“Math is Cool” Masters -- 2020-21

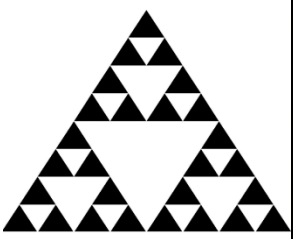
7th Grade

Mental Math Solutions

7th	Answer	Solution
1	756	What is the positive difference between one thousand and six and two hundred and fifty? 1006 – 250 = 756
2	6	What is the greatest common factor of eighteen and twenty-four? 18 = 6*3, 24 = 6*4, so 6 is the GCF
3	2100	What is the product of the numbers five, six, seven, and ten? 5*6*7*10 = 2100
4	[A + B =] 5	As a reduced common fraction, the probability of flipping no heads when you flip a fair coin twice is A over B. What is the value of A plus B? P(T,T) = 1/2*1/2 = 1/4, so 1 + 4 = 5
5	[x =] 12	Solve for X in the equation two X plus four equals twenty-eight. 2x + 4 = 28, 2x = 24, x = 12
6	7 [letters]	Sunday is one day after Saturday. How many letters are in the name of the day that is seventeen days after Saturday? Since 14 is a multiple of 7, 14 days from now will be Saturday, 15-Sun, 16-Mon, 17-Tue and Tuesday has 7 letters
7	[A + B =] 7	As a reduced common fraction, the slope of a line with points eight comma three and twelve comma six is A over B. What is the value of A + B? (8, 3) and (12, 6) Slope = (6 – 3)/(12 – 8) = 3/4, and 3 + 4 = 7

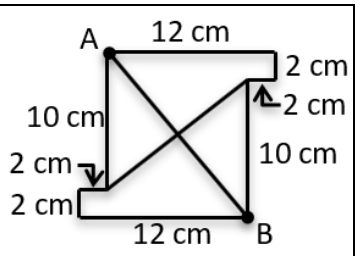
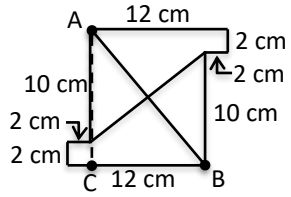
8	[A + B =] 45	As a decimal to the nearest hundredth, the number of square inches in the area of a square whose perimeter is eighteen inches is A point B, where A and B each represent two-digit integers. What is the value of A plus B? 18/4 = 4.5, 4.5^2 = 20.25 and 20 + 25 = 45

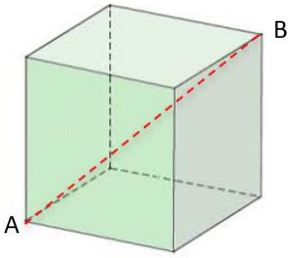
"Math is Cool" Masters -- 2020-21
7th Grade
Individual Test Solutions

7th	Answer	Solution
1	11	What is the smallest positive odd 2-digit number? 11
2	[A + B =] 3	As a reduced common fraction, the probability of drawing a red card from a standard deck of cards is A/B. What is the value of A + B? 26 red out of 52 total = 1/2 and 1 + 2 = 3
3	13 [triangles]	How many white triangles are in the figure shown?  4 * 3 + 1 = 13
4	[A =] 100	The circumference of a circle is 20π feet. In terms of π, the number of square feet in the area of the circle is Aπ. What is the value of A? C = 20π, so d = 20 and r = 10 $A = \pi r^2$, so $A = 10^2\pi = 100\pi$
5	315	A proper factor of a number is a factor that is not the number itself. The largest proper factor of 210 is 105. What is the first number greater than 210 that also has 105 as its largest proper factor? 315 = 3 x 105 Since 315 can't be divided evenly by 2, 105 must be the largest proper factor
6	[A + B =] 7	Mitch eats half of a cake and Lindsey eats one-third of the same cake. The reduced common fraction representing the portion of the cake that has not been eaten yet is A/B. What is the value of A + B? 1 – 1/2 - 1/3 = 1/6 and 1 + 6 = 7
7	1	Evaluate and express as a whole number: $\frac{12}{37} \cdot \frac{37}{12}$ The product of reciprocals is 1

8	[A + B =] 4	Reyna walks to her friend's house at an average rate of 1 mile per hour. It takes her 20 minutes to walk to her friend's house. As a reduced common fraction, the number of miles in the distance to her friend's house is A/B. What is the value of A + B? 20 minutes is 1/3 of an hour so the distance is 1/3 of a mile, and 1 + 3 = 4
9	984 [miles]	The side length of a regular octagon is 123 miles. What is the number of miles in the perimeter of the octagon? 8 x 123 = 984
10	79	Riddesh has scores of 92, 86, 88, and 90 on his last four rounds of golf. What score must he get on the next round in order to lower his average for the five rounds to exactly 87? 5 x 87 = 435 435 – (92 + 86 + 88 + 90) = 79
11	-15	Evaluate and express as an integer: $(2 - 6)^3 + (15 - 22)^2$ $(2 – 6)^3 + (15 – 22)^2 = (-4)^3 + (-7)^2 = -64 + 49 = -15$
12	5 [cm]	The length of the hypotenuse of a right triangle is 6 cm. One of the legs is $\sqrt{11}$ cm long. What is the number of centimeters in the length of the other leg? $(\sqrt{11})^2 + b^2 = 6^2$ $11 + b^2 = 36$, so $b^2 = 25$, so $b = 5$
13	3 [ft]	An 18-foot rope is cut 5 times at regular intervals so that each of the resulting pieces are the same length. What is the number of feet in the length of each piece? 5 cuts result in 6 pieces 18/6 equals 3
14	[x =] 18	Solve for x: $30 - 7.5x = -105$ $30 – 7.5x = -105$ $-7.5x = -135$ $x = 18$
15	4 [days]	Ten giants take ten days to build ten cities. How many days would it take five giants to build two cities? 10 G : 10 D : 10 C 10 G : 2 D : 2 C – the same number of giants take 1/5 the time to build 1/5 the cities 5 G : 4 D : 2 C – half of the giants take twice as long to build the same number of cities

16	[A + B + C =] 26	The given expression can be rewritten in the form Ax(Bx – C), where A, B, and C are positive integers and B and C have no common factors other than 1. What is the value of A + B + C? $38x^2 - 95x$ $38x^2 - 95x = 19x(2x) - 19x(5) = 19x(2x - 5)$ and 19 + 2 + 5 = 26
17	[A + B =] 9	Ahaan flips a nickel, a dime, and a quarter. As a reduced common fraction, the probability that the nickel is heads, the dime is heads, and the quarter is tails is A/B. What is the value of A + B? 1/2 x 1/2 x 1/2 = 1/8 and 1 + 8 = 9
18	1	What is the units digit of 7 to the power of 256? Powers of 7 cycle through 7, 9, 3, 1 and 256 is evenly divisible by four, so the units digit is 1
19	13 [cards]	Allison is holding 13 cards, Bella is holding 12 cards and Cam is holding 11 cards. Allison gives Bella and Cam 2 cards each. Bella gives Allison 1 card and Cam 3 cards. Cam gives Allison 3 cards and Bella 2 cards. After these trades, what is the number of cards held by the person holding the largest number of cards? A: 13, 9, 10, 13 B: 12, 14, 10, 12 C: 11, 13, 16, 11
20	15	All the prime numbers are removed from the set of integers from 10 to 20, inclusive. What is the mean of the remaining numbers? (10 + 12 + 14 + 15 + 16 + 18 + 20)/7 = 15
21	[A + B =] 59	The following expression reduced to a common fraction is A/B. What is the value of A + B? $\frac{2^8 \cdot 3^{11} \cdot 5^3 \cdot 7^{13}}{2^7 \cdot 3^{13} \cdot 5^4 \cdot 7^{12}}$ $\frac{2^8 \cdot 3^{11} \cdot 5^3 \cdot 7^{13}}{2^7 \cdot 3^{13} \cdot 5^4 \cdot 7^{12}} = \frac{2 \cdot 7}{3^2 \cdot 5}$ = 14/45 and 14 + 45 = 59

22	[C + D =] 63	In simplest radical form, the number of centimeters in the length of $\overline{AB}$ in the figure shown is $C\sqrt{D}$, where C and D are integers and D has no perfect square factors other than 1. What is the value of C + D?  Add $\overline{AC}$ to make a right triangle. The legs of this triangle are 10 and 12, so $AB^2 = 10^2 + 12^2 = 244$ and $AB = \sqrt{244} = 2\sqrt{61}$ and 2 + 61 = 63. 
23	15	On a coordinate plane, point A (-10, 21) is translated 3 units to the right and down 13 units resulting in point A'. Then point A' is reflected over the y-axis resulting in point A''. What is the sum of the coordinates of A''? A(-10, 21) → A'(-7, 8) → A''(7, 8) 7 + 8 = 15
24	98 [inches]	What is the smallest possible perimeter in inches of a rectangle with sides of integer length in inches and an area of 600 square inches? Closest two numbers that multiply to 600 are 24*25, so 2*(24 + 25) = 98
25	325000 [codes]	How many 5-character codes can be made if the first and last characters must be different letters of the alphabet, the three middle characters must be nonnegative single-digit integers that may repeat, and the third of the three integers must be odd? 26*25*10*10*5 = 325000

26	[D + E =] 12	Hero's Theorem states that the area of a triangle equals $\sqrt{s(s-a)(s-b)(s-c)}$, where s is the semi-perimeter (half of the perimeter) of the triangle, and a, b, and c are the three side lengths of the triangle. In simplest radical form, the number of square inches in the area of a triangle with side lengths of 5, 6, and 7 inches is D$\sqrt{\text{E}}$. What is the value of D + E? Perimeter is 18; Semi-perimeter is 9 so area is $\sqrt{9(9-5)(9-6)(9-7)}$ = $\sqrt{9(4)(3)(2)}$ = $\sqrt{9}\sqrt{4}\sqrt{6}$ = 3(2)$\sqrt{6}$ = 6$\sqrt{6}$ and 6 + 6 = 12
27	[C + D =] 10	The side length of the cube shown here is 7 meters. In simplest radical form, the number of meters in the length of $\overline{AB}$ is C$\sqrt{D}$. What is the value of C + D? If x is the length of the side of a cube, then the space diagonal is x$\sqrt{3}$, so the space diagonal is 7$\sqrt{3}$ meters and 7 + 3 = 10 
28	6 [ways]	How many ways are there to make a sum of 10 by adding together the digits 1, 3 and 7, or any combination of these digits? 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 1 + 1 + 1 + 1 + 1 + 1 + 1 + 3 1 + 1 + 1 + 1 + 3 + 3 1 + 1 + 1 + 7 1 + 3 + 3 + 3 3 + 7
29	90 [minutes]	An 80-gallon bathtub has a faucet and a drain. When the faucet is on and the drain is closed, it takes 15 minutes to fill the bathtub. When the bathtub is full, the faucet is off, and the drain is open, it takes 18 minutes to empty the bathtub. If the drain is open while the faucet is on, how long will it take to fill the bathtub? 80/15 – 80/18 = 16/3 – 40/9 = 48/9 – 40/9 = 8/9 gallons per minute 80/(8/9) = 80*9/8 = 90 minutes

30	[A + B =] 37	As a reduced common fraction, the sum of $\frac{\frac{3}{5}}{7} + \frac{3}{\frac{5}{7}}$ is A/B. What is the value of A + B? (3/5)/7 = 3/35 and 3/(5/7) = 21/5 3/35 + 21/5 = 150/35 = 30/7 and 30 + 7 = 37
31	16	Find the sum of all terms of the infinite geometric sequence beginning 4, 3, 9/4 . . . First term/(1-ratio) = 4/[1-(3/4)] = 4/(1/4) = 16
32	[A + B =] 338	A jar has four red marbles and some other marbles in it. When drawing two marbles out of the jar without replacement, the probability of getting one red and one blue marble is determined using the following calculation: $\left(\frac{4}{17}\right)\left(\frac{7}{16}\right)(2)$ Once the marbles are put back in the jar, as a reduced common fraction, the probability of drawing two blue marbles out of the jar with replacement is A/B, where A is a two-digit whole number and B is a three-digit whole number. What is the value of A + B? 7/17*7/17 = 49/289 and 49 + 289 = 338
33	[A + B =] 84	A data set has ten distinct positive whole numbers and a mean of 50. As a decimal to the nearest tenth, the largest possible median of the set is A.B, where A is a two-digit whole number and B is a single digit. What is the value of A + B? The sum of the ten numbers is 10 * 50 = 500 To make the median as large as possible, make the lowest 4 numbers, 1, 2, 3, and 4 so the other 6 numbers add up to 490. For the median to be as large as possible, the remaining 6 numbers need to be consecutive, or nearly consecutive and they need to add up to 490. 490/6 = 81 and 4/6, so the last 6 numbers would be 79, 80, 81, 82, 83, 85, because 79 + 80 + 81 + 82 + 83 + 85 = 490. The median of the set would be (79 + 80)/2 = 79.5 and 79 + 5 = 84

34

54

Let A = m/n, let B = p/q, and let m, n, p, and q be distinct single-digit positive whole numbers. If A + B = 12, what is the product of m and p?

9/1 + 8/2	13	9/2 + 8/1	12.5
9/1 + 7/2	12.5	9/2 + 7/1	11.5
9/1 + 6/2	12	9/2 + 6/1	10.5
9/1 + 5/2	11.5	8/1 + 7/2	11.5

The 1st column shows the decreasing pattern in the sums that are made when 9 is over 1. The 2nd column shows the decreasing pattern in the sums that are made when 9 is over 2, and the largest possible sum that does not include 9. From this sample, it is apparent that there is only one way for a sum of 12 to occur, namely if m = 9 and p = 6, or if m = 6 and p = 9. Either way, m * p = 9 * 6 = 54.

35

[A + B =] 27

Mike has the 13 hearts from a standard deck in his hand. Molly randomly chooses two cards from his hand. As a reduced common fraction, the probability that the two cards add up to 7 is A/B. What is the value of A + B? Note: the ace has a value of one and each of the face cards has a value of ten.

The probability of getting an Ace and a 6 is 1/13*1/12*2 = 1/78
The probability of getting a 2 and a 5 or a 3 and a 4 is the same so the answer is 1/78 + 1/78 + 1/78 = 3/78 = 1/26 and 1 + 26 = 27

36

24

If x + y = 26 and xy = 50, what is the positive value of $\sqrt{x^2 + y^2}$?

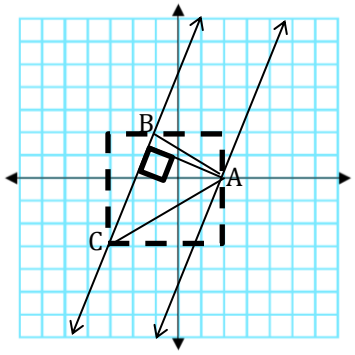
$(x + y)^2 = x^2 + 2xy + y^2 = 26^2 = 676$
$x^2 + y^2 = 676 - 2xy = 676 - 100 = 576$
$\sqrt{576} = 24$

37

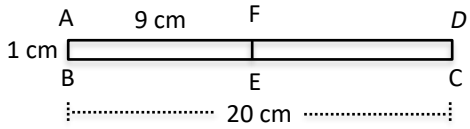
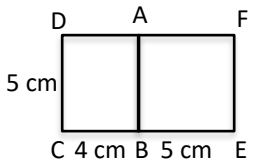
98 [lineups]

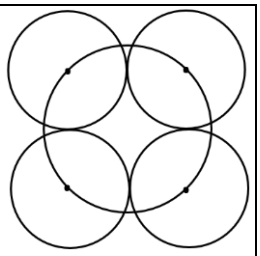
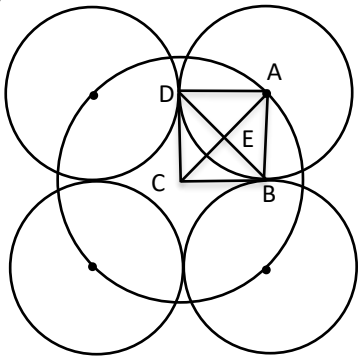
A certain sport has three positions: forward, middle, and back, and during a game each team has five players on the field at one time. If the Redtown Raptors team has two forwards, four middles, and three backs on their roster, how many different five-player lineups can they use if there must always be at least one of each position on the field?

Combos of Forwards, Middles & Backs				
F	M	B		
1	1	3	2*4*1	8
1	2	2	2*6*3	36
1	3	1	2*4*3	24
2	1	2	1*4*3	12
2	2	1	1*6*3	18
3	1	1	0*4*3	0
				98

38	41	Kennard's favorite 6-digit number is 720,720. What is the sum of the distinct prime factors of Kennard's number? 1001 x 720 1001 x 9 x 80 7 x 11 x 13 x 3^2 x 2^4 x 5 7 + 11 + 13 + 3 + 2 + 5 = 41
39	[A + B =] 4178	Alexa is trying to break a piñata. She needs to hit it on at least four of her five attempts to break it, and she has a 60% chance of hitting it on each attempt. As a reduced common fraction, the probability that she breaks the piñata is A/B, where A and B are each four-digit integers. What is the value of A + B? Case 1: 4/5 attempts hit -> $(3/5)^4 \times (2/5) \times {}_5C_4 = 162/625$ Case 2: 5/5 attempts hit -> $(3/5)^5 = 243/3125$ Case 1 + Case 2 = 1053/3125 and 1053 + 3125 = 4178
40	[A + B =] 48	On a coordinate plane, every point on the line with equation $y = \frac{5}{2}x - 5$ is translated left three units and up two units to create a second line. The number of units in the distance between the original line and the translated line is $A/\sqrt{B}$, where A and B are both prime numbers. What is the value of A + B? The height of ΔABC is the distance between the two lines. The area of ΔABC can first be calculated using the surround and conquer method: 25 – 5 – 3 – 7.5 = 9.5. The area is also equal to BC * height/2 or $h\sqrt{29}/2$ = 9.5. So, h = $19/\sqrt{29}$. 19 + 29 = 48 

41	1367	The sum of a number and its reciprocal is 37. What is the sum of the number squared plus its reciprocal squared? $x + \frac{1}{x} = 37$ $(x + \frac{1}{x})^2 = 1369$ $x^2 + 2 + \frac{1}{x^2} = 1369$ $x^2 + \frac{1}{x^2} = 1367$
42	115 [minutes]	Alan, Brooklyn, and Carol are doing a project for school tomorrow. Alan and Brooklyn know they can finish the project by themselves in 2 hours. Brooklyn and Carol can finish in 3 hours, and Alan and Carol can finish in 4 hours. They start working at 5:00 PM, but Carol leaves at 6 PM, and she does not return. Beginning at 5:00 PM, how many total minutes will it take to complete the project? A works at a base rate of $\frac{1}{A}$ of a project in 1 hour. B works at a base rate of $\frac{1}{B}$ of a project in 1 hour. C works at a base rate of $\frac{1}{C}$ of a project in 1 hour. We now have a system of equations. $\frac{1}{A} + \frac{1}{B} = \frac{1}{2}$ $\frac{1}{B} + \frac{1}{C} = \frac{1}{3}$ $\frac{1}{A} + \frac{1}{C} = \frac{1}{4}$ Adding all three equations yields $\frac{1}{A} + \frac{1}{B} + \frac{1}{C} = \frac{13}{24}$ This means all three of them working together can finish 13/24ths of a project in 1 hour. Carol leaves after 1 hour, so right as she leaves, 13/24 of the project has been completed. 11/24 of the project is left. Using the first equation, we know that Alan and Brooklyn can finish 1/2 of a job in an hour. Dividing 11/24 by 1/2 yields 11/12 hours. The total time of the project, from start to finish, is 1&11/12 hours, or 115 minutes.

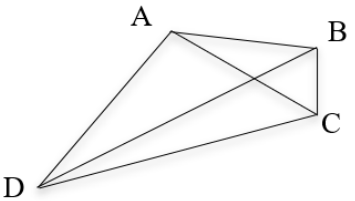
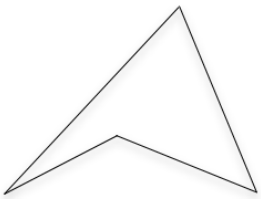
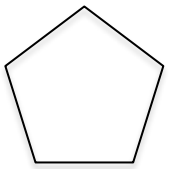
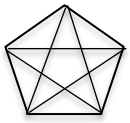
43	25 [cm^2]	The area of rectangle ABCD is 20 cm^2, and the perimeter of rectangle ABEF is 20 cm. Both rectangles have side lengths that are whole numbers. What is the positive difference between the maximum and minimum area of the resulting figure when these two rectangles are combined? Hint: the rectangles may overlap and the side lengths of ABCD and ABEF in the maximum area arrangement can be different than the side lengths of the two rectangles in the minimum area arrangement. The minimum area is achieved when the rectangles overlap, such that AB is 1, BC is 20, and AF is 9, and when they are arranged such that A, F, and D are collinear. See below. This minimum area would be 20 cm^2.  The maximum area is achieved when the rectangles have no overlap. However, the side lengths do not necessarily have to be the same as the rectangles in the minimum, as there is no restriction on the side lengths, only restrictions on what the area and perimeter can be. The maximum area is achieved when CD = EF = 5, BC is 4, and BE is 5. See below. This area would be 45 cm^2.  45 – 20 = 25 cm^2.

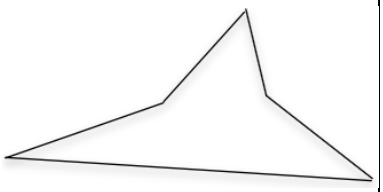
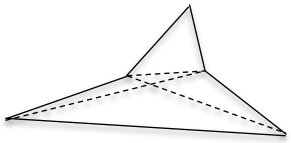
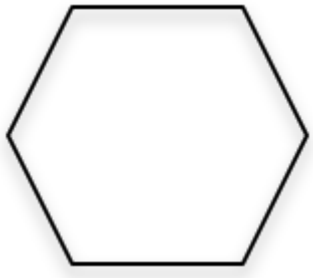
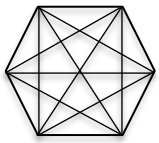
44	[G + F =] 7	 In the drawing shown, the centers of each of the four smaller circles are on the larger circle and the radius of the larger circle is 10 centimeters. The four smaller circles are tangent to each other. The number of centimeters in the radius of one of the smaller circles can be written in the form F$\sqrt{\mathrm{G}}$, where F and G are integers and G has no perfect square factors other than 1. What is the value of G + F? $\angle B$, $\angle D$ (tangent to a circle is perpendicular to a radius), and $\angle C$ (360/4) are right angles and BC = CD (tangent segments from the same exterior point to a circle are congruent), so ABCD is a square. AC = 10 so AB = 10/$\sqrt{2}$ = 5$\sqrt{2}$ and 5 + 2 = 7. 
45	[M + N + P + Q + R =] 47	 On rectangle ABCD, AD = 10 cm and DC = 20 cm. A point E is chosen along side $\overline{AB}$ to create ΔCDE. The positive difference between the number of centimeters in the longest and shortest possible perimeters of ΔCDE can be written in the form M + N$\sqrt{P}$ – Q$\sqrt{R}$, where M, N, P, Q, and R are positive integers and neither P nor R have perfect square factors other than 1. What is the value of M + N + P + Q + R? The longest perimeter occurs when E coincides with A or B to make a right triangle with 10 and 20 as the length of the legs. The hypotenuse is then $\sqrt{500}$ = 10$\sqrt{5}$ and the perimeter is 30 + 10$\sqrt{5}$. The shortest perimeter occurs when E is at the midpoint of $\overline{AB}$. This creates an isosceles right triangle where $\overline{CD}$ is the hypotenuse. The two legs are each 10$\sqrt{2}$ so the perimeter is 20 + 20$\sqrt{2}$. 30 + 10$\sqrt{5}$ – (20 + 20$\sqrt{2}$) = 10 + 10$\sqrt{5}$ - 20$\sqrt{2}$ and 10 + 10 + 5 + 20 + 2 = 47

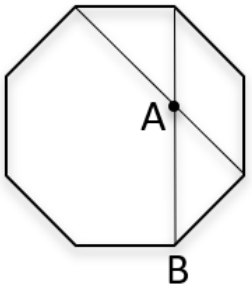
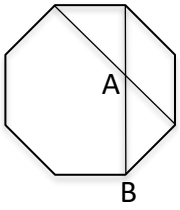
"Math is Cool" Masters -- 2020-21

7th Grade

Multiple Choice Solutions

7th	Answer	Solution
REFER TO THE FOLLOWING INFORMATION FOR PROBLEMS #1 THROUGH #4. A "diagonal" is any segment that has two vertices of a polygon as endpoints, that is not also a side of the polygon. For example, $\overline{AC}$ is a diagonal in the convex quadrilateral shown to the right, while $\overline{AB}$ is not. The diagonals of a polygon divide the polygon into a certain number of non-overlapping polygonal regions. For example, the diagonals of a convex quadrilateral divide the quadrilateral into 4 triangular regions as shown. The word "convex" means that all the interior angles of the polygon are less than 180°, as shown in polygon ABCD above. The word "concave" means that at least one of the interior angles of a polygon is more than 180°, as shown in the unlabeled polygon here. The word "regular" means that all the interior angles and all the sides of the polygon are congruent, as in, for example, a square. All regular polygons are convex. Convex Quadrilateral  Concave Quadrilateral 		
1	B	What is the number of non-overlapping polygonal regions created by the five diagonals of a regular pentagon?  A) 10 B) 11 C) 12 D) 14 E) 15 There are 11 non-overlapping regions, 10 triangles and 1 pentagon. 

2	C	Any pentagon, whether convex or concave, has five diagonals. In a concave pentagon, some of the diagonals are partly or entirely outside the original polygon. In the figure shown below, add only the three diagonals that can be drawn completely inside the polygon. What is the number of non-overlapping polygonal regions created as a result?  A) 1 B) 3 C) 5 D) 6 E) 7 There are 5 non-overlapping triangular regions created by the three diagonals that can be drawn completely inside the pentagon. 
3	E	What is the ratio of non-overlapping triangular regions to non-overlapping quadrilateral regions created by all of the diagonals in a regular hexagon?  A) 1:2 B) 2:1 C) 5:2 D) 1:3 E) 3:1 There are 18 non-overlapping triangular regions and 6 non-overlapping quadrilateral regions. 18:6 = 3:1 

4	A	The figure shown here is a regular octagon with a perimeter of 24 cm. What is the length of $\overline{AB}$?  A) $3\sqrt{2}$ cm B) 4 cm C) 4.3 cm D) $\sqrt{19}$ cm E) 4.5 cm The two triangles formed by the two diagonals are 45-45-90 right triangles with legs of 3 cm, so the hypotenuse or AB will be $3\sqrt{2}$. 

REFER TO THE FOLLOWING INFORMATION FOR PROBLEMS #5 THROUGH #7.

A formula for adding together the terms of a finite arithmetic sequence is $\frac{n}{2}(a_1 + a_n)$, where n = the number of terms in the sequence, a_1 is the first term of the sequence and a_n is the last term of the sequence. This formula works because in an arithmetic sequence, the sum of the first and last terms is the same as the sum of the 2nd and the 2nd to last terms, and the same as the sum of the 3rd and 3rd to last terms, and so on.

Also, if there are n terms in the sequence, there are $\frac{n}{2}$ pairs of terms each with the same sum. For example, the sum of the terms in the sequence {1, 2, 3, 4, 5, 6, 7, 8, 9} can be determined using this formula. The calculations would be $\left(\frac{9}{2}\right)(1 + 9)$ = (4.5)(10) = 45 and 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45.

Note: when there is an odd number of terms in the sequence, then there is not a whole number of equal pairs, but the formula still works. In this example there are 4.5 pairs that add up to 10. The extra 0.5 of a pair is always equal to the median of the sequence, which must be the same as $\frac{(a_1+a_n)}{2}$, so you can also use the formula in the form $\frac{n}{2}(a_1 + a_n)$.

5	A	What is the sum of the terms in the given arithmetic sequence? {34, 39, 44, 49, 54, 59, 64, 69, 74, 79, 84} A) 649 B) 704 C) 1298 D) 1408 E) 15708 (11/2)(34 + 84) = 5.5(118) = 11(59) = 590 + 59 = 649

6	D	What is the sum of the terms in the given arithmetic sequence? {-1.3, 3.9, 9.1, . . . , 61.1, 66.3, 71.5} A) 263.25 B) 456.3 C) 491.4 D) 526.5 E) 561.6 The common difference is 5.2. -1.3 + 5.2(14) = -1.3 + 72.8 = 71.5, so there are a total of 15 terms. (15/2)(-1.3 + 71.5) = 7.5(70.2) = 15(35.1) = 526.5
7	C	Let A be the sum of all two-digit integers. Let B be the sum of all two-digit multiples of 10. Let C be the sum of all two-digit multiples of 10 that are also divisible by 3. What is A – (B – C) ? A) 4580.5 B) 4630.5 C) 4635 D) 4685 E) 5230 A = (90/2)(10 + 99) = 45(109) = 4500 + 405 = 4905 B = (9/2)(10 + 90) = 4.5(100) = 450 C = (3/2)(30 + 90) = 1.5(120) = 180 A – (B – C) = 4905 – (450 – 180) = 4905 – 270 = 4635

USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #8 THROUGH #10.

Tides for Seattle (Madison St.), Elliott Bay on Saturday, April 18, 2020.

Day	High/Low	Tide Time	Height (ft)	Sunrise	Sunset
Sa 18	Highest	3:40 AM	10.9	6:13 AM	8:04 PM
Sa 18	Low	10:02 AM	4.2		
Sa 18	High	3:14 PM	8.4		
Sa 18	Low	9:12 PM	2.2		

Tides for Tacoma, Commencement Bay, Sitcum Waterway on April 18, 2020.

Day	High/Low	Tide Time	Height (ft)	Sunrise	Sunset
Sa 18	Highest	3:44 AM	11.3	6:14 AM	8:04 PM

Tides for Budd Inlet, Olympia Shoal on April 18, 2020.

Day	High/Low	Tide Time	Height (ft)	Sunrise	Sunset
Sa 18	Highest	4:23 AM	14.1	6:17 AM	8:05 PM

8	B	In feet, what was the average of the highest tide at each of the three locations on April 18, 2020? A) $12.0\overline{6}$ ft B) 12.1 ft C) $12.1\overline{3}$ ft D) $12.1\overline{6}$ ft E) 12.2 ft (10.9 + 11.3 + 14.1)/3 = 36.3/3 = 12.1

9	D	Let S equal the number of minutes from sunrise to sunset in Seattle. Let T equal the number of minutes from sunrise to sunset in Tacoma. Let O equal the number of minutes from sunrise to sunset in Olympia. What was S + T + O on April 18, 2020? A) 2485 min B) 2487 min C) 2488 min D) 2489 min E) 2493 min 13:51 = 780 + 51 = 831 = S T = S – 1 = 830 O = S – 3 = 828 831 + 830 + 828 = 2489
10	C	The surface area of Elliot Bay is calculated to be approximately 8.1 square miles. What was the number of cubic feet in the volume of water that flowed into Elliot Bay between the low tide at 10:02 AM and the high tide at 3:14 PM on April 18, 2020? Assume any change in the surface area of Elliot Bay during the changing tide levels is zero. (1 mile = 5280 feet) A) 34.02 ft^3 B) 179625.6 ft^3 C) 948423168 ft^3 D) 5007674327040 ft^3 E) Answer not given. 4.2(8.1)(5280)(5280) = 34.02(27878400) = 948423168

"Math is Cool" Masters -- 2020-21

7th Grade

Team Test Solutions

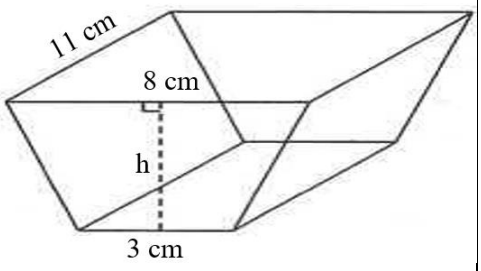
<table>
<tr><th>7th</th><th>Answer</th><th>Solution</th></tr>
<tr><td>1</td><td>16 [inches]</td><td>A rectangle is 32 inches long by 46 inches wide. What is the number of inches in the radius of the largest circle that can fit in its entirety inside the rectangle?

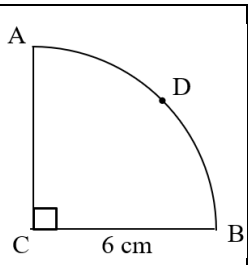
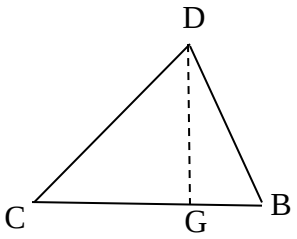
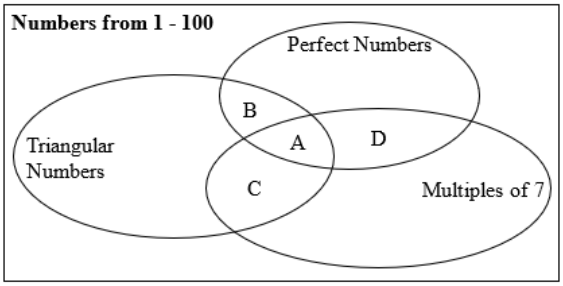
The shortest dimension of the rectangle determines the longest possible diameter of the circle, which is 32, so the radius would be 16.</td></tr>
<tr><td>2</td><td>487 cents</td><td>Elizabeth has 13 quarters, 11 dimes, 9 nickels, and 7 pennies. What is the number of cents in the value of Elizabeth's coins?

13x.25 + 11x.10 + 9x.05 + 7x.01 = 4.87, so 487 cents</td></tr>
<tr><td>3</td><td>[A + B =] 97</td><td>Aditri runs a mile in 370 seconds. Paola runs a mile in 90 percent of the time that it takes Aditri. As a decimal to the nearest tenth, Aditri takes A.B seconds longer than Paola to run two and a half miles, assuming they continue running at these rates. If A represents a two-digit integer and B represents a single digit, what is the value of A + B?

37 seconds longer per mile, so 2.5 x 37 = 92.5, so 92 + 5 = 97</td></tr>
<tr><td>4</td><td>[3 + 20 =] 23</td><td>According to the data in the two-way table, as a reduced common fraction, the probability that a randomly selected student plays an instrument, but does not play a team sport is A/B. What is the value of A + B?

Middle School Music and Sports Survey
<table><tr><th></th><th>Plays Team Sport</th><th>Does Not Play Team Sport</th><th>Total</th></tr><tr><td>Plays Instrument</td><td>8</td><td>3</td><td>11</td></tr><tr><td>Does Not Play Instrument</td><td>2</td><td>7</td><td>9</td></tr><tr><td>Total</td><td>10</td><td>10</td><td>20</td></tr></table>3 represents the students who play an instrument, but do not play a team sport. 20 is the total number of students, so the probability is 3/20. 3 + 20 = 23</td></tr>
</table>

5	[A + B =] 651	As a decimal to the nearest thousandth, the value of y when x = 4 for the equation below is A.B, where A represents a 2-digit whole number and B represents a 3-digit whole number. What is the value of A + B? $y = \frac{3x^2}{2} + \frac{3x}{4} - \frac{3}{8}$ $3x4^2/2 + 3x4/4 - 3/8 = 24 + 3 - 3/8 = 27 - .375 = 26.625$, and 26 + 625 = 651
6	4 [cm]	The volume of a trapezoidal prism is 242 cm^3. The height of the prism is 11 cm and the bases of the trapezoidal base are 3 cm and 8 cm as shown. What is the number of centimeters in the height of the trapezoidal base, h?  (8 + 3)h/2 x 11 = 242 121h/2 = 242 121h = 484 h = 4
7	32 [three-digit numbers]	What is the number of three-digit positive numbers whose tens digit equals the product of the ones and the hundreds digits? 100, 200, . . . , 900, 111, 122, 221, 133, 331, 144, 242, 441, 155, 551, 166, 263, 362, 661, 177, 771, 188, 284, 482, 881, 199, 393, 991
8	231	For the equation $\frac{2}{3}x - \frac{3}{4}y = 3$, there are 6 ordered pair solutions, (x, y), when -3 < x < 48 and in which x and y are both integers. What is the total sum of all the x-values and y-values of these 6 solutions? Multiplying the equation on both sides by 12 results in 8x – 9y = 36. For every integer substituted for x, the equation will ultimately be solved by dividing by -9, so the x-values with corresponding integer y-values will be spaced apart by 9. The smallest x-value in the given domain that has a corresponding y-value that is an integer is x = 0, y = -4. Counting by 9s from there give the other solutions where both values are integers: (9, 4), (18, 12), (27, 20), (36, 28), and (45, 36). 0 + 9 + 18 + 27 + 36 + 45 + -4 + 4 + 12 + 20 + 28 + 36 = 231

9	[E + F =] 11	The figure shown is a quarter of a circle with a radius of 6 cm. A point D is placed exactly halfway between A and B along $\widehat{AB}$ as shown. In simplest radical form, the number of square centimeters in the area of ΔBCD is E$\sqrt{F}$. What is the value of E + F?  If you sketch triangle BCD and add the height DG, then triangle CDE is a 45-45-90 triangle with a hypotenuse of 6. The legs would be 6/rt 2 or 3rt 2 and this would be the height of triangle BCD. Area = (3rt 2 x 6)/2 = 9rt 2, so 9 + 2 = 11. 
10	[E + F =] 7	A formula to derive Perfect Numbers is $2^{p-1}(2^p - 1)$, where p is any positive prime number. Triangular Numbers can be derived with the formula $\frac{n(n+1)}{2}$, where n is a positive whole number. According to the following Venn Diagram, as a reduced common fraction, the probability that a randomly drawn number from inside one of the three ovals is in one of the four regions labeled A, B, C, and D, is E/F. What is the value of E + F?  Perfect numbers: 6, 28 Triangular numbers: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91 Multiples of 7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98 Both perfect numbers are also triangular number so they will be in A, B, C, or D. In addition to 28, multiples of 7 that are also triangular numbers are 21 and 91, so there are a total of 4 numbers in A, B, C, or D and that will be the top number in the probability fraction. To get the bottom number, there are 13 triangular numbers and 14 multiples of 7. There are 3 numbers that are both, 21, 28, and 91, so the bottom number will be 13 + 14 – 3 = 24. 4/24 = 1/6, and 1 + 6 = 7

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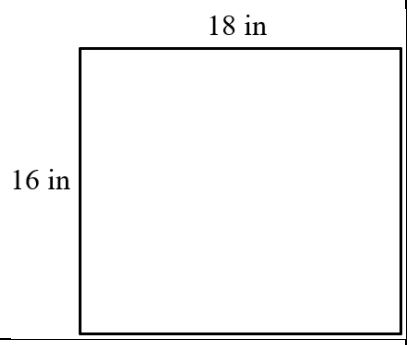
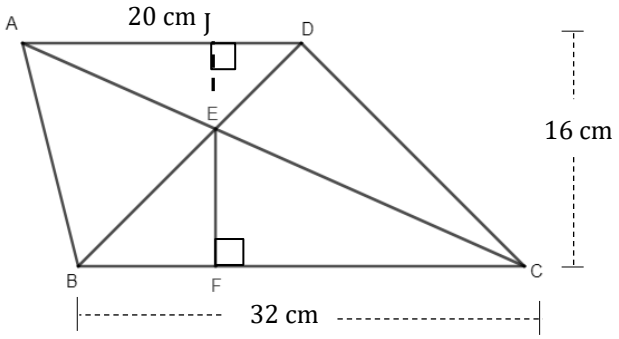
7th Grade

Triple Jump Solutions

7th	Answer	Solution
1	8	What is the sum of the given sequence? 1 + 1 – 1 + 2 – 2 + 3 – 3 + 4 – 4 + 5 – 5 + 6 – 6 + 7? 1 + 1 – 1 + 2 – 2 + 3 – 3 + 4 – 4 + 5 – 5 + 6 – 6 + 7 = 8
2	19	What is the largest prime factor of 323? 323 = 17 * 19, so the answer is 19
3	12 [rectangles]	What is the maximum number of 4 inch by 6 inch rectangles that will fit inside a 16 inch by 18 inch rectangle? 16/4 = 4 and 18/6 = 3, so 4 * 3 = 12 
4	9 [marbles]	A jar has 21 marbles and there are six different colors, including red. As a reduced common fraction, the probability that a randomly chosen marble is red is 3/7. How many red marbles are in the jar? P(red) = r/21 = 3/7, so 7r = 63, and r = 9
5	884	What is the positive difference between the largest three-digit multiple of 37 and the smallest three-digit multiple of 23? 999 is the largest three-digit multiple of 37 (37 * 27) and 115 is the smallest three-digit multiple of 23 (23 * 5). 999 – 115 = 884
6	[x =] 56	For the following inequality, what is the largest integer solution? $7x \leq 5x + 113$ $7x \leq 5x + 113 \rightarrow 2x \leq 113 \rightarrow x \leq 56.5$, so the largest integer solution is 56

7	25 [ordered pairs]	Let A and B each represent a whole number between 1 and 50, inclusive. It is possible for A and B to represent the same whole number. How many ordered pairs in the form (A, B) are there, such that A + B = 76? (26, 50), (27, 49), . . . , (49, 27), (50, 26) From 26 to 50 is 25 numbers, so 25 ordered pairs
8	[A + B =] 22	Jen rides her bike to the grocery store and back along the same route. Her total travel time is 28 minutes. Because of hills, her average speed in miles per hour on the way to the store is 3/5 of her average speed in miles per hour on the way back home. The number of minutes it takes her to ride to the store is A.B, where A is a 2-digit number and B is a single digit. What is the value of A + B? Using the d = rt formula, $d_{tostore}$ = (3r/5)(28 – t)/60 and d_{home} = rt/60, so (3r/5)(28 – t) = rt → 3/5(28 – t) = t → 84/5 = 8t/5 → t = 10.5 minutes on the way home and 17.5 minutes on the way to the store. 17 + 5 = 22
9	[G + H =] 141	In Trapezoid ABCD, AD = 20 cm, BC = 32 cm, point E is the intersection of $\overline{AC}$ and $\overline{BD}$, and $\overline{EF} \perp \overline{BC}$. The height of the trapezoid is 16 cm. As a reduced common fraction, the height of ΔBCE, $\overline{EF}$, is equal to G/H, where G is a three-digit integer and H is a two-digit integer. What is the value of G + H? ΔADE ~ ΔBCE, so the sides and the altitudes are proportional and in a ratio of 20/32 = 5/8. Since 5 + 8 = 13, EJ = 5/13*16 and EF = 8/13*16 = 128/13 and 128 + 13 = 141. 

10	$[\alpha + \beta =] 3$	Let A = $\left(\frac{-7}{4}\right)^2 + \frac{-29}{16}$, let B = $A^2 + \frac{-29}{16}$, let C = $B^2 + \frac{-29}{16}$, and so on through the 26 letters of the alphabet. As a reduced common fraction, Z = α/β. If α represents a negative integer, what is the value of $\alpha + \beta$? A = 49/16 – 29/16 = 20/16 = 5/4 B = 25/16 – 29/16 = -4/16 = -1/4 C = 1/16 – 29/16 = -28/16 = -7/4 Since -7/4 was the input resulting in A and now it will be the input resulting in D, D = A, E = B, F = C, and so on with these three values repeating every three letters. Twenty-four is a multiple of 3, so the 24th letter X = C, the 25th letter Y = A, and the 26th letter Z = B = -1/4 and -1 + 4 = 3

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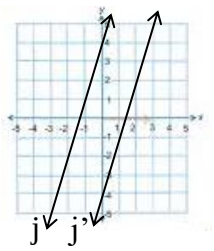
7th Grade

College Bowl Round #1 Solutions

7th	Answer	Solution
1	150 [cents]	Prisha goes to the store and buys a thirty-two-pack of energy drinks. The case costs forty-eight dollars. How many cents does one energy drink cost? 48/32 = 1.5 or $1.50 = 150 cents
2	[x =] 15	When a number, X, is multiplied by three, then by four, then by ten, the result is one thousand eight hundred. What is the value of X? 3*4*10*x = 1800 120x = 1800, x = 15
3	4 [days]	Five people can do one job in eight days. How many days would it take twenty people to do two of the same job? 5 people:1 job:8days = 20 people:1 job: 2 days = 20 people: 2 jobs:4 days
4	[M + B =] -1	A line with the equation y = 3x + 4 is rotated one hundred eighty degrees around the origin on a coordinate plane. When the equation of the new line is written in y = mx + b form, what is the value of m + b? (x, y) → (-x, -y) is the rule for a 180° rotation about the origin. The points with coordinates (-1, 1) and (-2, -2) on line j have images at (1, -1) and (2, 2) on line j’. The equation for line j’ is y = 3x – 4. So, 3 + -4 = -1. 

5	7 [snaps]	A bacteria population is one million. Every time Thanos snaps his fingers, the population is cut in half. How many times would he need to snap his fingers for the population to drop below ten thousand? If (x, y) = (snap #, pop), then (1, 500000), (2, 250000), (3, 125000), (4, 62500), (5, 31250), (6, 15625), (7, 7812.5). On the 7^{th} snap it goes below 10000.
6	32	Row zero of Pascal's triangle is one. Row one is one-one. Row two is one-two-one. What is the sum of the numbers in the fifth row of Pascal's triangle? 1 + 5 + 10 + 10 + 5 + 1 = 32 or $2^5 = 32$
7	[A + B =] 37	Mei rolls three fair six-sided dice. As a reduced common fraction, the probability of getting the same number on all three dice is A/B. What is the value of A plus B? 1/6*1/6*1/6*6 = 6/216 = 1/36 The extra *6 is because there are 6 ways the three numbers could be the same. So, 1 + 36 = 37.
8	11 [cards]	Abel has six cards, Bonita has seven cards, and Cherise has eight cards. Cherise gives half of her cards to Abel and half of her cards to Bonita. How many cards does Bonita now have? 8/2 = 4, 4 + 7 = 11
9	[A + B =] 39	As a decimal, the mean of the two-digit integers from thirty-one to thirty-eight inclusive is A.B, where A is a two-digit whole number and B is a single digit. What is the value of A plus B? Since they are consecutive integers, it's an arithmetic sequence and the mean of all the numbers is the same as the average of the first and last number, so (31 + 38)/2 = 69/2 = 34.5. So 34 + 5 = 39
10	350 [min]	Let seventy divided by twelve equal X. How many minutes are in X hours? 70/12*60 = 350

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7th Grade

College Bowl Round #2 Solutions

7th	Answer	Solution
1	72 [centimeters]	A vine grows three centimeters every two days. How many centimeters will it grow in forty-eight days? 3*24 = 72
2	[A =] 45	As a decimal, 1/5 plus 1/4 is 0.A (zero point A), where A represents a two-digit whole number. What is the value of A? 1/5 + 1/4 = 5/20 + 4/20 = 9/20 = 0.45, so A = 45
3	84 [in^2]	What is the number of square inches in the area of a rectangle if the length is twelve inches and the width is seven inches? 12*7 = 84
4	8 [letters]	Tony Stark was born on a Friday in nineteen seventy, a non-leap year. How many letters are in the day of the week of his first birthday? 7*52 = 364 and there are 365 days in a non-leap year, so his 1^{st} birthday would have been one day after Friday, or Saturday, and Saturday has 8 letters
5	2 [numbers]	One set of numbers contains positive composite numbers less than twenty. A second set of numbers contains positive odd numbers less than twenty. How many numbers are members of both sets? Set 1 = {4, 6, 8, 9, 10, 12, 14, 15, 16, 18} Set 2 = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19} 9 and 15 are members of both sets
6	1,260 [ways]	How many distinct ways are there to rearrange the letters in the word RAINING? 7!/(2!*2!) = 5040/4 = 1260

7	[x=] 6	Solve for x: $\frac{5x-6}{x-3} = 8$ (5x – 6)/(x – 3) = 8 5x – 6 = 8x – 24, 3x = 18, x = 6
8	27 [square feet]	A rectangle has dimensions of three feet by three yards. How many square feet are in its area? 3 yards = 9 feet, so 3*9 = 27
9	32 [kids]	Forty kids are loaded onto a bus and twenty percent of them forgot to put on their seatbelts. How many kids remembered to put their seatbelts on? 0.8*40 = 32
10	784	Let A = 1 + 2 + 3 + 4 + 5 + 6 + 7. What is A squared? 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28 28^2 = 784

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7th Grade

College Bowl Round #3 Solutions

7th	Answer	Solution
1	62 [days]	How many days are there from April eighteenth to June eighteenth inclusive? 4/18 to 4/30 = 13 days May has 31 days 6/1 to 6/18 = 18 days 13 + 31 + 18 = 62
2	8421	In a certain four-digit number, the digit in the ones place is half the digit in the tens place, which is half the digit in the hundreds place, which is half the digit in the thousands place. What is this four-digit number? 1 is in the ones place, 2 is in the tens place, 4 is in the hundreds place, and 8 is in the thousands place, so the number is 8421
3	8 [inches]	The area of a triangle is 124 square inches. The base of the triangle is 31 inches. How many inches are in the height of the triangle? 31h/2 = 124, 31h = 248, h = 8
4	6	Let A/B and C/D represent two fractions. A, B, C, and D are each replaced with one of the digits from one through four (each digit is used only one time). What is the largest possible value of A/B times C/D? 4/2*3/1 = 6
5	[A =] 225	A circle has an area of 144π square centimeters. Its radius is increased by 25 percent to make a new circle. In terms of π, the number of square centimeters in the area of the new circle is $A\pi$. What is the value of A? A = 144π, r = 12. Increasing 12 by 25% means the new radius is 15. $15^2\pi = 225\pi$, so A = 225

6	22 [min]	Fernando averaged thirty miles per hour driving to work. On his drive back home along the same route, he averaged twenty-five miles per hour, and it took two minutes longer than the drive to work. What was the total number of minutes spent driving to and from work? D = rt, D = 30*t/60 and D = 25(t + 2)/60, so 30t/60 = 25(t + 2)/60 or 30t = 25(t + 2), 30t = 25t + 50, 5t = 50, t = 10 and t + 2 = 12, 10 + 12 = 22
7	9,000,000 or 9 million [numbers]	How many positive seven-digit numbers are there? The number of numbers from 1000000 to 9999999 is 9999999 – 999999 = 9000000
8	63	The first positive odd number is one. What is the thirty-second positive odd number? The first positive even number is 2. The 32nd positive even number is 64, so the 32nd positive odd number is 63.
9	[A + B =] 18	Two cards are drawn from a standard deck without replacement. As a reduced common fraction, the probability that both are hearts is A/B. What is the value of A + B ? 13/52*12/51 = 1/4*4/17 = 1/17, so A + B = 18
10	400 [minutes]	How many minutes will Pavarotti sing if he sings for six and two-thirds hours? $6\frac{2}{3}$ = 20/3 and 20/3*60 = 400