>18</td></tr>
<tr><td>3</td><td>1</td><td>1</td><td>0*4*3</td><td>0</td></tr>
<tr><td></td><td></td><td></td><td></td><td>98</td></tr>
</table>
</td></tr>
<tr><td>38</td><td>[N =] 39</td><td>When N! is evaluated the result ends with 8 zeros. What is the largest possible value of N? Assume N is an integer.

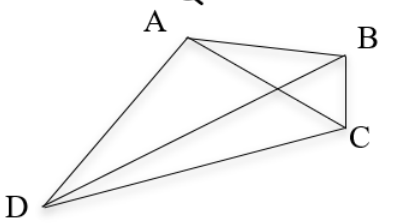
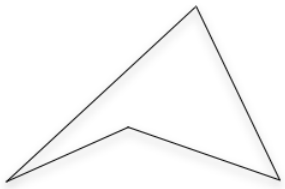
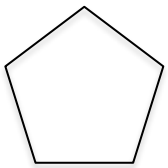
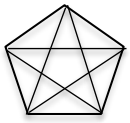
Every multiple of 5 results in one terminal zero, and every multiple of 25 results in two terminal zeros, so there should be 6 multiples of 5 and 1 multiple of 25 in N! The largest value of N having this number of multiples of 5 and 25 is 39.</td></tr>
</table>

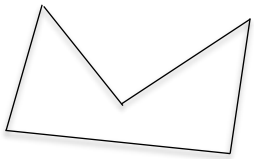
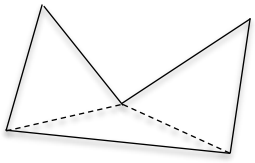
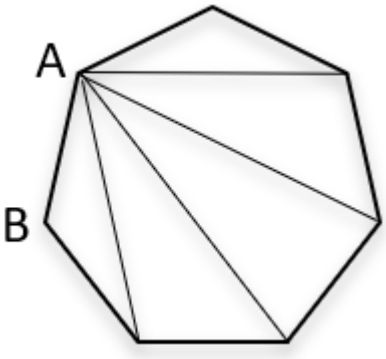
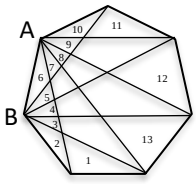
39	[A + B =] 53	As a reduced common fraction, the probability of getting 3 or more heads when you flip a fair two-sided coin 6 times is A/B. What is the value of A + B? Using Pascal's triangle, we have (1+6+15+20)/64, which simplifies to 21/32 and 21 + 32 = 53
40	[A =] 29	On a coordinate plane, every point on the line with equation $y = \frac{2}{5}x - 4$ is translated left two units and up five units to create a second line. The number of units in the distance between the original line and the translated line is $\sqrt{A}$. What is the value of A? The slope of $\overline{AB}$ is the reciprocal of the slope of the lines, so AB equals the distance between the two lines. $AB^2 = 2^2 + 5^2$. AB = $\sqrt{29}$, so A = 29 

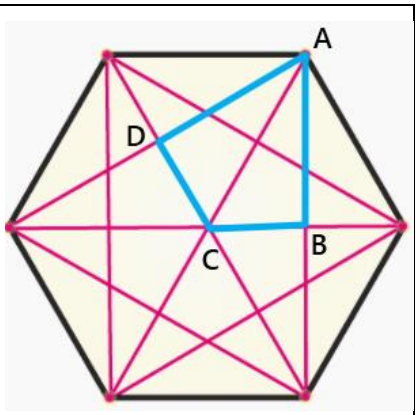
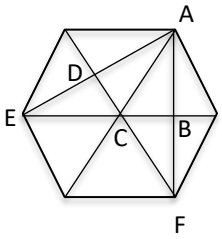
"Math is Cool" Masters -- 2020-21

6th Grade

Multiple Choice Solutions

6th	Answer	Solution
REFER TO THE FOLLOWING INFORMATION FOR PROBLEMS #1 THROUGH #4.		
A "diagonal" is any segment that has two vertices of a polygon as endpoints, that is not also a side of the polygon. For example, $\overline{AC}$ is a diagonal in the convex quadrilateral shown to the right, while $\overline{AB}$ is not. The diagonals of a polygon divide the polygon into a certain number of non-overlapping polygonal regions. For example, the diagonals of a convex quadrilateral divide the quadrilateral into 4 triangular regions as shown. The word "convex" means that all the interior angles of the polygon are less than 180°, as shown in polygon ABCD above. The word "concave" means that at least one of the interior angles of a polygon is more than 180°, as shown in the unlabeled polygon here. The word "regular" means that all the interior angles and all the sides of the polygon are congruent, as in, for example, a square. All regular polygons are convex.		Convex Quadrilateral  Concave Quadrilateral 
1	**B**	What is the number of non-overlapping polygonal regions created by the five diagonals of a regular pentagon?  A) 10 B) 11 C) 12 D) 14 E) 15 There are 11 non-overlapping regions, 10 triangles and 1 pentagon. 

2	**C**	Any pentagon, whether convex or concave, has five diagonals. In a concave pentagon, some of the diagonals are partly or entirely outside the original polygon. In the figure shown here, add only the two diagonals that can be drawn completely inside the polygon. What is the number of non-overlapping polygonal regions created as a result?  A) 1 B) 2 C) 3 D) 5 E) 10 There are 3 non-overlapping triangular regions created by the two diagonals that can be drawn completely inside the pentagon. 
3	**E**	On the heptagon below four diagonals can be drawn having vertex A as an endpoint. If the four diagonals having vertex B are also drawn in the figure, some non-overlapping regions will be created that are either triangles or quadrilaterals. How many of these regions are triangles? 
 A) 9 B) 10 C) 11 D) 12 E) 13 There are 13 non-overlapping triangular regions. 

4	**A**	The figure shown here is a regular hexagon with all its diagonals included. If the area of the hexagon is 60 square centimeters, what is the area of quadrilateral ABCD, shown here outlined in blue?  A) 10 cm^2 B) 31/3 cm^2 C) $6\sqrt{3}$ cm^2 D) 10.5 cm^2 E) 32/3 cm^2 A regular hexagon consists of 6 congruent equilateral triangles, and in this case each has an area of 10 cm^2. Diagonals $\overline{AE}$ and $\overline{AF}$ divide two of those equilateral triangles in half. The quadrilateral ABCD is made up of two of these half equilateral triangles, so together they have the same area as one of the equilateral triangles, or 10 cm^2. 

REFER TO THE FOLLOWING INFORMATION FOR PROBLEMS #5 THROUGH #7.

A formula for adding together the terms of a finite arithmetic sequence is $\frac{n}{2}(a_1 + a_n)$, where n = the number of terms in the sequence, a_1 is the first term of the sequence and a_n is the last term of the sequence. This formula works because in an arithmetic sequence, the sum of the first and last terms is the same as the sum of the 2nd and the 2nd to last terms, and the same as the sum of the 3rd and 3rd to last terms, and so on.

Also, if there are n terms in the sequence, there are $\frac{n}{2}$ pairs of terms each with the same sum. For example, the sum of the terms in the sequence {1, 2, 3, 4, 5, 6, 7, 8, 9} can be determined using this formula. The calculations would be $\left(\frac{9}{2}\right)(1 + 9)$ = (4.5)(10) = 45 and 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45.

Note: when there is an odd number of terms in the sequence, then there is not a whole number of equal pairs, but the formula still works. In this example there are 4.5 pairs that add up to 10. The extra 0.5 of a pair is always equal to the median of the sequence, which must be the same as $\frac{(a_1+a_n)}{2}$, so you can also use the formula in the form $\frac{n}{2}(a_1 + a_n)$.

5	**A**	What is the sum of the terms in the given arithmetic sequence? {34, 39, 44, 49, 54, 59, 64, 69, 74, 79, 84} A) 649 B) 704 C) 1298 D) 1408 E) 15708 (11/2)(34 + 84) = 5.5(118) = 11(59) = 590 + 59 = 649
6	**D**	What is the sum of the terms in the given arithmetic sequence? {1.3, 4.9, 8.5, . . . , 26.5, 30.1, 33.7} A) 87.5 B) 140 C) 157.5 D) 175 E) 192.5 The common difference is 3.6. 1.3 + 3.6(9) = 1.3 + 32.4 = 33.7, so there are a total of 10 terms. (10/2)(1.3 + 33.7) = 5(35) = 175

7	**C**	Let A be the sum of all two-digit integers. Let B be the sum of all two-digit multiples of 10. Let C be the sum of all two-digit multiples of 10 that are also divisible by 3. What is $A - (B - C)$? A) 4580.5 B) 4630.5 C) 4635 D) 4685 E) 5230 A = (90/2)(10 + 99) = 45(109) = 4500 + 405 = 4905 B = (9/2)(10 + 90) = 4.5(100) = 450 C = (3/2)(30 + 90) = 1.5(120) = 180 A – (B – C) = 4905 – (450 – 180) = 4905 – 270 = 4635

USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #8 THROUGH #10.

Tides for Seattle (Madison St.), Elliott Bay on Saturday, April 18, 2020.

Day	High/Low	Tide Time	Height (ft)	Sunrise	Sunset
Sa 18	Highest	3:40 AM	10.9	6:13 AM	8:04 PM
Sa 18	Low	10:02 AM	4.2		
Sa 18	High	3:14 PM	8.4		
Sa 18	Low	9:12 PM	2.2		

Tides for Tacoma, Commencement Bay, Sitcum Waterway on April 18, 2020.

Day	High/Low	Tide Time	Height (ft)	Sunrise	Sunset
Sa 18	Highest	3:44 AM	11.3	6:14 AM	8:04 PM

Tides for Budd Inlet, Olympia Shoal on April 18, 2020.

Day	High/Low	Tide Time	Height (ft)	Sunrise	Sunset
Sa 18	Highest	4:23 AM	14.1	6:17 AM	8:05 PM

8	**B**	In feet, what was the average of the highest tide at each of the three locations on April 18, 2020? A) $12.0\overline{6}$ ft B) 12.1 ft C) $12.1\overline{3}$ ft D) $12.1\overline{6}$ ft E) 12.2 ft (10.9 + 11.3 + 14.1)/3 = 36.3/3 = 12.1

9	**D**	Let S equal the number of minutes from sunrise to sunset in Seattle. Let T equal the number of minutes from sunrise to sunset in Tacoma. Let O equal the number of minutes from sunrise to sunset in Olympia. What was S + T + O on April 18, 2020? A) 2485 min B) 2487 min C) 2488 min D) 2489 min E) 2493 min 13:51 = 780 + 51 = 831 = S T = S – 1 = 830 O = S – 3 = 828 831 + 830 + 828 = 2489
10	**C**	The surface area of Elliot Bay is calculated to be approximately 8.1 square miles. How many more cubic feet of water were in Elliot Bay at 10:02 AM compared with the volume of water in Elliot Bay at 9:12 PM on April 18, 2020? Assume any change in the surface area of Elliot Bay during the changing tide levels is zero. (1 mile = 5280 feet) A) 16.2 ft^3 B) 85536 ft^3 C) 451630080 ft^3 D) 2384606822400 ft^3 E) Answer not given. 2(8.1)(5280)(5280) = 16.2(27878400) = 451630080

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6th Grade

Team Test Solutions

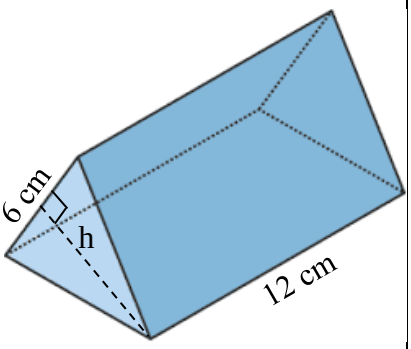
<table>
<tr><th>6th</th><th>Answer</th><th>Solution</th></tr>
<tr><td>1</td><td>16 [inches]</td><td>A rectangle is 32 inches long by 46 inches wide. What is the number of inches in the radius of the largest circle that can fit in its entirety inside the rectangle?

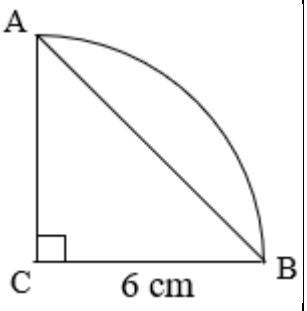
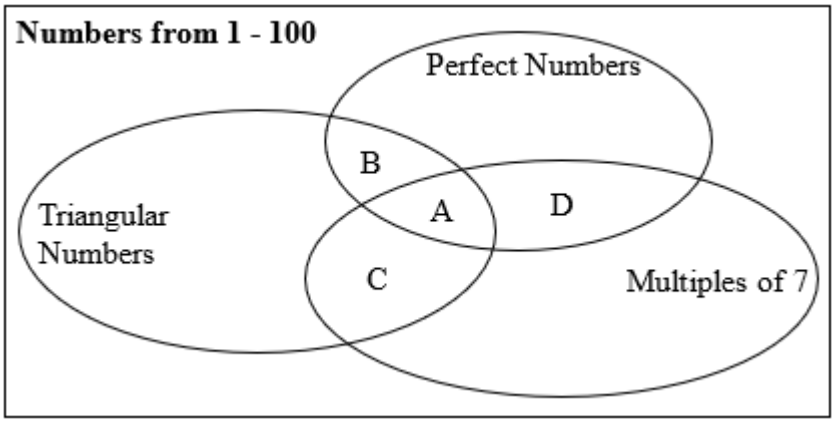
The shortest dimension of the rectangle determines the longest possible diameter of the circle, which is 32, so the radius would be 16.</td></tr>
<tr><td>2</td><td>487 cents</td><td>Elizabeth has 13 quarters, 11 dimes, 9 nickels, and 7 pennies. What is the number of cents in the value of Elizabeth's coins?

13x.25 + 11x.10 + 9x.05 + 7x.01 = 4.87, so 487 cents</td></tr>
<tr><td>3</td><td>50 [seconds]</td><td>Aditri runs a mile in 370 seconds. Paola runs a mile in 350 seconds. How many seconds longer than Paola will it take Aditri to run two and a half miles, if they continue running at these rates?

20 seconds longer per mile, so 2.5 x 20 = 50</td></tr>
<tr><td>4</td><td>[3 + 20 =] 23</td><td>Middle School Music and Sports Survey
<table><tr><th></th><th>Plays Team Sport</th><th>Does Not Play Team Sport</th><th>Total</th></tr><tr><td>Plays Instrument</td><td>8</td><td>3</td><td>11</td></tr><tr><td>Does Not Play Instrument</td><td>2</td><td>7</td><td>9</td></tr><tr><td>Total</td><td>10</td><td>10</td><td>20</td></tr></table>According to the data in the two-way table, as a reduced common fraction, the probability that a randomly selected student plays an instrument, but does not play a team sport is A/B. What is the value of A + B?

3 represents the students who play an instrument, but do not play a team sport. 20 is the total number of students, so the probability is 3/20. 3 + 20 = 23</td></tr>
</table>

5	[A + B =] 651	As a decimal to the nearest thousandth, the value of y when x = 4 for the equation below is A.B, where A represents a 2-digit whole number and B represents a 3-digit whole number. What is the value of A + B? $y = \frac{3x^2}{2} + \frac{3x}{4} - \frac{3}{8}$ $3x4^2/2 + 3x4/4 - 3/8 = 24 + 3 - 3/8 = 27 - .375 = 26.625$, and 26 + 625 = 651
6	10 [cm]	The volume of a triangular prism is 360 cm^3. The height of the prism is 12 cm and the base of the triangular base of the prism is 6 cm as shown. What is the number of centimeters in the height of the triangular base, h?  6h/2 x 12 = 360 36h = 360 h = 10
7	36 [three-digit numbers]	What is the number of three-digit positive numbers whose tens digit is half the ones digit? 148, . . . , 948 – 9 136, . . . , 936 – 9 124, . . . , 924 – 9 112, . . . , 912 – 9 9 x 4 = 36
8	-20	For the equation 5x + 3y = 24, there are 5 ordered pair solutions, (x, y), when 10 < x < 25 and in which x and y are both whole numbers. What is the total sum of all the x-values and y-values of these 5 solutions? For every integer substituted for x, the equation will ultimately be solved by dividing by 3, so the x-values with corresponding integer y-values will be spaced apart by 3. Start by substituting 0 for x, then y = 8. Counting by 3s from there give the solutions where both values are integers in the given domain: (12, -12), (15, -17), (18, -22), (21, -27), and (24, -32). 12 + -12 + 15 + -17 + 18 + -22 + 21 + -27 + 24 + -32 = -20

9	27	The figure shown is a quarter of a circle with a radius of 6 cm. Line segment $\overline{AB}$ is added to create ΔABC as shown. The area of the region (in square centimeters) that is inside the quarter circle, but outside ΔABC can be expressed in the form Dπ – E, where D and E are positive integers. What is the value of D + E?  Quarter circle area = 36π/4 = 9π Area of ΔABC = 6x6/2 = 18 Area of the region inside the quarter circle but outside the triangle = 9π – 18, so D = 9 and E = 18, and D + E = 27.
10	[E + F =] 7	A formula to derive Perfect Numbers is $2^{p-1}(2^p - 1)$, where p is any positive prime number. Triangular Numbers can be derived with the formula $\frac{n(n+1)}{2}$, where n is a positive whole number. According to the following Venn Diagram, as a reduced common fraction, the probability that a randomly drawn number from inside one of the three ovals is in one of the four regions labeled A, B, C, or D, is E/F. What is the value of E + F?  Perfect numbers: 6, 28 Triangular numbers: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91 Multiples of 7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98 Both perfect numbers are also triangular number so they will be in A, B, C, or D. In addition to 28, multiples of 7 that are also triangular numbers are 21 and 91, so there are a total of 4 numbers in A, B, C, or D and that will be the top number in the probability fraction. To get the bottom number, there are 13 triangular numbers and 14 multiples of 7. There are 3 numbers that are both, 21, 28, and 91, so the bottom number will be 13 + 14 – 3 = 24. 4/24 = 1/6, and 1 + 6 = 7

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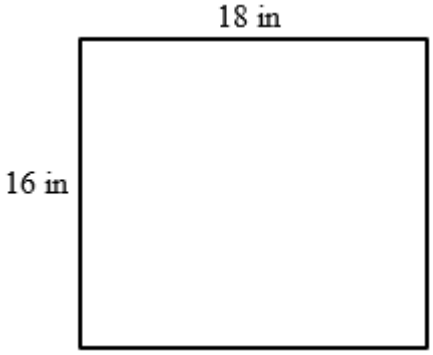
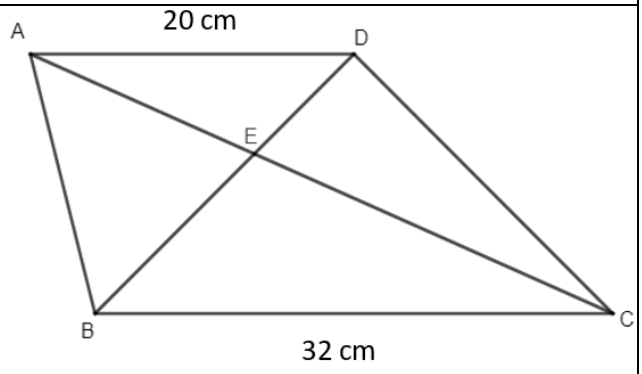
6th Grade

Triple Jump Solutions

6th	Answer	Solution
1	8	What is the sum of the given sequence? 1 + 1 – 1 + 2 – 2 + 3 – 3 + 4 – 4 + 5 – 5 + 6 – 6 + 7? 1 + 1 – 1 + 2 – 2 + 3 – 3 + 4 – 4 + 5 – 5 + 6 – 6 + 7 = 8
2	37	What is the largest prime factor of 111? 111 = 3 * 37, so the answer is 37.
3	4 [rectangles]	What is the maximum number of 8 inch by 9 inch rectangles that will fit inside a 16 inch by 18 inch rectangle? 16/8 = 2 and 18/9 = 2, so 2 * 2 = 4 
4	9 [marbles]	A jar has 21 marbles and there are six different colors, including red. As a reduced common fraction, the probability that a randomly chosen marble is red is 3/7. How many red marbles are in the jar? P(red) = r/21 = 3/7, so 7r = 63, and r = 9
5	884	What is the positive difference between the largest three-digit multiple of 37 and the smallest three-digit multiple of 23? 999 is the largest three-digit multiple of 37 (37 * 27) and 115 is the smallest three-digit multiple of 23 (23 * 5). 999 – 115 = 884
6	[x =] 45	What is the largest whole number value of x that satisfies the following inequality? 2x + 7 < 98 2x + 7 < 98 → 2x < 91 → x < 45.5, so the largest whole number solution is 45

7	25 [ordered pairs]	Let A and B each represent a whole number between 1 and 50, inclusive. It is possible for A and B to represent the same whole number. How many ordered pairs in the form (A, B) are there, such that A + B = 76? (26, 50), (27, 49), . . . , (49, 27), (50, 26) From 26 to 50 is 25 numbers, so 25 ordered pairs
8	[A + B =] 22	Jen rides her bike to the grocery store and back along the same route. Her total travel time is 28 minutes. Because of hills, her average speed in miles per hour on the way to the store is 3/5 of her average speed in miles per hour on the way back home. The number of minutes it takes her to ride to the store is A.B, where A is a 2-digit number and B is a single digit. What is the value of A + B? Using the d = rt formula, $d_{tostore}$ = (3r/5)(28 – t)/60 and d_{home} = rt/60, so (3r/5)(28 – t) = rt → 3/5(28 – t) = t → 84/5 = 8t/5 → t = 10.5 minutes on the way home and 17.5 minutes on the way to the store. 17 + 5 = 22
9	160 [cm^2]	In Trapezoid ABCD, AD = 20 cm, BC = 32 cm, and point E is the intersection of $\overline{AC}$ and $\overline{BD}$. The area of ΔADE is 62.5 cm^2. What is the number of square centimeters in the area of ΔBCE?  ΔADE ~ ΔBCE, so the sides are proportional and in a ratio of 20/32 = 5/8. The areas are in a ratio of $5^2/8^2$ = 25/64, so 25/64 = 62.5/x, where x is the area of ΔBCE. Two and a half times 25 is 62.5, so x = 2.5*64 = 160.

10	[$\alpha + \beta$ =] 3	Let A =$\left(\frac{-7}{4}\right)^2 + \frac{-29}{16}$, let B = $A^2 + \frac{-29}{16}$, let C = $B^2 + \frac{-29}{16}$, and so on through the 26 letters of the alphabet. As a reduced common fraction, Z = α/β. If α represents a negative integer, what is the value of $\alpha + \beta$? A = 49/16 – 29/16 = 20/16 = 5/4 B = 25/16 – 29/16 = -4/16 = -1/4 C = 1/16 – 29/16 = -28/16 = -7/4 Since -7/4 was the input resulting in A and now it will be the input resulting in D, D = A, E = B, F = C, and so on with these three values repeating every three letters. Twenty-four is a multiple of 3, so the 24th letter X = C, the 25th letter Y = A, and the 26th letter Z = B = -1/4 and -1 + 4 = 3

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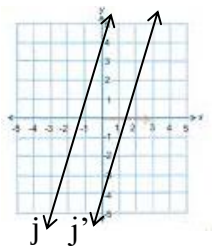
6th Grade

College Bowl Round #1 Solutions

6th	Answer	Solution
1	150 [cents]	Prisha goes to the store and buys a thirty-two-pack of energy drinks. The case costs forty-eight dollars. How many cents does one energy drink cost? 48/32 = 1.5 or $1.50 = 150 cents
2	[x =] 15	When a number, X, is multiplied by three, then by four, then by ten, the result is one thousand eight hundred. What is the value of X? 3*4*10*x = 1800 120x = 1800, x = 15
3	4 [days]	Five people can do one job in eight days. How many days would it take twenty people to do two of the same job? 5 people:1 job:8days = 20 people:1 job: 2 days = 20 people: 2 jobs:4 days
4	[M + B =] -1	A line with the equation y = 3x + 4 is rotated one hundred eighty degrees around the origin on a coordinate plane. When the equation of the new line is written in y = mx + b form, what is the value of m + b? (x, y) → (-x, -y) is the rule for a 180° rotation about the origin. The points with coordinates (-1, 1) and (-2, -2) on line j have images at (1, -1) and (2, 2) on line j’. The equation for line j’ is y = 3x – 4. So, 3 + -4 = -1. 

5	7 [snaps]	A bacteria population is one million. Every time Thanos snaps his fingers, the population is cut in half. How many times would he need to snap his fingers for the population to drop below ten thousand? If (x, y) = (snap #, pop), then (1, 500000), (2, 250000), (3, 125000), (4, 62500), (5, 31250), (6, 15625), (7, 7812.5). On the 7^{th} snap it goes below 10000.
6	32	Row zero of Pascal's triangle is one. Row one is one-one. Row two is one-two-one. What is the sum of the numbers in the fifth row of Pascal's triangle? 1 + 5 + 10 + 10 + 5 + 1 = 32 or $2^5 = 32$
7	[A + B =] 37	Mei rolls three fair six-sided dice. As a reduced common fraction, the probability of getting the same number on all three dice is A/B. What is the value of A plus B? 1/6*1/6*1/6*6 = 6/216 = 1/36 The extra *6 is because there are 6 ways the three numbers could be the same. So, 1 + 36 = 37.
8	11 [cards]	Abel has six cards, Bonita has seven cards, and Cherise has eight cards. Cherise gives half of her cards to Abel and half of her cards to Bonita. How many cards does Bonita now have? 8/2 = 4, 4 + 7 = 11
9	[A + B =] 39	As a decimal, the mean of the two-digit integers from thirty-one to thirty-eight inclusive is A.B, where A is a two-digit whole number and B is a single digit. What is the value of A plus B? Since they are consecutive integers, it's an arithmetic sequence and the mean of all the numbers is the same as the average of the first and last number, so (31 + 38)/2 = 69/2 = 34.5. So 34 + 5 = 39
10	350 [min]	Let seventy divided by twelve equal X. How many minutes are in X hours? 70/12*60 = 350

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6th Grade

College Bowl Round #2 Solutions

6th	Answer	Solution
1	72 [centimeters]	A vine grows three centimeters every two days. How many centimeters will it grow in forty-eight days? 3*24 = 72
2	[A =] 45	As a decimal, 1/5 plus 1/4 is 0.A (zero point A), where A represents a two-digit whole number. What is the value of A? 1/5 + 1/4 = 5/20 + 4/20 = 9/20 = 0.45, so A = 45
3	84 [in^2]	What is the number of square inches in the area of a rectangle if the length is twelve inches and the width is seven inches? 12*7 = 84
4	8 [letters]	Tony Stark was born on a Friday in nineteen seventy, a non-leap year. How many letters are in the day of the week of his first birthday? 7*52 = 364 and there are 365 days in a non-leap year, so his 1^{st} birthday would have been one day after Friday, or Saturday, and Saturday has 8 letters
5	2 [numbers]	One set of numbers contains positive composite numbers less than twenty. A second set of numbers contains positive odd numbers less than twenty. How many numbers are members of both sets? Set 1 = {4, 6, 8, 9, 10, 12, 14, 15, 16, 18} Set 2 = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19} 9 and 15 are members of both sets
6	1,260 [ways]	How many distinct ways are there to rearrange the letters in the word RAINING? 7!/(2!*2!) = 5040/4 = 1260

7	[x=] 6	Solve for x: $\frac{5x-6}{x-3} = 8$ (5x – 6)/(x – 3) = 8 5x – 6 = 8x – 24, 3x = 18, x = 6
8	27 [square feet]	A rectangle has dimensions of three feet by three yards. How many square feet are in its area? 3 yards = 9 feet, so 3*9 = 27
9	32 [kids]	Forty kids are loaded onto a bus and twenty percent of them forgot to put on their seatbelts. How many kids remembered to put their seatbelts on? 0.8*40 = 32
10	784	Let A = 1 + 2 + 3 + 4 + 5 + 6 + 7. What is A squared? 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28 28^2 = 784

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6th Grade

College Bowl Round #3 Solutions

6th	Answer	Solution
1	62 [days]	How many days are there from April eighteenth to June eighteenth inclusive? 4/18 to 4/30 = 13 days May has 31 days 6/1 to 6/18 = 18 days 13 + 31 + 18 = 62
2	8421	In a certain four-digit number, the digit in the ones place is half the digit in the tens place, which is half the digit in the hundreds place, which is half the digit in the thousands place. What is this four-digit number? 1 is in the ones place, 2 is in the tens place, 4 is in the hundreds place, and 8 is in the thousands place, so the number is 8421
3	8 [inches]	The area of a triangle is 124 square inches. The base of the triangle is 31 inches. How many inches are in the height of the triangle? 31h/2 = 124, 31h = 248, h = 8
4	6	Let A/B and C/D represent two fractions. A, B, C, and D are each replaced with one of the digits from one through four (each digit is used only one time). What is the largest possible value of A/B times C/D? 4/2*3/1 = 6
5	[A =] 225	A circle has an area of 144π square centimeters. Its radius is increased by 25 percent to make a new circle. In terms of π, the number of square centimeters in the area of the new circle is $A\pi$. What is the value of A? A = 144π, r = 12. Increasing 12 by 25% means the new radius is 15. $15^2\pi = 225\pi$, so A = 225

6	22 [min]	Fernando averaged thirty miles per hour driving to work. On his drive back home along the same route, he averaged twenty-five miles per hour, and it took two minutes longer than the drive to work. What was the total number of minutes spent driving to and from work? D = rt, D = 30*t/60 and D = 25(t + 2)/60, so 30t/60 = 25(t + 2)/60 or 30t = 25(t + 2), 30t = 25t + 50, 5t = 50, t = 10 and t + 2 = 12, 10 + 12 = 22
7	9,000,000 or 9 million [numbers]	How many positive seven-digit numbers are there? The number of numbers from 1000000 to 9999999 is 9999999 – 999999 = 9000000
8	63	The first positive odd number is one. What is the thirty-second positive odd number? The first positive even number is 2. The 32nd positive even number is 64, so the 32nd positive odd number is 63.
9	[A + B =] 18	Two cards are drawn from a standard deck without replacement. As a reduced common fraction, the probability that both are hearts is A/B. What is the value of A + B ? 13/52*12/51 = 1/4*4/17 = 1/17, so A + B = 18
10	400 [minutes]	How many minutes will Pavarotti sing if he sings for six and two-thirds hours? $6\frac{2}{3}$ = 20/3 and 20/3*60 = 400