"Math is Cool" Masters -- 2022-23

High School

Mental Math Solutions

	Answer	Solution
1	-11	The sum of the x and y coordinates of a point is 11. What is the sum of the x and y coordinates after the point is rotated 180° around the origin? **For example, if the point is (11, 0), it will rotate to (-11, 0). Any point (x, y) such that x + y = 11 will rotate to (-x, -y), where –(x + y) = -11.**
2	140 [dimes]	How many dimes have the same monetary value as 56 quarters? **56/4 = 14$** **14$ x 10 = 140 dimes**
3	-10	What is the sum of the first ten odd positive integers minus the sum of the first ten even positive integers? **1st 10 odd: 10^2 = 100** **1st 10 even: 10(11) = 110** **100 – 110 = -10**
4	70 [inches]	A 5-inch by 7-inch rectangle is cut out of a corner of a 16-inch by 19-inch rectangle. In inches, what is the perimeter of the remaining figure? **The perimeter of the original rectangle doesn't change, it's just been moved. P = 2(16+19) = 2(35) = 70.**
5	3 [sets]	How many distinct sets of three positive integers have a mean of 6, a median of 7, and no mode? **All numbers must be different, and 7 must be in the middle. The numbers must sum to 18. The only possibilities are:** **3, 7, 8** **2, 7, 9** **1, 7, 10**
6	25 [feet]	On each bounce after being dropped from a height, a bouncy ball rises to 5/4 of its previous height. If the ball is initially dropped from a height of 16 feet, how high in feet does the ball rise after its second bounce? **After 1st bounce: 16(5/4) = 20 ft** **After 2nd bounce: 20(5/4) = 25 ft**

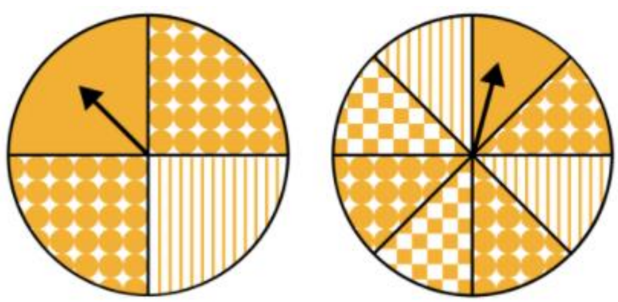
7	6 [years]	One-third of Biff's current age is twice that of Biff's age five years ago. What is Biff's current age in years? **(1/3)B = 2(B – 5)** **(1/3)B = 2B – 10** **10 = (5/3)B** **B = 30/5 = 6**
8	72 [arrangements]	Six people are to be seated at a round table. Two of the people refuse to sit next to each other. How many distinct arrangements of the six people are there? Rotations of an arrangement are not distinct to one another. **Call the two stubborn people A and B. Put A in a particular position at the table. B cannot be seated on either side of A, so there are only 3 spots left for B. If B chooses one of those spots, the remaining four people have 4! = 24 ways to arrange themselves. Multiply by 3 for the 3 different spots that B can choose. 24x3 = 72.** **Alternatively, circular permutations = (n – 1)! = 5! = 120 total. Subtract off 4! = 24 for B seated on one side of A, and 4! for B seated on the other side of A. 120 – 48 = 72.**

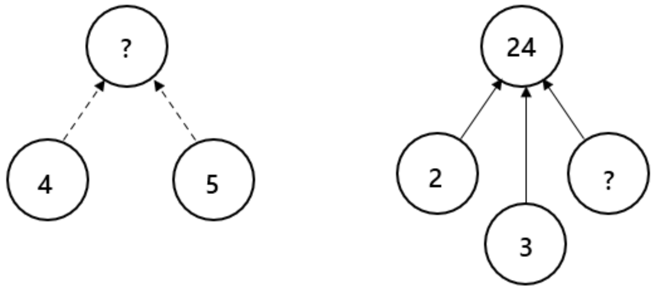
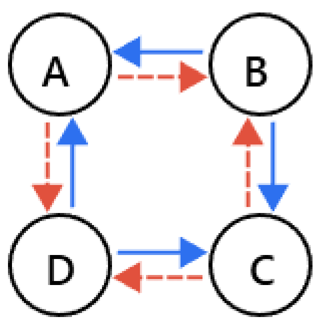
"Math is Cool" Masters -- 2022-23
High School
Individual Test Solutions

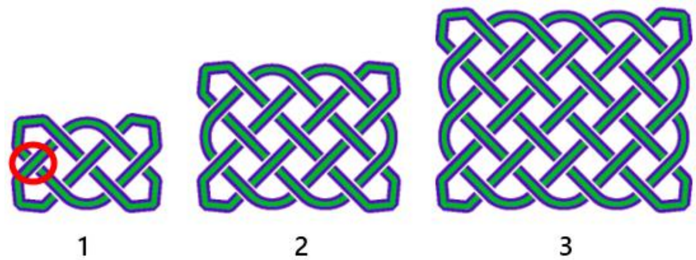
	Answer	Solution
1	36	Find r(3), given that: r(q) = 8q + 12 **r(3) = 8(3) + 12 = 36**
2	65	Thirteen is 20% of what number? **13 = 10% of 130** **13 = 20% of 130/2 = 65**
3	31 [°]	Two lines are perpendicular to each other. One line is rotated 13° clockwise, and the other line is rotated 46° counterclockwise. What is the measure, in degrees, of the smaller angle between the two lines after they have been rotated? **Start at 90° + 13° + 46° = 149°, which is the larger angle. The smaller angle is 180 – 149 = 31°.** **Equivalently, 90 – 13 – 46 = 31, which is one less step.**
4	2	The mean of a set of 5 numbers is 6. Four of the numbers are 3, 5, 8 and 12. What is the fifth number? **Sum of the 5 numbers = 30** **3+5+8+12 = 28** **Therefore, 5th number = 2**
5	[discriminant] = -7	What is the value of the discriminant of the following quadratic equation? $F(x) = x^2 - 5x + 8$ **Discriminant =** $b^2 - 4ac = (-5)^2 - 4(1)(8) = 25 - 32 = -7$
6	9	What is the greatest common factor of 36 and 27? **9 is the largest integer that divides into both 27 and 36.**
7	50 [%]	When a single fair coin is flipped twice, what is the probability in percent that the second flip shows heads? **A single coin flip is always 50/50 if the coin is fair, and coin flips are independent of each other.**
8	32	Simplify: $128^{\left(\frac{5}{7}\right)}$ $\mathbf{128^{\frac{1}{7}} = 2, 2^5 = 32}$
9	[x =] 12	Solve for x: $\frac{x!}{(x-1)!} = 12$ **12!/11! = 12**

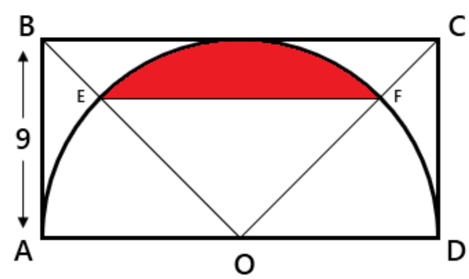
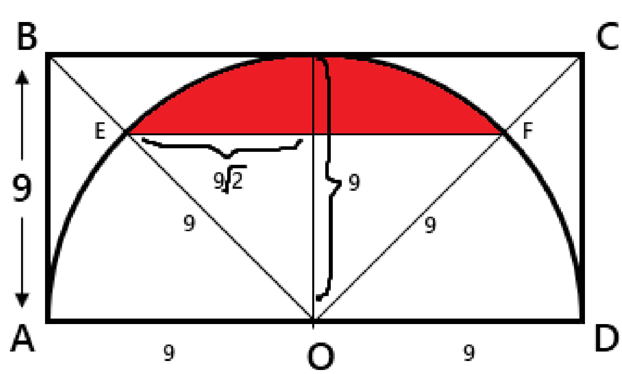
10	8 [times]	A solid brass sphere has a diameter of 5 inches and a smaller solid brass sphere of the same density has a diameter of 2.5 inches. How many times the weight of the smaller sphere is the weight of the larger sphere? **$V = (4/3)\pi r^3$, therefore the ratio of the volumes (and also weights) will be the cubes of the radii. The bigger radius is twice the smaller radius, therefore the ratio will be $2^3 = 8$.**
11	[sum] = 5	Find the sum of the zeros of the following function. $f(x) = 5x - x^2$ **$X(5 - x) = 0$** **$X = 0$ or $5 - x = 0$** **$X = 0$ or $x = 5$**
12	165 [mg]	Connie drank 6 cans of Red Bull, which have 110 mg of caffeine per can. If caffeine has a half-life of 5 hours, how much caffeine in mg will be left in her body after 10 hours, assuming that she started at 0 mg of caffeine? **Starting amount = 110x6 = 660 mg of caffeine. After 5 hours, reduce to 330 mg. After 10 hours, reduce to 165 mg.**
13	25 [% increase]	An iPhone case that used to cost $40 has been repriced at $50. What was the percent increase in the price? **% change = (new – old)/old * 100** **(50 – 40)/40 * 100 = 25**
14	17	When the sum of 3x and 4 is divided by 5, the result is 11. What is the value of x? **$(3x+4)/5 = 11$** **$3x + 4 = 55$** **$3x = 51$** **$x = 17$**
15	[range =] 50	The range of a data set {a, b, c, d, e} is 10. What is the range of the data set {5a, 5b, 5c, 5d, 5e}? **Range = $e - a = 10$** **Therefore, $5e - 5a = 5(e - a) = 5(10) = 10$.**
16	120 [flavor combinations]	The corner ice cream shop has 10 flavors to choose from. Their specialty is a bowl with three scoops, each with a different flavor. How many distinct flavor combinations are possible? **10C3 = 10!/(7!3!) = (10x9x8)/(3x2x1) = 720/6 = 120**
17	6 [minutes]	How many minutes does it take a car averaging 24 miles per hour to travel 2.4 miles? **24 miles in 60 minutes is equivalent to 2.4 miles in 6 minutes.**
18	129[°]	In degrees, what is the measure of the supplement of the complement of an angle whose measure is 39°? **90 – 39 = 51 is the complement. 180 – 51 = 129 is the supplement**

19	[x + y =] -7	The midpoint of the line segment connecting the points (-7, -3) and (3, -7) is (x, y). What is x + y? **(-7+3)/2, (-3 + -7)/2** **-4/2, -10/2** **(-2, -5)**
20	1331	Evaluate: $\frac{11^7}{121^2}$ $\frac{\mathbf{11^7}}{\mathbf{121^2}} = \frac{\mathbf{11^7}}{(\mathbf{11^2})^2} = \frac{\mathbf{11^7}}{\mathbf{11^4}} = \mathbf{11^3}$ **11x11 = 121** **121x11 = 1331**
21	13 [people]	There were 100 mints in a candy box. The box was passed down along a row of people. The first person took one mint. Each subsequent person took more mints than the person before them, until the box was empty. What is the largest number of people that could have been in the row? **The largest number of people will occur if each person takes the minimal number of mints, which would be just one more than the person before them. Therefore the number of mints taken will be 1+2+3+…, which must be ≤ 100. The sum of the 1st 13 integers is (13)(14)/2 = 91, and the sum of the 1st 14 integers is (14)(15)/2 = 105, which is too many. Therefore the maximum no. of people is 13.**
22	24 [ways]	In how many distinct ways can 3 different vases be arranged onto four shelves such that no two vases are on the same shelf? **This is equivalent to arranging 3 different letters and a blank spot in a row. 4! = 24**
23	100	Compute the number of zeros at the end of the following product: $1^1 \cdot 2^2 \cdot 3^3 \cdot 4^4 \cdot \ldots \cdot 24^{24} \cdot 25^{25}$ **Each factor of 5, when multiplied by a 2, will contribute a 0 at the end. There are loads of factors of 2, so the limiting value will be how many factors of 5 are there.** **5, 10, 15, and 20 each have 1 factor of 5. Raised to their respective powers, that will add 5+10+15+20 = 50 zeros to the end. 25 has 2 factors of 5, so 25^{25} = $(5^2)^{25} = 5^{50}$, which adds another 50. Therefore a total of 100 zeros.**
24	1	The following list of numbers consists of powers of two, going up to 64, where each number 'n' appears in the list 'n' times. What is the positive difference between the median and range of this set? {1, 2, 2, 4, 4, 4, 4, …, 64} **There are a total of 127 numbers in the list, so position #64 is the exact middle, which is also equal to the number 64. The range is 63, so the difference = 1**

<table>
<tr><td>25</td><td>180 [units2]</td><td>A region in the shape of a parallelogram is enclosed by the four lines with equations:
y = 0.4x + 9
x = -10
y = 0.4x – 3
x = 5
What is the number of square units in the area of the parallelogram-shaped region?
The four vertices of the parallelogram are (-10, 5), (-10, -7), (5, -1), and (5, 11). The distance between (-10, 5) and (-10, -7), or between (5, -1) and (5, 11), is 12, which can be the base of the parallelogram and the distance between the two vertical lines is 15, which can be the height. So, the area is 12*15 = 180.</td></tr>
<tr><td>26</td><td>5 [dimes]</td><td>Laysha has 12 coins, each of which is a nickel or a dime. There are exactly 17 different monetary values that can be obtained as combinations of one or more of her coins. How many dimes does Laysha have?
With 5 dimes and 7 nickels there are 17 values: 5 cents through 85 cents. With dimes and nickels, every multiple of 5 is possible, hence the maximum value must be 17*5=85.</td></tr>
<tr><td>27</td><td>34 [putts]</td><td>Sahir has made 82 out of 198 putts. How many consecutive putts must he make in order to raise his putt-making rate to 50%?
(82+x)/(198+x)=1/2
164 + 2x = 198 + x
X = 34</td></tr>
<tr><td>28</td><td>41</td><td>A game consists of spinning two spinners. The spinner on the left is divided into four equal sections, and the spinner on the right is divided into eight equal sections. You win the game if both arrows land on a section with the same pattern. The probability of winning can be written as a reduced common fraction A/B. What is A + B?

S = solid
T = stripe
C = circles
P(S&S or T&T or C&C) =
(1/4)(1/8) + (1/4)(1/4) + (1/2)(3/8)
= 1/32 + 1/16 + 3/16 = 9/32
9 + 32 = 41</td></tr>
</table>

29	[A + B =] 65	In a circle puzzle like the ones shown here, dashed arrows mean to add and solid arrows mean to multiply. For example, the unknown value in the first puzzle is 9, because 4 + 5 = 9. The unknown value in the second puzzle is 4, because 2x3x4 = 24.  In the following circle puzzle, each circle contains a positive number. The product of the numbers in all four circles can be written as a reduced common fraction A/B. What is A + B? **Assign variables, and write the following equations:** **(1) B = A + C** **(2) D = A + C ➔ B = D** **(3) A = BC** **(4) C = BD ➔ A = C** **From (1): B = 2A** **From (3) A = B^2** **Substitute: B = $2B^2$** **B = ½** **A = ¼** **ABCD = (1/2)(1/2)(1/4)(1/4) = 1/64** 
30	40320 [ways]	I have nine different pictures of my son taped to the shelf of my desk. How many different ways could I line them up from left to right, assuming that I always keep my favorite picture in the center position? **The position of one picture is already fixed, therefore there are 8! = 40320 ways to arrange the other 8 pictures.**

31	881 [crossings]	The following figure shows the first three iterations of a Celtic knot pattern. Iteration #1 contains 7 crossings, where a 'crossing' is indicated by the circle. If the pattern illustrated here continues, how many crossings will Iteration #20 contain? **There are multiple ways to solve. For the first three iterations, notice the number of crossings:** **N = 1: (2 + 3) + 2** **N = 2: (3 + 4) + (3 + 4) + 3** **N = 3: (4 + 5) + (4 + 5) + (4 + 5) + 4** **This can be written as:** **# crossings =** **n[(n + 1) + (n + 2)] + (n + 1)** **20[21 + 22] + 21 = 881** 
32	30 [%]	A 10-sided die has sides numbered 0 through 9. Assume that two of the fair 10-sided dice are thrown. What is the probability in percent that the sum of the numbers showing on the two dice is less than 10, given that the first die is showing a 7? **If the first die is showing a 7, then there are 10 possible outcomes (sums of 7 through 16). Three of those outcomes are less than 10.** **P(sum < 10 | 1st die = 7) = 3/10 = 0.300.**
33	4 [obstacles]	The rule of an "obstacle course" specifies that at the n^{th} obstacle, a person has to toss a 6-sided die (numbered 1 – 6) 'n' times. If the sum of the numbers showing on the 'n' tosses is greater than 2^n, the person is said to have crossed the obstacle. At most, how many obstacles can a person cross? **N = 1: 2^1 = 2, can roll up to 6** **N = 2: 2^2 = 4, can roll up to 12.** **N = 3: 2^3 = 8, can roll up to 18.** **N = 4: 2^4 = 16, can roll up to 24.** **N = 5: 2^5 = 32, can only roll up to 30, not possible.**

34	[product] = 500	The sum of two numbers is 45. The sum of the quotient of the two numbers and its reciprocal is 2.05. Find the product of the two numbers. **a + b = 45** **a/b + b/a = 2.05** **$a^2/(ab) + b^2/(ab) = 2.05$** **$a^2 + b^2 = 2.05ab$** **$a^2 + b^2 + 2ab = 2.05ab + 2ab$** **$45^2 = 2025 = 4.05ab$** **ab = 500**
35	247	Semicircle O, with diameter AD, is inscribed in rectangle ABCD, which has side length AB = 9. Lines have been drawn from B and C to point O, intersecting the semicircle at points E and F. The area of the shaded region, in square units, of the semicircle above EF can be written as a reduced fraction: $\frac{A\pi - B}{C}$, where A, B and C are positive integers. What is A + B + C?  **The height of the rectangle = radius of the semicircle = 9. Therefore, AO and OD = 9. The rectangle is divided into two squares, with side length 9. Area of the semicircle = $81\pi/2$. The two pie-slice shaped sectors are each ¼ of that, or $81\pi/8$ each. The two central triangles are 45-45-90, with a hypotenuse = radius = 9. Therefore, the side lengths are $9/\sqrt{2}$. The area of each triangle is $1/2*(9/\sqrt{2})^2 = 81/4$. The shaded area = semicircle – 2 sectors – 2 triangles = $81\pi/2 - 2(81\pi/8) - 2(81/4) = \frac{81\pi - 162}{4}$** **A = 81, B = 162, C = 4** **A + B + C = 247** 

<table>
<tr><td>36</td><td>[A+B+C=] 153</td><td>

The following number pattern continues infinitely. Find the sum: A + B + C

1	3	9	3	11	18	13	19	27	55	A	...
2	6	2	7	15	8	17	24	34	29	B	...
3	1	5	12	5	13	21	21	23	30	C	...

A = 31
B = 33
C = 89
31+33+89 = 153
Yellow diagonals = Fibonacci
Red diagonals = primes
Blue diagonals = multiples of 3

1	3	9	3	11	18	13	19	27	55	A	...
2	6	2	7	15	8	17	24	34	29	B	...
3	1	5	12	5	13	21	21	23	30	C	...

</td></tr>
<tr><td>37</td><td>228</td><td>

The sum of four consecutive even integers is a 3-digit perfect square. What is the largest possible integer that could be the greatest of the four?
Write the 4 integers as: n – 3, n – 1, n + 1 and n + 3, where n is an odd integer. Summing them = 4n, which is a 3-digit perfect square. 4 is a perfect square, therefore n must be an odd perfect square. 4n < 1000, therefore n < 250. 15^2 = 225, therefore n = 225 and n + 3 = 228.

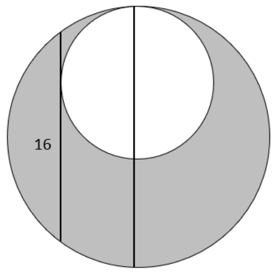
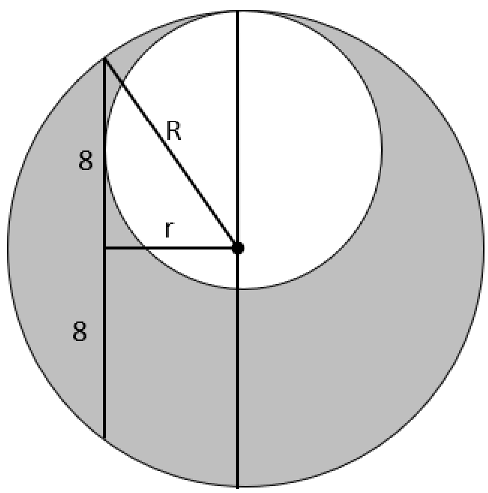
</td></tr>
<tr><td>38</td><td>3</td><td>

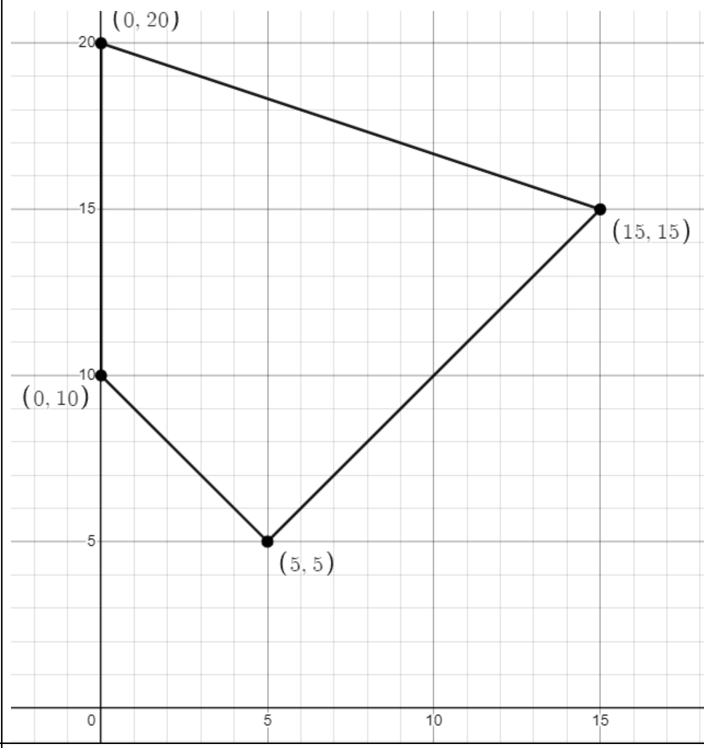
Evaluate: $-\log_2 \log_2 \sqrt{\sqrt{\sqrt{2}}}$

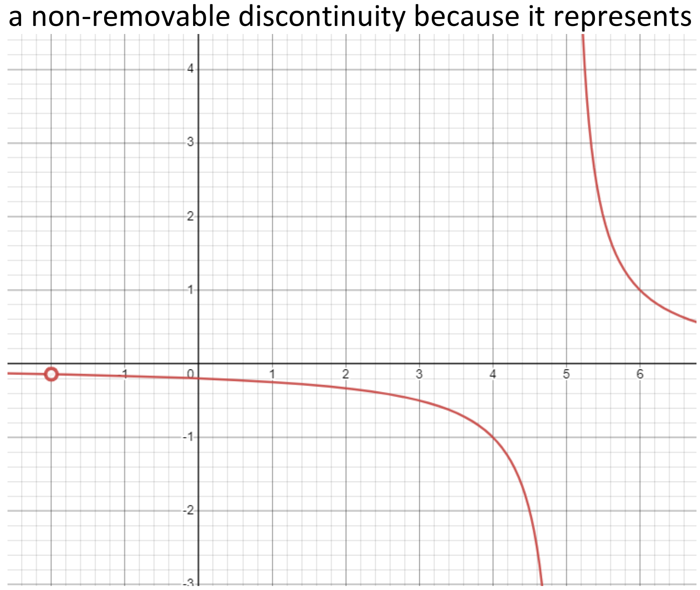
$\sqrt{\sqrt{\sqrt{\mathbf{2}}}} = \mathbf{2^{1/8}}$

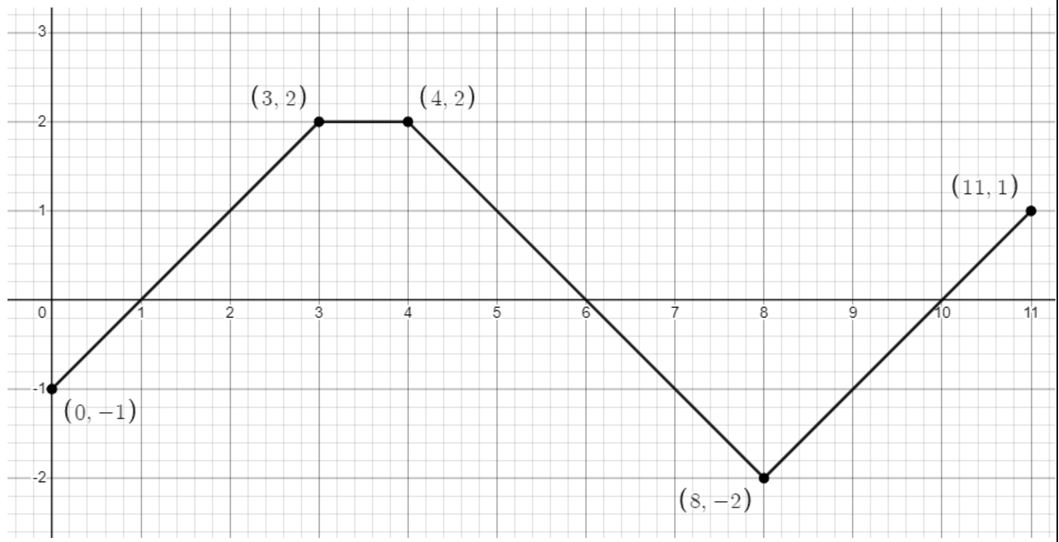
$-\mathbf{log_2\, log_2\, 2^{1/8}} = -\mathbf{log_2}\left(\frac{\mathbf{1}}{\mathbf{8}}\right) = -\mathbf{log_2\, 2^{-3}} = -(-\mathbf{3}) = \mathbf{3}$

</td></tr>
</table>

39	64	In the figure shown here, the larger circle has two parallel chords: one is a diameter and the other has length 16 and is tangent to the smaller circle. The diameter of the larger circle is collinear with the diameter of the smaller circle. The shaded area can be written as $A\pi$, where A is a positive integer. What is the value of A?  **Let r = diameter of the smaller circle, and R = diameter of the larger circle.** $\mathbf{r^2 + 8^2 = R^2}$ $\mathbf{R^2 - r^2 = 64}$ $\mathbf{Area_{large\ circle} - Area_{small\ circle}}$ $\mathbf{= \pi R^2 - \pi r^2 = \pi(R^2 - r^2) = 64\pi}$ 
40	[A + B =] 343	The rule of an "obstacle course" specifies that at the nth obstacle, a person has to toss a 6-sided (numbered 1 – 6) die 'n' times. If the sum of the numbers showing on the 'n' tosses is greater than 2^n, the person is said to have crossed the obstacle. The probability, as a reduced fraction, that a person crosses the first three obstacles can be written as a reduced common fraction A/B. What is A + B? **At the first obstacle, the person needs to toss > 2^1, or a 3, 4, 5 or 6. The probability of this is 4/6 = 2/3. At the second obstacle, the person needs to toss a sum on two dice > 2^2. The probability of this is 30/36 = 5/6. At the third obstacle, the person needs to toss a sum of three dice > 2^3. The probability of this is 160/216 = 20/27.** **Therefore, (2/3)(5/6)(20/27) = 100/243**

41	[z =] 300	A function z is defined as follows: z = 5x + 15y The function z is subject to the following conditions: x + 3y ≤ 60 x + y ≥ 10 x – y ≤ 0 x, y ≥ 0 What is the maximum possible value of z under these constraints? **The linear constraints, when plotted, set up a feasible region contained within the following boundaries. Evaluating the function for z at each of these points shows that the maximum value of z = 300 occurs at (0, 20) and (15, 15)** 
42	[x + y =] -4	An x-y plane curve is described parametrically by the following equations, for $-\infty < \theta < \infty$: $x = 8cos\theta$ $y = 4sin\theta$ When $\theta = \frac{3\pi}{2}$, what is the sum x + y of the corresponding point (x, y) on the x-y plane? $\boldsymbol{x = 8 \cdot cos\frac{3\pi}{2} = 8 \cdot 0 = 0}$ $\boldsymbol{y = 4 \cdot sin\frac{3\pi}{2} = 4 \cdot (-1) = -4}$ **x + y = -4**

43	[x =] 0	In the following matrix, find the value of x such that the determinant of the matrix is equal to 12. $\begin{bmatrix} 2 & -2 & x \\ -3 & 1 & 2 \\ 1 & -3 & -1 \end{bmatrix}$ $\begin{vmatrix} \mathbf{2} & \mathbf{-2} & \boldsymbol{x} \\ \mathbf{-3} & \mathbf{1} & \mathbf{2} \\ \mathbf{1} & \mathbf{-3} & \mathbf{-1} \end{vmatrix} = \mathbf{12}$ **Expanding on the first row:** $\mathbf{2}\begin{vmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{-3} & \mathbf{-1} \end{vmatrix} - (-2)\begin{vmatrix} \mathbf{-3} & \mathbf{2} \\ \mathbf{1} & \mathbf{-1} \end{vmatrix} + \boldsymbol{x}\begin{vmatrix} \mathbf{-3} & \mathbf{1} \\ \mathbf{1} & \mathbf{-3} \end{vmatrix}$ $= \mathbf{2(-1 - -6) + 2(3-2) + \boldsymbol{x}(9-1) = 2(5) + 2 + 8\boldsymbol{x} = 12}$ **12 + 8x = 12** **8x = 0** **X = 0**
44	-2	Find the sum of any x-values at which the given function f(x) has a removable discontinuity. If there are no removable discontinuities, then enter 0 as your answer. $\boldsymbol{f(x) = \frac{x+2}{x^2 - 3x - 10}}$ $\boldsymbol{f(x) = \frac{x+2}{x^2 - 3x - 10} = \frac{x+2}{(x+2)(x-5)}}$ Therefore, x = -2 and 5 are not in the domain. However, x = -2 represents a removable discontinuity, because it is a hole in the graph, versus x = 5 which is a non-removable discontinuity because it represents a vertical asymptote. 

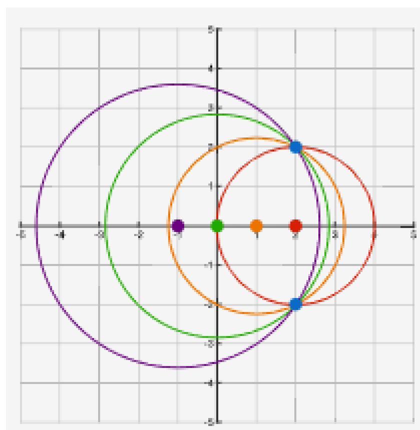
45	2	The graph of f(x) consists of line segments, as shown here. Evaluate the following definite integral: $$\int_0^{11} f(x)dx$$  **Use geometry to calculate all of the areas below the axis (negative contribution) and above the axis (positive contribution).** **From x = 0 – 1: -1/2** **From x = 1 – 3: +2** **From x = 3 – 4: +2** **From x = 4 – 6: +2** **From x = 6 – 10: -4** **From x = 10 – 11: +1/2** **Sum of areas = 2**

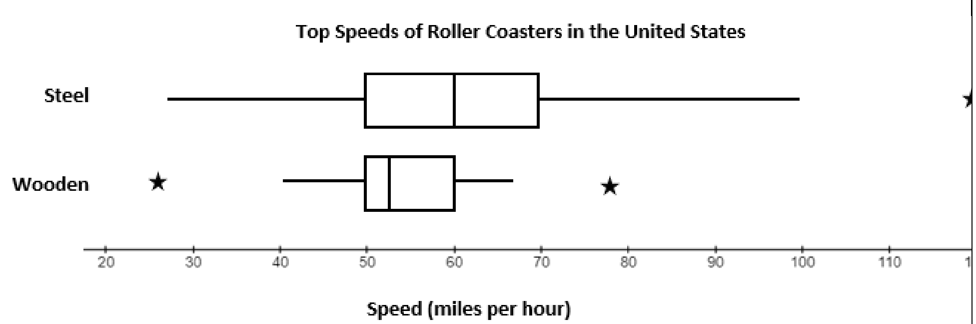
"Math is Cool" Masters -- 2022-23

High School

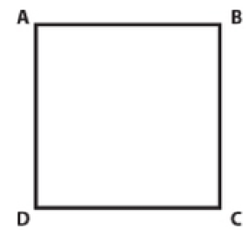
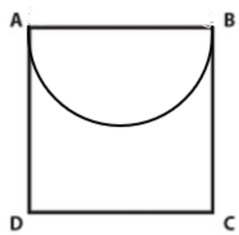
Individual Multiple Choice Solutions

9/10th	11/12th	Answer	Solution
1	1	E	The lengths of all sides of a rectangle are multiplied by 5/3. As a result, how many times larger is the area of the new rectangle compared to the original rectangle? A) 5/3 B) 10/3 C) 9/25 D) 10/9 E) Answer not given. **If the sides are multiplied by 5/3, the area is multiplied by $(5/3)^2$ = 25/9.**
2		D	If a = 30, b = 9/2, and c = 25, what is the value of: (a + c)(a – b)(a – c) A) 2023 B) 2575 C) 5212.5 D) 7012.5 E) Answer not given. **(55)(25.5)(5) = 7012.5** **OR** **$(a + c)(a - c) = a^2 - c^2 = 900 - 625 = 275$** **(275)(25.5) = 7012.5**
	2	C	The value of $\log_2 14$ is: A) Between 1 and 2 B) Between 2 and 3 C) Between 3 and 4 D) Between 4 and 5 E) Answer not given. **$Y = \log_2 14$** **$2^Y = 14$** **$2^3 = 8,\ 2^4 = 16$**

3		B	Which of the following equations describes a circle that is completely contained in the first quadrant of the coordinate plane? A) $(x - 8)^2 + (y - 9)^2 = 100$ B) $(x - 10)^2 + (y - 7)^2 = 36$ C) $(x + 10)^2 + (y + 12)^2 = 81$ D) $(x + 6)^2 + (y - 9)^2 = 25$ E) Answer not given. **Circle B is centered at (10, 7) in Q1, with a radius of 6, therefore completely contained in the first quadrant.**
	3	D	How many circles can be constructed that pass through the two points (2, 2) and (2, -2) on the coordinate plane? A) 1 B) 2 C) 3 D) Infinitely many E) Answer not given **By sliding the center of the circle perpendicularly away from the line connecting the two points, infinitely many circles can be constructed.** 
4	**4**	D	In the cryptogram shown here, each letter corresponds to a different digit, and 0 cannot be a leading digit. What is the sum EH + AH? $\begin{array}{r} H\ A \\ +\ H\ A \\ +\ H\ A \\ +\ H\ A \\ \hline E\ H \end{array}$ A) 84 B) 86 C) 92 D) 124 E) Answer not given. **Notice that it is the same as HA x 4 = EH. Since EH < 100, HA < 25, therefore H = 1 or 2. Since H is the last digit of HA x 4, H = 2. A quick guess and check shows that A = 3. Therefore HA = 23 and EH = 92.** **92 + 32 = 124**

5		*C*	Mrs. Stephenson collects data from her Statistics students every quarter to use in various analyses. She has the ages in years of all 28 of her Winter 2023 students, as follows: Three students are younger than eighteen, 18, 18, 18, 18, 18, 19, 20, 21, 21, 21, 21, 22, 22, 23, 23, 23, 24, 27, 28, 28, five students are older than thirty. What is the median age of her students, in years? A) 18 B) 21 C) 21.5 D) 22 E) Impossible to determine without having all of the data. **With an even number of data values (28), the median will be halfway between the two "middle" values, which are values #14 and #15 in the list. Fourteen data values on the bottom, and fourteen on the top. From the list, data value numbers 14 and 15 are 21 and 22 respectively, therefore the median is 21.5.**
	5	*C*	The Roller Coaster DataBase contains extensive data on roller coasters around the world. The following boxplots show the distributions of top speeds for coasters in the United States, classified by whether the coaster is wooden or steel. The stars indicate outliers. Which type of coaster has a higher percentage of coasters with a top speed of 50 miles per hour or more?  A) Steel B) Wooden C) They are the same. D) Impossible to determine from the boxplots E) Answer not given. **Both Steel and Wooden have their Q1 located right at 50 miles per hour, indicating that they both have 75% of their data at 50 miles per hour or more. Therefore, they are the same.**

6	6	A	Arnav is making lemonade. He currently has 6 liters of lemonade that is 10% lemon juice. If he wants his final drink to be 20% lemon juice, how many liters of pure lemon juice should he add? A) 0.75 B) 1.0 C) 1.25 D) 1.5 E) Answer not given. **X = amount of 20% to add** **0.6 + x = 0.2(x + 6)** **0.8x = 0.6** **X = 0.75**
7	7	D	Kuniko is driving to visit a friend in another city that is 560 miles away. She begins driving at noon going 50 miles per hour. At some point she realizes that she is going to be late, so she speeds up to 60 miles per hour. She arrives at her friend's house at 10:00 pm that night. At what time did she decide to speed up? A) 2 pm B) 2:30 pm C) 3:30 pm D) 4 pm E) Answer not given. **50t + 60(10 – t) = 560** **-10t + 600 = 560** **40 = 10t** **T = 4 hours** **4 hours after noon = 4 pm**
8		B	What is the sum of the terms in the following infinite geometric sequence? 875, 625, 3125/7, … A) 5435 B) 6125/2 C) 7225/4 D) 8735/2 E) Answer not given. **The common ratio r is 625/875 = 5/7.** **S = a1/(1 – r) = 875/(1 – 5/7) = 875/(2/7) = (875)(7)/2 = 6125/2**
	8	A	The sum of an infinite geometric sequence is 2022, and its first term is 1685. What is the common ratio, r, of the sequence? A) 1/6 B) 335/2022 C) 47/7225 D) 3/5 E) Answer not given. **S = a1/(1 – r)** **2022 = 1685/(1 – r)** **1 – r = 1685/2022** **r = 1 – 1685/2022 = 337/2022 = 1/6**

9	9	D	Consider a unit square ABCD. If a point Q is chosen at random inside the square, what is the probability that angle $\angle AQB$ is obtuse?  A) $\frac{\pi}{24}$ B) $\frac{\pi}{16}$ C) $\frac{5\pi}{16}$ D) $\frac{\pi}{8}$ E) Answer not given. **$\angle AQB$ will be a right triangle when Q lies along the semi-circle with diameter AB. For Q outside the semi-circle, $\angle AQB$ will be acute, and for Q inside the semi-circle, $\angle AQB$ will be obtuse.** **P = area of semi-circle / area of square = $\frac{\frac{1}{2}\pi(\frac{1}{2})^2}{1} = \frac{\pi}{8}$** 
10	10	D	If p and q are the zeroes of the polynomial: $x^2 - x + 4$ what is the value of $\frac{1}{p^3} + \frac{1}{q^3}$? A) $\frac{-15}{64}$ B) $\frac{-5}{32}$ C) $\frac{-3}{16}$ D) $\frac{-11}{64}$ E) Answer not given **For quadratic $ax^2 + bx + c$:** **Sum of roots: p + q = -b/a = 1** **Product of roots: pq = c/a = 4** $\frac{1}{p^3} + \frac{1}{q^3} = \frac{p^3+q^3}{p^3q^3} = \frac{(p+q)^3-3pq(p+q)}{(pq)^3} = \frac{1^3-3\cdot 4\cdot 1}{4^3} = \frac{-11}{64}$

"Math is Cool" Masters -- 2022-23

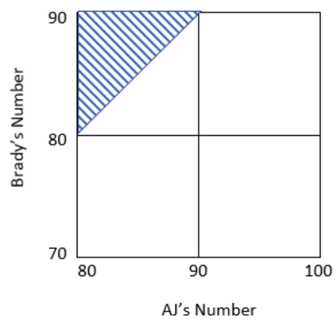
High School

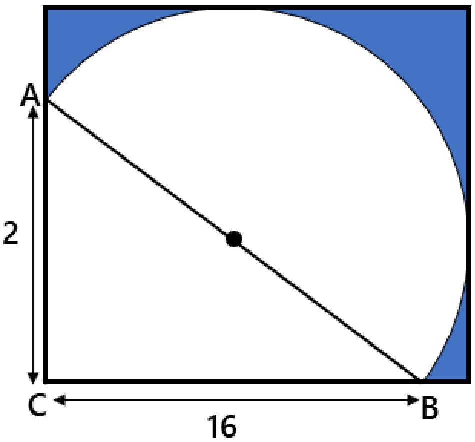
Team Test Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	5	Let the 3rd and 5th terms of an arithmetic sequence be 14 and 23 respectively. What is the 1st term of that sequence? **The difference between terms 3 and5 will be the same as the difference between terms 1 and3. 14-(23-14)=14-9=5**
2		39856	Evaluate: $(2^6)(5^4) - (2^4)(3^2)$ **$(2^6)(5^4) - (2^4)(3^2) = (2^2)(10^4) - (2^2)(6^2) = (2^2)(10^4 - 6^2) = 4(10^2 - 6)(10^2 + 6) = 4(94)(106) = 424(94) = 424(100) - 424(6) = 42400 - 2544 = 39856$** **Or** **$64(625) - 16(9) = 40000 - 144 = 39856$**
	2	9720000	Simplify: $32^{\left(\frac{6}{5}\right)} \cdot 81^{\left(\frac{5}{4}\right)} \cdot 125^{\left(\frac{4}{3}\right)}$ **$= 2^6 * 3^5 * 5^4 = 2^2 * 3 * 30^4 = 4 * 3 * 810000 = 10(810000) + 2(810000) = 8100000 + 1620000 = 9720000$**
3		38	Let $f(x) = 3x^2 - 2x + 5$ and $g(x) = -4x + 17$. Find the value of $f(g(5))$. **$g(5) = -4(5) + 17 = -3$, so $f(g(5)) = f(-3) = 3(-3)2-2(-3) + 5 = 38$**
	3	6	In radians, what is the period of the graph of the following function? $f(x) = 5 - 3cos\left[\frac{\pi}{3}\left(x - \frac{\pi}{4}\right)\right]$ **period = $2\pi/B = 2\pi/(\pi/3) = 6$**

4	4	36 [°]	Three exterior angles of a triangle, one at each vertex, have degree measures in the ratio of 5:7:8. What is the measure in degrees of the triangle's smallest interior angle? **The sum of the exterior angles = 360, therefore:** **$5x + 7x + 8x = 360$** **$X = 18$** **Each exterior angle is a supplement to an interior angle, therefore the smallest interior angle will be supplementary to 8x, or 8(18) = 144°. 180 – 144 = 36°.**
5		[$]144 [dollars]	Hardwood boards are typically sold in terms of a unit called board-feet. A board-foot is equivalent to a volume whose face measures one square foot and whose depth measures one inch. If domestic cherry is priced at $9 per board-foot, what would be the total cost of a board measuring 8 inches wide, 12 feet long, and 2 inches deep, in dollars? **A board-foot is 1ft x 1ft x 1in, so we express the measurements in terms of these dimensions.** **8 in / 12 in (ie 1 ft) becomes 2/3, 12 ft / 1 ft becomes 12, and 2 in / 1 in = 2.** $$2/3 \times 12 \times 2 = 16$$ **And 16 * $9 = $144**
	5	-6	What is the smallest integer value of x that satisfies the following inequality? $4x^2 - 105 < 64$ **Rearrange to:** **$4x^2 < 169$** **$X^2 < 169/4$** **$X < 13/2$, or $x > -13/2$** **The smallest integer value of x in that range is -6.**

6	**6**	15 [guests]	The Overlook Hotel has 20 rooms in a row along one side of a hallway. Mr. Ullman, the front desk manager, assigns each guest to one of these rooms at random. What is the minimum number of guests that would guarantee that at least one group of three adjacent rooms are occupied? **Worst case scenario, the guests are assigned: room, room, empty, room, room, empty, … This would result in 14 rooms being occupied, with no three adjacent. The 15th guest would therefore cause at least one group of three adjacent rooms to be occupied.**
7	**7**	313	What is the smallest positive integer that has a remainder of 3 when divided by 5, a remainder of 5 when divided by 7, and a remainder of 7 when divided by 9? **The numbers that have a remainder of 3 when divided by 5 end in 3 or 8 only.** **3, 8, 13, 18, 23, 28, 33, 38, 43, 48, 53, 58, 63, 68, 73, 78, 83, 88, 93, 98, 103, 108, . . .** **The first number that has a remainder of 5 when divided by 7 that ends in a 3 or an 8 is 33 and then adding multiples of 35 results in further numbers that have both of these in common.** **33, 68, 103, 138, 173, 208, 243, 278, 313** **The first number that has a remainder of 7 when divided by 9 that ends in a 3 or an 8 is 43 and then adding multiples of 45 results in further numbers that have both of these in common.** **43, 88, 133, 178, 223, 268, 313** **The first number that all three have in common is 313.** **OR,** **LCM(5, 7, 9) = 315 and since the remainders are all two less than the divisors, the answer is 315 – 2 = 313.**

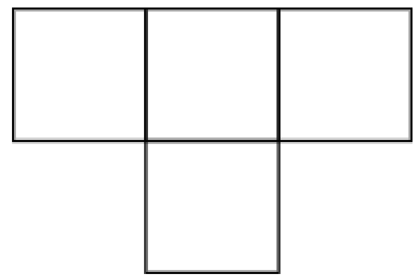
8	**8**	[A + B =] 9	AJ randomly chooses a real number between 80 and 100, and Brady randomly chooses a real number between 70 and 90. The probability that Brady's number is greater than AJ's number can be written as a reduced common fraction A/B. What is A + B? **The possible numbers that the two choose are indicated by the area in the following graph. The shaded area indicates the pairs of numbers where Brady's number is greater than AJ's. If each square has an area of 2, then the shaded area over the total area = P = 1/8.** 
9	**9**	165 [values of n]	A 3-digit positive integer n = abc, where a, b, and c represent the three digits of 'n', which are not necessarily distinct. An isosceles (or equilateral) triangle will be constructed using a, b, and c as the lengths of the sides. How many different values of the 3-digit positive integer 'n' are possible? **First, you can create 9 equilateral triangles: 111, 222, ..., 999.** **Next, let's consider the non-equilateral isosceles triangles:** **If the two equal sides = 2: third side can equal 1 or 3. For each, there are 3 numbers abc, i.e. 122, 212, 221. Therefore, 2 sides times 3 numbers = 6** **If the two equal sides = 3: third side can equal 1, 2, 4, 5. For each, and for all such cases, there are again 3 numbers abc. Therefore, 4 sides times 3 numbers = 12** **If the two equal sides = 4, third side can equal 1, 2, 3, 5, 6, 7. 6 sides times 3 numbers = 18** **If the two equal sides = 5 through 9, there are 8 possible values for the third side. 8 sides times 3 numbers times 5 values = 120** **Grand total: 120 + 18 + 12 + 6 + 9 equilateral = 165**

10	**10**	242 [X + Y =]	A semicircle with diameter AB is inscribed in a rectangle. The area of the shaded region between the semicircle and the rectangle, given that AC = 12 and CB = 16, can be written as: X – Yπ, where X and Y are positive integers. What is the value of X + Y?  **Triangle ABC is a right triangle, and we can see that it is a scaled-up 3-4-5 triangle, by a factor of 4. Therefore the hypotenuse AB = 20, and the radius of the semicircle = 10. Triangles AOQ and OBP are congruent, and similar to triangle ABC, so with a hypotenuse of 10, they are both 6-8-10 right triangles. Therefore, BF = 2, and AD = 4. The area of the rectangle = (16)(18) = 288. The area of triangle ABC = 1/2(12)(16) = 96. The area of the semicircle = $\frac{1}{2}\pi(10^2) = 50\pi$. Therefore, the shaded area = 288 – 96 - 50π = 192 - 50π. 192 + 50 = 242.**

“Math is Cool” Masters -- 2022-23

High School

Pressure Round Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	108	 In the following figure, composed of four unit squares, each square can be colored either apricot, bronze, crimson or emerald. A color may be used more than once. How many distinct ways are there to color in each square if no two adjacent squares can be the same color? Adjacent means sharing an edge, not just a vertex. **Suppose that the left-most square is apricot. Therefore there are three choices for the middle box, three choices for the far box, and 3 choices for the bottom box, for a total of 3x3x3 = 27 ways. There are 4 different colors that could go in the first box, so 4x27 = 108 ways**
2		1152	What is the product of the digits of the quantity 67^2? **67^2 = 4489** **4x4x8x9 = 1152**
	2	19	Evaluate: $\frac{20!}{19!+18!}$ **19! = 19(18!)** $\frac{\mathbf{20!}}{\mathbf{19(18!)+18!}} = \frac{\mathbf{20!}}{\mathbf{18!\,(19+1)}} = \mathbf{19}$

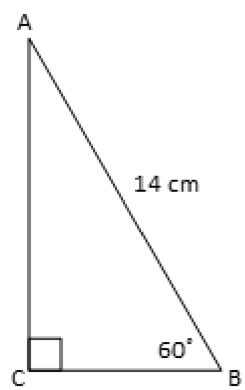
3		1082	A new sequence is obtained from the sequence of the positive integers {1, 2, 3, …} by deleting all of the perfect squares. What is the 1050th term in the new sequence? **32^2 = 1024, so if the squares from 1^2 through 32^2 are removed from a list of integers 1 - 1024, then there are 1024 – 32 = 992 numbers left in the list, and the 992nd number is 1023. The 993rd term is 1025, since 1024 has been deleted. 993 + 57 = 1050th term, so 1025 + 57 = 1082.**
	3	2068	A new sequence is obtained from the sequence of the positive integers {1, 2, 3, …} by deleting all of the perfect squares. What is the 2023rd term in the new sequence? **45^2 = 2025, so if the squares from 1^2 through 45^2 are removed from a list of integers 1 - 2025, then there are 2025 – 45 = 1980 numbers left in the list, and the 1980th number is 2024. The 1981st term is 2026, since 2025 has been deleted. 1981 + 42 = 2023rd term, so 2026 + 42 = 2068.**
4	**4**	441	In the following equation, how many times should 7^2 appear under the square root sign on the left-hand side to make the equation true? $\sqrt{7^2+7^2+\cdots+7^2+7^2} = 7^2+7^2+7^2$ **Rewrite:** $\boldsymbol{\sqrt{n\times 7^2} = 3\times 7^2}$ $\boldsymbol{n\times 7^2 = 3^2\times (7^2)^2}$ **n = 9 x 49 = 441**

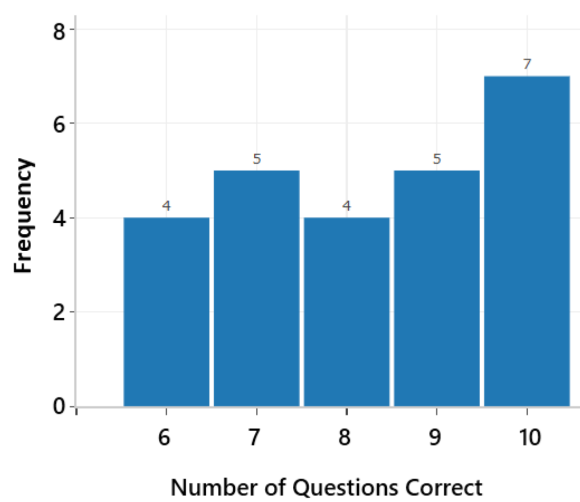
5	**5**	5 [cards]	A box contains eight cards numbered 1 through 8. Mateo needs a pair of cards that sum to 9, but may only select cards from the box at random. What is the minimum number of cards that Mateo must pull from the box to guarantee that he has a pair that sum to 9? **Worst case scenario, draw 1, 2, 3, 4 first. On the 5th draw, Mateo will be guaranteed to get a number (5, 6, 7 or 8) that will form a pair summing to 9.**

"Math is Cool" Masters -- 2022-23

High School

College Bowl Round #1 Solutions

	Answer	Solution
1	32 [years]	Today, Nidhi is 6 times as old as Quinn. Four years ago, Quinn was 2 years old. How old in years was Nidhi 4 years ago? **N = 6Q** **Q – 4 = 2** **Q = 6, N = 36 now, N = 32 four years ago**
2	-48	Evaluate: $-6-[-6(-5+(-2))]$ **-6 – (42) = -48**
3	40	A friend draws one card from a standard 52-card deck and truthfully reports that it is not a diamond. The probability that the card is the 2 of clubs can be written as a reduced common fraction A/B. What is A + B? **There are 52 – 13 = 39 cards that are not diamonds. Only one of them is the 2 of clubs, therefore P = 1/39.**
4	10	For right $\triangle ABC$, $m\angle B = 60°$ and $\overline{AB}$ = 14 cm. The length of $\overline{AC}$ in simplest radical form is $X\sqrt{Y}$ cm. What is the sum X + Y?  **Since it's a 30-60-90 triangle, if AB = 14, then BC = 7, and AC = $7\sqrt{3}$. 7 + 3 = 10**
5	360	How many 3-digit counting numbers have a tens digit that is a positive multiple of 2? **The possible tens digits are 2, 4, 6, 8. For each, the first digit of the number can be 1-9, and the units digit can be 0-9. 9x10 = 90, and 90 x 4 possible tens digits = 360**

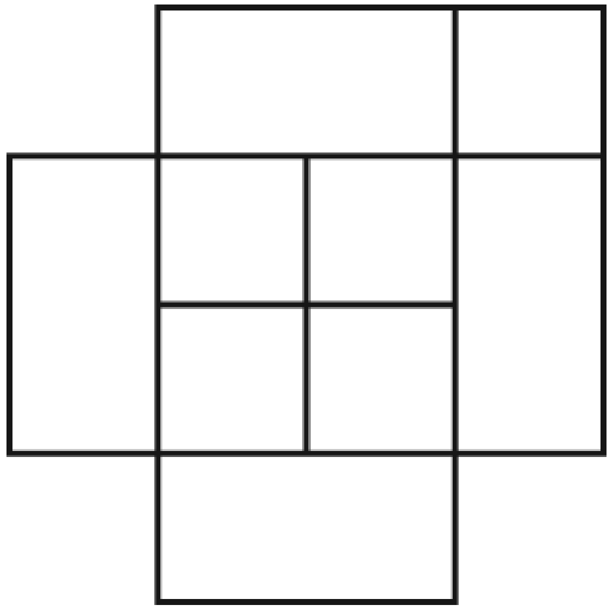
6	73	A sequence is defined as $a_n = \frac{1}{n} - \frac{1}{n+1}$ for n = 1, 2, 3, … The sum $a_3 + a_4 + \ldots + a_{18}$ can be written as a reduced common fraction A/B. What is A + B? **The sum will be: (1/3 – ¼) + (1/4 – 1/5) + … + (1/18 – 1/19). All terms cancel except for 1/3 – 1/19 = 16/57**
7	4	A solid cube will be intersected by a plane. The 2-dimensional region of intersection is called the cross-section. For example, the intersection of a sphere and a plane forms a circle. For a cube being intersected by a plane, how many different-sided polygons can be formed? Do not consider different specific shapes, only the number of sides. **A cube has six faces, and a plane can be positioned to intersect exactly 3, 4, 5, or 6 of the faces. Therefore, a triangle, quadrilateral, pentagon and hexagon can be created.**
8	95059	What is the largest palindrome less than 95113? A palindrome is a number that reads the same forwards or backwards, such as 121 or 3443. **Changing the middle number to 0 results in 95059.**
9	778	Function G is defined as: $G(a, b) = a^2 + b$ Evaluate: $G(G(5, 3), G(-1, -7))$ **$G(5, 3) = 5^2 + 3 = 28$** **$G(-1, -7) = (-1)^2 - 7 = -6$** **$G(28, -6) = 28^2 - 6 = 778$**
10	8	The following histogram shows the results for students on a 10-question quiz. What was the median number of questions that were correct?  **The frequencies add up to a total of 25 scores. The 13th (middle) value will be the median, which corresponds to a value of 8 questions correct.**

“Math is Cool” Masters -- 2022-23

High School

College Bowl Round #2 Solutions

	Answer	Solution
1	9 [mph]	Grace rode her bike 48 miles over the course of 5 hours, then rode for 3 more hours at an average speed of 8 miles per hour. What was her average speed for the entire journey, in miles per hour? **For the second part of the journey, she traveled 3*8 = 24 miles. Total distance was 48 + 24 = 72 miles divided by total time = 8 hours ➔ average rate = 9 mph.**
2	10 [$]	What is the total value of 31 quarters, 12 dimes, 18 nickels, and 15 pennies expressed in dollars? **$7.75 in quarters + $1.20 in dimes + 0.90 in nickels + 0.15 in pennies = $10.00**
3	230	What is the arithmetic mean of the following numbers: 232, 226, 228, 230, 234, 229, 231 **In order:** **226, 228, 229, 230, 231, 232, 234** **By inspection, the numbers are symmetrically arranged around the center, 230.**
4	149	On a coordinate plane, the point (7, 10) is rotated 360 degrees about the origin tracing the path of a circle. In terms of π, the area of the circle is Aπ square units. What is the value of A? **The radius is $\sqrt{7^2+10^2}=\sqrt{149}$, so the area is 149π and A = 149**
5	465	What is the sum of the positive integer factors of 200? **200 = $2^3 5^2$** **Sum of factors = (1+2+4+8)(1+5+25) = 465**
6	207	What is the next term of the following quadratic sequence? 2, 5, 27, 68, 128, … **Look at the first and second difference between each two terms. The second differences = 19, therefore work backwards to add 128 + 79 = 207.**
7	3 [hours]	Five bakers can prepare 30 pies in 2 hours. Working at the same rate, how many hours will 7 bakers need to prepare 63 pies? **5b:30p:2h ➔ 10 baker-hours/30 pies = 1/3 baker-hour per pie x 63 pies = 21 baker-hours / 7 bakers = 3 hours**
8	66	The surface area of a right circular cylinder with a base diameter of 6 meters and a height of 8 meters can be written as Aπ square meters. What is the value of A? **SA = $2\pi r^2 + 2\pi rh$** **= $2\pi(3)^2 + 2\pi(3)(8) = 18\pi + 48\pi = 66\pi$**

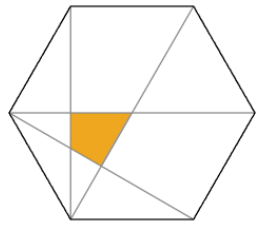
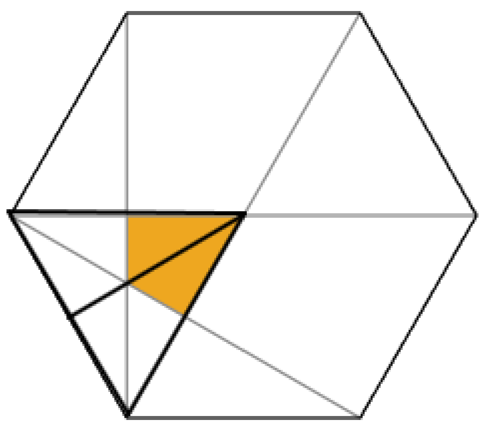
9	54	A data set has a mean of 36, a median of 18, a range of 60 and a unique mode of 24. If each element in the data set is multiplied by 3, what is the median of the new data set? **The median will be 3 times the original median. 3x18 = 54**
10	11 [squares]	The following figure is composed of unit squares (1x1) and rectangles (1x2). In total, how many squares appear in the figure?  **5 unit squares.** **1 square in middle composed of 4 unit squares.** **4 squares composed of 2 unit squares + 1 rectangle.** **1 big square composed of 5 unit squares + 2 rectangles.**

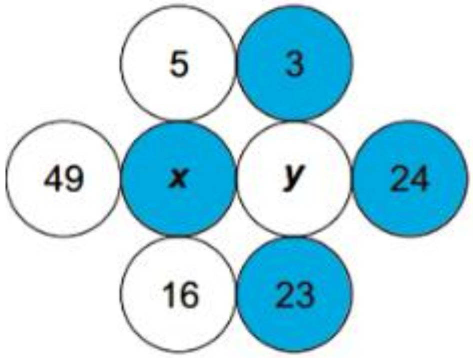
“Math is Cool” Masters -- 2022-23

High School

College Bowl Round #3 Solutions

	Answer	Solution
1	-2	What is the slope of the line through the points (5, -2) and (6, -4)? **M = (-4 - -2)/(6 – 5) = -2/1 = -2**
2	8	In the following addition problem, X and Y represent digits from 1 through 9. If X ≠ Y, what is the value of Y? $\begin{array}{r} X \\ +\ XX \\ \hline Y4 \end{array}$ **X must = 7, and 7 + 77 = 84**
3	2 [%]	Eho bought two different types of scratch-off lottery tickets. The first has a 10% chance of winning a prize, and the second has a 20% chance of winning a prize. What is the probability in percent that both of Eho’s lottery tickets are winners? **P(A&B) = P(A)xP(B) = (0.10)(0.20) = 0.02**

4	19	A regular hexagon is shown, with some of its diagonals drawn. The fraction of the total area of the hexagon that is shaded can be written as a reduced common fraction A/B. What is A + B?  **Adding a line from the center to the side midpoint shows that the outlined triangle is divided into 6 congruent triangles, 2 of which are shaded. There are 6 of the "big" (outlined) triangles in the hexagon, so 36 total little triangles. 2/36 = 1/18.** 
5	72	Five people ran a race. One competitor, Foster, was neither the first nor the last finisher. In how many different ways could the competitors have finished the race? **Foster was 2nd, 3rd or 4th. In each case, there are 4! = 24 ways to order the other competitors. 24x3 = 72 total ways.** **Or, using the multiplication principal: $4 \cdot 3 \cdot 3 \cdot 2 \cdot 1 = 72$ ways. The order of ways to fill places is first, fifth, second, third, fourth.**
6	194	What is the greatest integer x ‹ 200 that has exactly 4 factors? **Based on the prime factorization method for determining the number of factors, if x has exactly 4 factors, then x either has one unique prime factor raised to the third power, or two distinct prime factors each raised to the first power.** **If x has exactly one unique prime factor raised to the third power, then the largest value x < 200 is 5^3 = 125.** **If x has two distinct prime factors, since x < 200, the prime factors of x must be < 100. The pair that gets you closest to 200 without going over is 2 times 97 =194.** **Or, just brute-force it by trying integers starting with 199 on down.**

7	4	In the diagram, the value of x is the mean of the four numbers in the white circles. The value of y is the mean of the four numbers in the blue circles. What is the value of x – y?  **X = (70 + y)/4** **Y = (50 + x)/4** **4x = y + 70** **4y = x + 50** **4x – y = 70** **X – 4y = -50** **5x – 5y = 20** **X – y = 4**
8	13	A polygon with 'n' sides has one exterior angle of 72°, one exterior angle of 35°, and the remaining exterior angles all measure 23°. What is the value of 'n'? **The sum of the exterior angles = 360°. Therefore, 72 + 35 + 23x = 360, where x is the number of 23° angles. Solve for x = 11, therefore there are a total of 13 angles and 13 sides.**
9	840	What is the sum of the following finite arithmetic sequence? 4 + 8 + 12 + … + 72 + 76 + 80 **Factor: 4(1 + 2 + 3 + … + 20)** **Sum of integers 1 … 20 = (20)(21)/2 = 210** **4(210) = 840**
10	-1	Given that the following equation is true, and 'a' and 'b' are non-zero numbers, what is the value of $\frac{a}{b}$? $(5^a)(5^b) = 1$ $\mathbf{(5^a)(5^b) = 5^{a+b} = 1 = 5^0}$ **Therefore, a + b = 0, a = -b** **a/b = -b/b = -1**