"Math is Cool" Championships -- 2022-23

High School

Mental Math Solutions

	Answer	Solution
1	32 [%]	Due to inflation, the owners of Hill's Bait shop raised the prices of their nightcrawlers by 10% in July. In August, they raised the prices by an additional 20%. What was the overall increase in percent? **100(1.1) = 110, 110(1.2) = 132**
2	13	In the equation $3^x = 1000$ (read as: 3 raised to the x equals 1000), the value of x that makes the equation true lies between two consecutive integers A and B. What is A + B? **$3^6 = 729$, $3^7 = 2187$, 6+7 = 13**
3	-24	What is the y-coordinate of the vertex of the parabola described by the function: $f(x) = x^2 - 10x + 1$ (read as: x-squared minus 10x plus 1) **x-coordinate = -b/2a = 10/2 = 5** **$f(5) = 5^2 - 50 + 1 = -24$**
4	80 [units]	What is the sum, in units, of the lengths of all of the edges of a rectangular prism that measures 4 units wide by 7 units tall by 9 units deep. **Each dimension will appear 4 times on an edge: 4(4 + 7 + 9) = 4(20) = 80**
5	15	An arithmetic sequence has a common difference of 4. The sum of the first two terms in the sequence is 10. What is the value of the 4^{th} term in the sequence? **$a_1 + (a_1 + 4) = 10$, therefore $a_1 = 3$. Sequence is 3, 7, 11, 15.**
6	9	Two cards are randomly selected from a standard deck of playing cards, with replacement. The probability that one of them is a heart and one of them is a diamond can be written as a reduced common fraction A/B (read as: A over B). What is A + B? **P(H&D or D&H) = (1/4)(1/4) + (1/4)(1/4) = 2/16 = 1/8**
7	61 [cows]	A farmer was counting his cows. The first herd was consecutively numbered 45 through 83. The second herd was consecutively numbered 191 through 213, except cow number 211 was missing. How many total cows were there? 83 – 44 = 39 **213 – 190 – 1 = 22, 39 + 22 = 61**
8	624 [base 10]	Expressed as a base 10 number, what is the largest 4 digit base 5 number? Do not include the base 10 in your answer. **4*125 + 4*25 + 4*5 + 4*1 = 624** **4444_5 = 624 base 10**

"Math is Cool" Championships -- 2022-23
High School
Individual Test Solutions

	Answer	Solution
1	-4	What is the slope of the line given by the following equation: 12x + 3y = 2 **3y = -12x + 2** **Y = -4x + 2/3**
2	1035	What is 5/3 of 621? **(5/3)*621 = 1035**
3	540 [cm^3]	The volume of a right rectangular pyramid is 20 cm^3. If the length, width and height of the pyramid are all tripled to create a larger, similar pyramid, what is the volume of the larger pyramid in cubic centimeters? $\mathbf{20 * 27 = 540}$
4	50 [%]	A 12-sided die contains the numbers 1 through 12. When the die is rolled once, what is the probability in percent that the number showing is a factor of 12? **Factors of 12: 1, 2, 3, 4, 6, 12** **6/12 = ½ = 50%**
5	5	What value of 'a' satisfies the following equation? $\frac{7!}{4!\,(7-a)!} = 105$ $\mathbf{\frac{7!}{4!\,(7-5)!} = 105}$
6	2	The parabola described by the following function has its vertex at the point (x, y). What is x + y? F(x) = $(x - 3)^2 - 1$ **The vertex is at (3, -1)**
7	43	The lengths of the legs of a right triangle are 10 and 8 units. The length of the hypotenuse can be written in simplified radical form as $A\sqrt{B}$. What is A + B? **Hypotenuse =** $\mathbf{\sqrt{164} = 2\sqrt{41}}$
8	48 [inches]	Andrea is stacking up her identical patio chairs. One chair by itself is 30 inches tall. Five chairs stacked together are 38 inches tall. How tall in inches will a stack of 10 chairs be? **The additional 4 chairs added 2 inches each. Therefore, adding 5 more chairs will add 10 inches.**

9	94	The digits 2, 3, 4, 5 and 9 are used to form the smallest possible five-digit even integer N (each digit is used exactly once). What is the value of N mod 100? **The number will have to end in 2 or 4, but to make it the smallest it should start with the 2: 23594 mod 100 = 94.**
10	360 [potato chips]	If one potato chip weighs 1/5 of an ounce, how many potato chips would it take to weigh 4.5 pounds? **4.5 lbs x 16 ounces/lb = 72 ounces x 5 chips/ounce = 360 chips**
11	8	PJ solved the equation $ax^2 + bx + c = 0$, where a, b, and c are integers that are all relatively prime to each other, and a > 0, and got the solutions: $x = \frac{-7 \pm \sqrt{57}}{4}$ What is the value of a + b + c? **Working backwards from the solutions, the quadratic is: $2x^2 + 7x - 1 = 0$**
12	120 [sequences]	How many four letter sequences can be made from the letters in the word STATISTICS, if letters cannot be repeated? For example, STTA is not a valid sequence. **Throw out any repeated letters, which leaves us with: STAIC, 5 letters total. Therefore there are 5*4*3*2 = 120 different sequences that can be made.**
13	16 [scores]	Aditi's first four chemistry lab scores are 72, 81, 85 and 99. All lab scores are integer values from 0 to 100 inclusive. How many possible scores can Aditi get on the 5th lab assignment so that the median of all five lab scores will be exactly 85? **For 85 to be the median, it must be in the middle, which will be accomplished if the 5th score is from 85 to 100, inclusive. That means 100 – 84 = 16 possible values.**
14	[A + B =] -3	The function $f(x) = x^3 + x^2 - 46x + 80$ has three x-intercepts, one of which is at (2, 0). The other two x-intercepts are at (A, 0) and (B, 0). What is the value of A + B? **$(x^3 + x^2 - 46x + 80)/(x - 2) = x^2 + 3x - 40 = (x + 8)(x - 5)$, so the other two x-intercepts are (-8, 0) and (5, 0) and -8 + 5 = -3**
15	30	What is 80% of 150% of 25? **0.8(1.5)(25) = 30**
16	55	What is the next term in the sequence that begins: -10, -5, 4, 17, 34, … **The differences between the terms are: 5, 9, 13, 17, so they are increasing by 4 each time. Therefore the next difference will be 17+4 = 21, and 34 + 21 = 55.**

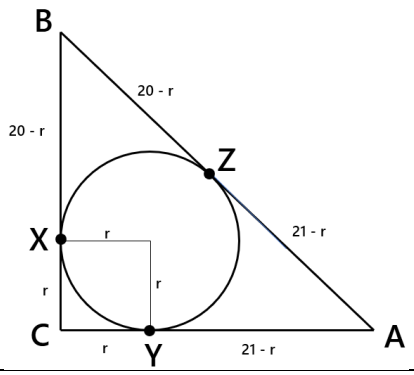
17	20 [miles]	One train leaves the station heading due west at an average rate 50 miles per hour. A second train leaves the same station at the same time at an average rate of $50\sqrt{2}$ miles per hour heading northwest, at 45° to the first train. In miles, what is the shortest distance between the two trains after 24 minutes? **After 24 minutes, the trains will have traveled (24/60)50 = 20 miles and (24/60)$50\sqrt{2}$ = $20\sqrt{2}$ miles respectively. The distance traveled by the first train will be the length of a leg of a 45-45-90 triangle, so the distance between the trains is the length of the other leg, which is also 20 miles.**
18	10	At a math contest, the four finalists are Alge-Bro, Joe-Mom-etry, Math Master G and The Denominator. Their respective probabilities of winning the contest are 11/28, 5/12, 3/28 and 1/12. The day of the contest, The Denominator sprained his brain and was unable to compete. The new probability for Alge-Bro to win can be written as a reduced common fraction A/B. What is A + B? **Without The Denominator, the new sum of the probabilities is 1 – 1/12 = 11/12. Therefore, Alge-Bro's probability of winning is now (11/28)/(11/12) = 12/28 = 3/7**
19	9 [cm]	Kaylee, Lu, Mayra and Nathan are comparing their heights in centimeters. We know that: • Nathan is 3 cm taller than Kaylee • Mayra is 6 cm taller than Lu • Mayra's height is the arithmetic average of Lu's and Nathan's heights What is the positive difference in height, in centimeters, between the shortest person and the second tallest person? **Nathan must be 6 cm taller than Mayra, to make her height the average of Nathan's and Lu's. Kaylee is 3 cm shorter than Nathan, so she must be the 2nd tallest, and the difference is 3 + 6 = 9 cm.**
20	40	Two full decks of playing cards are combined (standard 52-card decks). What is the fewest number of cards that must be selected to be guaranteed of having at least one four-of-a-kind? **If you get 3 cards each of all 13 denominations, then the next card (40th) will make a four of a kind.**

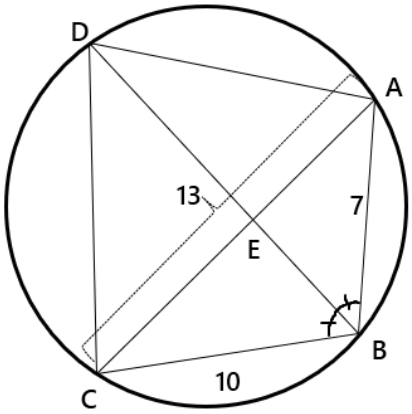
21	7	The function g(x) is created by transforming the following function f(x) with a horizontal shift of 4 units to the right and a vertical shift of 3 units down. The resulting function g(x) can be written: $g(x) = A(x - B)^2 + C$, where A, B and C and all non-negative integers. What is A + B + C? $f(x) = 2(x - 1)^2 + 3$ **$G(x) = 2(x - 5)^2 + 0$** **A = 2, B = 5, C = 0**
22	24	The tenth term in an arithmetic sequence is equal to four times the first term. The sum of the first 10 numbers of the sequence is 450. What is the second term in the sequence? **$Sn = n/2(a1 + a10)$** **$450 = 10/2(a1 + 4a1)$** **a1=18, a10 = 72, d = 6** **a2 = 18+6 = 24**
23	9	A square has an area of 40 square inches. The length of the diagonal, in inches, can be written as a fully-reduced radical $A\sqrt{B}$. What is A + B? **Side length = $\sqrt{40}$.** **Diagonal length = $\sqrt{40} \cdot \sqrt{2} = 4\sqrt{5}$.**
24	3	Evaluate: $\log(1750_8)$ **1750_8 = 1*512 + 7*64 + 5*8 + 0*1 = 1000.** **Log 1000 = 3**
25	[A + B =] 157	As a reduced common fraction, the mean of the following data set is A/B. What is the value of A + B? $\left\{\frac{2}{5}, \frac{5}{3}, \frac{13}{12}, \frac{7}{10}\right\}$ **[2/5 + 5/3 + 13/12 + 7/10]/4 = (1/4)(24 + 100 + 65 + 42)/60) = 231/240 = 77/80, and 77 + 80 = 157**
26	77	Mr. Saari made a stem-and-leaf plot of the first test scores from his Geometry students, where the Stem represents the tens digit and the Leaf represents the units digit. Find the value of: $Range * (Median - Mode)$ **Range = 99 – 22 = 77** **Median = 48, the middle value** **Mode = 47, repeated 3 times** **77(48 – 47) = 77**

Stem	Leaf
2	2 5 6
3	3 5 6 8
4	2 5 5 6 7 7 7 8 8 9
5	2 4 6 8 8
6	5 8 9
7	4 5
8	7
9	9

27	-3	A triangle with vertices (3, -2), (4, 1) and (-2, 4) is rotated 90° counterclockwise around the origin. What is the sum of all the x-coordinates of the vertices of the new triangle? **(3, -2) ➔ (2, 3)** **(4, 1) ➔ (-1, 4)** **(-2, 4) ➔ (-4, -2)** **2 – 1 – 4 = -3**
28	0	What is the sum of the coefficients of all terms in the remainder (including any constant term) when the polynomial $P(x) = x^5 + 2x^3 - 3$ is divided by the polynomial $x^2 + x - 2$? **The remainder will be linear: Ax + B. So, P(x) = (x+2)(x-1)Q(x) + Ax + B** **P(1) = 0 = A + B** **Alternatively, use long division to find that the remainder is 17x – 17.**
29	17	Sterling is filling a box with numbered ping-pong balls. He puts in one ball with the number 1, two balls with the number 2, three balls with the number 3, and so on, ending with ten balls with the number 10. He reaches in and pulls out a single ping-pong ball at randomly. The probability that it contains an even number can be written as a reduced fraction A/B. What is A + B? **There are 55 total balls. Thirty of them are even. 30/55 = 6/11**
30	20 [pairs]	How many pairs of nonnegative integers x and y are solutions to the following equation: $$\frac{x}{19} + \frac{y}{95} = 1$$ **Multiply through by 95 and rearrange to get:** **Y = -5x + 95** **This is a linear equation with y-intercept (0, 95) and x-intercept (19, 0). Therefore, all values of x = 0 to 19 will yield integer (x, y) coordinates.**
31	1	Solve for y, given the following infinite expression: $$y = \frac{4}{3 + \log_y\left(\frac{4}{3 + \log_y\left(\frac{4}{3 + \log_y(\dots)}\right)}\right)}$$ **Substitute as follows:** $$\boldsymbol{y = \frac{4}{3 + \log_y y} = \frac{4}{3 + 1} = 1}$$

32	10	Mr. Foster has five pairs of socks in his drawer. Each pair consists of two identical socks, and each pair is distinguishable from every other pair. On Monday, he randomly grabs two socks from the drawer and wears them. Without returning those socks to the drawer, he randomly grabs two socks from the drawer on Tuesday. The probability that he gets a matching pair on Tuesday can be written as a reduced common fraction A/B. What is A + B? **A total of 4 socks are being selected out of the 10, call them AABBCCDDEE.** **The 1st sock on Monday can be any sock, and there is a 1/9 probability of getting a match on the 2nd. Assuming there is a match, then the 3rd sock (on Tues) can be any, and there is a probability of 1/7 of getting a match. Thus a total P = (1/9)(1/7) = 1/63 for this path.** **Going back to the 2nd sock on Mon, there is 8/9 probability that it is not a match. For example, the first sock was A and the second was B. Therefore, for the 3rd sock selection, there are 2 out of 8 socks that it will be impossible to match, and there are 6 out of 8 socks where it is possible to match. For the 4th sock, there is 1/7 probability of a match and 6/7 probability of not a match. Following the path, there is (8/9)(6/8)(6/7) = 6/63 probability. Add this to the 1/63 = 7/63 = 1/9 probability of a matching pair on Tuesday.**
33	[a + b + c + d =] -512	The expression $(3x - 11)^3$ can be expanded, resulting in a polynomial in the form $ax^3 + bx^2 + cx + d$. What is the value of a + b + c + d? **Let x = 1: $(3 - 11)^3 = (-8)^3 = 512 = a + b + c + d$**
34	1	What is the remainder when 2^{100} is divided by 101? **In other words, what is 2^{100} (mod 101). 101 is prime and 2 is relatively prime to 101. Therefore, by Fermat's little theorem, $2^{100} \equiv 1$ (mod 101).**
35	2400 [ways]	How many ways are there to arrange the letters in the word G-E-N-E-R-A-T-E, if no two of the three Es may be adjacent to each other? **None of the S's can be next to each other. For example: S _ S _ S _ _ _** **In that case, there are 5! = 120 ways to arrange the other 5 letters.** **There are a total of 20 ways to arrange the S's so that none of them are next to each other, therefore 120 x 20 = 2400.**

36	12 [units]	Given that triangle ABC has side lengths of 20, 21 and 29 units, find the diameter of the circle, in units, that is inscribed in the triangle. **20-21-29 is a Pythagorean Triple, which makes ABC a right triangle. Point of tangency X, Y and Z are indicated. Let r = radius of the inscribed circle. Therefore, BX = BZ = 20 – r, and AY = AZ = 21 – r. But, AZ + BZ = 29, so:** **(20-r)+(21-r)=29, r = 6** **Therefore the diameter = 12.** 
37	936	What is the largest 3-digit number such that all of the digits are different and the number is divisible by each of the digits? **Start with a 9 in the hundreds place. Therefore the number needs to be divisible by 9, so the sum of the other 2 digits must be 9. Possible pairs are: 8 and 1, 7 and 2, 6 and 3, 5 and 4. Use trial and error to discover that 981, 972, 963, 954, and 945 are not divisible by all 3 digits. 936 is the first one that is divisible by all 3 digits.**

38	5	The sum of the following infinite series can be written as a reduced common fraction A/B. What is A + B? $$\frac{1}{2\cdot 4}+\frac{1}{4\cdot 6}+\frac{1}{6\cdot 8}+\cdots$$ **The sum can be written as:** $$\sum_{n=1}^{\infty}\frac{1}{(2n)(2n+2)}$$ **The expression can be decomposed into partial fractions:** $$\frac{\frac{1}{2}}{2n}-\frac{\frac{1}{2}}{2n+2}$$ **Therefore, the summation can be rewritten as:** $$\frac{1}{2}\left(\sum_{n=1}^{\infty}\frac{1}{2n}-\sum_{n=1}^{\infty}\frac{1}{2n+2}\right)$$ $$\frac{1}{2}\left(\frac{1}{2}+\sum_{n=2}^{\infty}\frac{1}{2n}-\sum_{n=1}^{\infty}\frac{1}{2n+2}\right)$$ **The two summations are now equal, so the sum is (1/2)(1/2) = ¼.**
39	108	ΔABC is inscribed in a circle. D is a point on arc $\widehat{AC}$ such that AD = CD and points B and D are on opposite sides on $\overline{AC}$. Let $\overline{BD}$ intersect $\overline{AC}$ at point E. If AB = 7, BC = 10, and AC = 13, the length of AE can be written as a reduced common fraction A/B. What is A + B? **Because AD = CD, it implies that ∠ABC = ∠CBD, making BD the angle bisector of ∠ABC. Point E is the point where the angle bisector of ∠ABC intersects AC. By the Angle Bisector Theorem,** $$\frac{BA}{AE}=\frac{BC}{CE},\rightarrow \frac{AE}{CE}=\frac{BA}{BC}=\frac{7}{10}$$ **Because AE/CE = 7/10, let AE = 7x and CE = 10x. Since AE + CE = AC, and AC = 13:** **7x + 10x = 13, x = 13/17.** **AE = 7(13/17) = 91/17.** 

40	65	The positive integers are arranged in the following infinite pattern. Each integer has a certain row and column position, indicated by (r, c). For example, the number 9 is in position (2, 3). Find the location (r, c) that the number 2022 is in. What is r + c? **Notice that the numbers in the first row are the triangular numbers:** **T(n) = n(n + 1)/2** **To find the diagonal containing 2022, find a T(n) that is close to 2022. T(63) = 2016. Therefore, position (64, 1) will be 2017. Count up and over to arrive at position (59, 6) for 2022. 59 + 6 = 65**

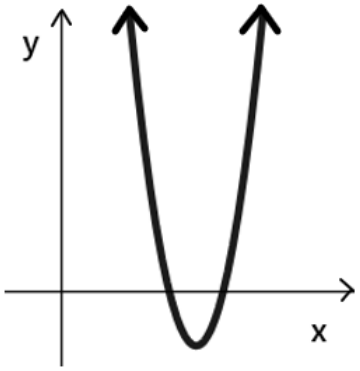
		Column Number:					
		1	**2**	**3**	**4**	**5**	**6**
Row Number:	**1**	1	3	6	10	15	…
	2	2	5	9	14	20	…
	3	4	8	13	19	26	…
	4	7	12	18	25	33	…
	5	11	17	24	32	41	…
	6	…	…	…	…	…	…

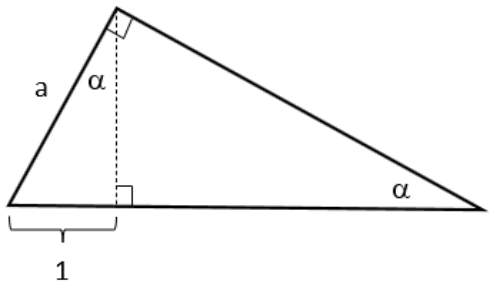
"Math is Cool" Championships -- 2022-23

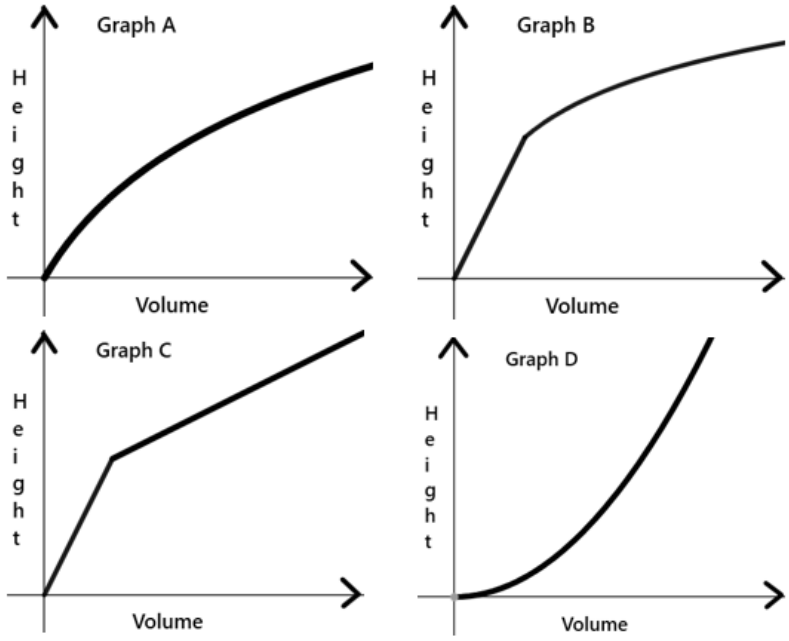
High School

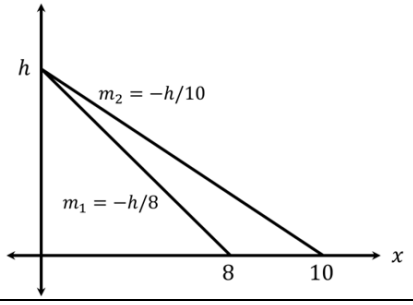
Multiple Choice Solutions

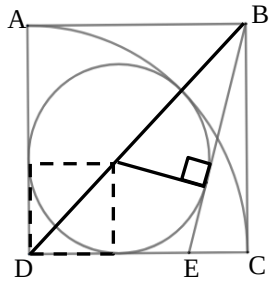
9/ 10th	11/ 12th	Answer	Solution
1	1	B	Given the following data set, which quantity cannot be changed by adding a single integer to the set? {2, 5, 18, 10, 5, 6, 13, 5} A) Median B) Mode C) Mean D) Range E) Answer not given. **Put in order: {2, 5, 5, 5, 6, 10, 13, 18}** **The mode cannot be changed, because there are three 5s. The other values can all be changed.**
2	2	A	If x = $3+\sqrt{8}$, find the value of: $x+\frac{1}{x}$ A) 6 B) $2\sqrt{10}$ C) $3\sqrt{8}$ D) $2\sqrt{12}$ E) Answer not given $\mathbf{3+\sqrt{8}+\frac{1}{3+\sqrt{8}}\cdot\frac{3-\sqrt{8}}{3-\sqrt{8}}}$ $\mathbf{3+\sqrt{8}+\frac{3-\sqrt{8}}{9-8}}$ **= 6**
3	3	E (28/169)	Mrs. Stephenson has two bags of dice. The first bag has 7 red dice and 6 white dice. The second bag has 4 red dice and 9 white dice. If she chooses one die randomly from each bag, what is the probability that they are both red? A) 7/13 B) 11/13 C) 11/169 D) 4/257 E) Answer not given. **P(R&R) = (7/13)(4/13) = 28/169**

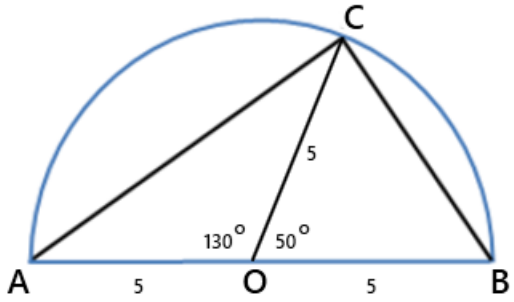
4		B	Which of the following functions could describe the graph shown here?  A) $f(x) = (x + 12)^2 + 4$ B) $f(x) = (8 - x)(12 - x)$ C) $f(x) = (2x + 3)^2 - 5$ D) $f(x) = 2(-x + 2)(x + 3)$ E) None of them. **The graph of function B is the only one where both zeros (x = 8, 12) are positive values.**
	4	B	How many years will it take for an investment of $1200 to grow to $3600 if the interest rate is 5% interest compounded yearly? A) $\log_3(1.05)$ B) $\frac{\log(3)}{\log(1.05)}$ C) 20.2 D) $12 \cdot \log(3)$ E) Answer not given. **$3600 = 1200(1 + 0.05)^t$** **$3 = 1.05^t$** **Log 3 = t log 1.05** **T = log 3 / log 1.05**

5	5	D	A clothing shop sells the same shirt in red, blue or green. All three colors cost the same amount of money to make, but they are sold at different markups. The red shirt is marked up 20%, the blue shirt 25% and the green shirt 10%. During a clearance sale, all shirts were offered at a discount. The red shirts were 35% off, the blue shirts were 40% off, and the green shirts were 30% off. Which of the following is the correct comparison of the final price of each shirt? A) red < blue < green B) green < red < blue C) blue < red < green D) blue < green < red E) Answer not given. **Assume the cost for each shirt is $100. With the markup:** **Red = $120** **Blue = $125** **Green = $110** **With the discounts:** **Red = 120 – 42 = $78** **Blue = 125 – 50 = $75** **Green = 110 – 33 = $77**
6		B	For the right triangle shown here, what is the length of side 'a' in terms of the angle α? A) a = sin(α) B) a = csc(α) C) a = cos(α) D) a = sec(α) E) Answer not given. **Use the similarity of the triangles to also label angle α as shown. SOH, therefore:** **sin α = 1/a, a = 1/sin α = csc α** 

	6	B	A funnel is sealed shut at the bottom, and is being filled with water at a constant rate. A cross-section of the funnel is shown. The bottom portion of the funnel is cylindrical, and the top portion is conical. Which of the following graphs could represent the height of the water in the funnel as a function of the volume of water in the funnel?  A) Graph A B) Graph B C) Graph C D) Graph D E) None of them. **Because the bottom of the funnel is a cylinder, it has a constant cross-section as that portion is being filled, therefore it will fill at a constant, or linear rate. For the top conical part of the funnel, the cross-section is not constant, therefore the rate of change of the height will not be linear. This can only be described by Graph B, which has steep linear increase initially, followed by a non-linear decreasing rate of change.**

7	7	A	Two candles of equal heights but different thicknesses are lit. The first burns off in 8 hours and the second in 10 hours. How long after lighting, in hours, will the first candle be half the height of the second candle? The candles are lit simultaneously and each burns at a constant linear rate. A) 20/3 B) 6 C) 43/6 D) 49/6 E) Answer not given. **Equations can be written for both candles:** **Y1 = (-h/8)x + h** **Y2 = (-h/10)x + h** **Want the time (x) when y2 = 2y1** **(-h/10)x + h = 2[(-h/8)x + h]** **Solve for x = 20/3** 
8	8	B	How many two-digit positive integers have the property that the integer is equal in value to the sum of its digits multiplied by 4? A) 3 B) 4 C) 5 D) 6 E) Answer not given. **A 2-digit number AB = 4(A + B). Can write AB as 10A + B, where A > 0. Therefore, 10A + B = 4A + 4B.** **6A = 3B** **B = 2A** **The possibilities for A are 1, 2, 3, 4, which means there are four such possible integers. Alternatively, can brute force it by listing out the 2-digit numbers that are divisible by 4, and discovering that 12, 24, 36, and 48 meet the requirements.**

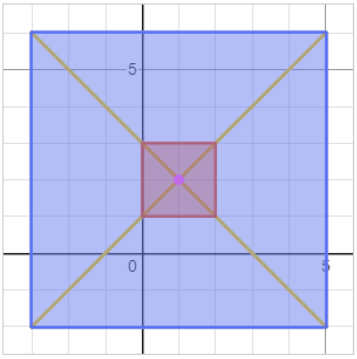
9		*C*	A quarter-circle is inscribed in square ABCD as shown below, with its center at vertex D. A smaller circle is tangent to the quarter-circle and to the bottom and left-hand sides of the square and $\overline{BE}$ is tangent to the smaller circle. What is the number of degrees in the measure of ∠BED? A) 95° B) 100° C) 105° D) 120° E) Answer not given. **Add diagonal $\overline{BD}$, which contains the center of the small circle. Let DC = R, which is the radius of the quarter-circle and let r be the radius of the small circle. The diagonal of the dashed square is then $r\sqrt{2}$, so this means $R = r + r\sqrt{2}$, and this is also the side length of square ABCD. The diagonal of square ABCD would then be $(r + r\sqrt{2})\sqrt{2} = r\sqrt{2} + 2r$. This means the distance from the center of the small circle to vertex B is 2r and the radius to the point of tangency of $\overline{BE}$ is r. The radius to the point of tangency is also perpendicular to $\overline{BE}$, so it must be a 30-60-90 triangle. This means that $m\angle DBE = 30^\circ$, $m\angle BDE = 45^\circ$, and 180 – 30 – 45 = 105 so $m\angle BED = 105^\circ$** 

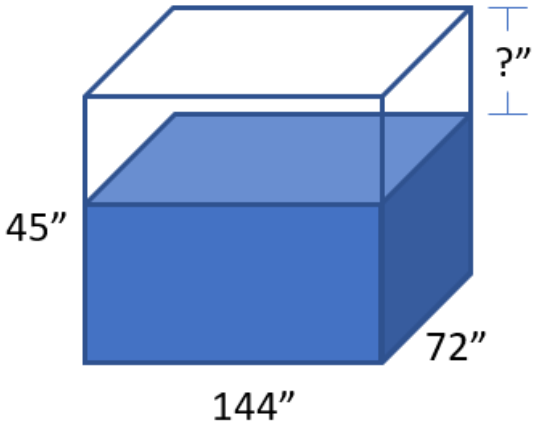
	9	*C*	There is a semicircle O with a radius of 5 and diameter AB. Point O is at the center of diameter AB. Point C lies on the semicircle so that ∠AOC = 130°. Find $(\overline{AC})^2 - (\overline{BC})^2$. A) 50·cos(130°) B) 50·cos(50°) C) 100·cos(50°) D) 100·cos(130°) E) Answer not given. **From the Law of Cosines, BC^2 = 50 – 50cos50. Also, AC^2 = 50 – 50cos130.** **$AC^2 - BC^2$ = 50(cos50 – cos130) = 50(cos50 – [-cos50]) = 50(2cos50) = 100cos50.** 
10	**10**	*B*	Let a sequence be defined as follows: $R_1 = 1$, $R_2 = 1$, R_n equals the remainder when $R_{n-1} + R_{n-2}$ is divided by 5, for $n \geq 3$. What is R_{2022}? A) 0 B) 1 C) 2 D) 3 E) 4 **Write down the first several values and look for a pattern.** **1, 1, 2, 3, 0, 3, 3, 1, 4, 0, 4, 4, 3, 2, 0, 2, 2, 4, 1, 0, 1, 1, …** **The pattern repeats itself every 20 values.** **Therefore, $R_{2022} = R_2 = 1$.**

"Math is Cool" Championships -- 2022-23

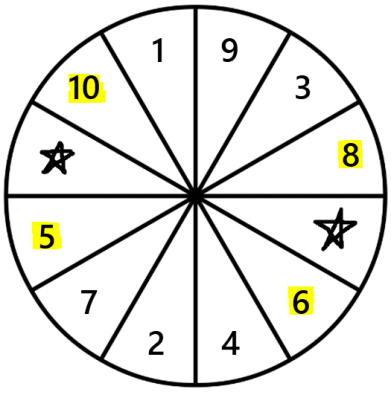
High School

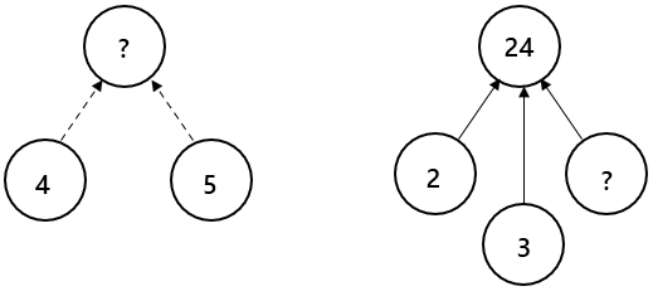
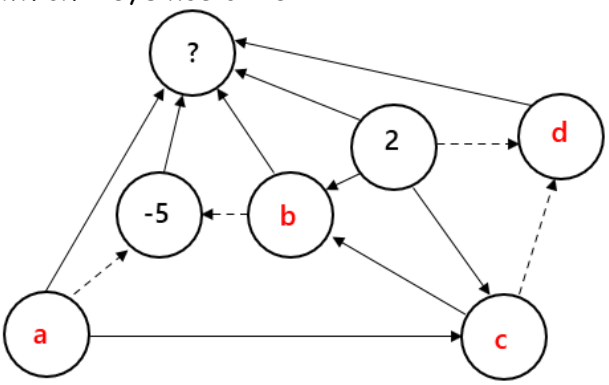
Team Test Solutions

9/ 10th	11/ 12th	Answer	Solution
1	**1**	1	Solve for x: 4 – 2(2 – 3x) – 6 = 9 + 2(5x – 4.5) – 10x **4 – 4 + 6x – 6 = 9 + 10x – 9 – 10x** **6x – 6 = 0** **x = 1**
2		-4	On the coordinate plane, the point (3, 7) is rotated counterclockwise 90° about the origin to a new location with coordinates (a, b). What is the value of a + b? **It will be rotated to the point (-7, 3).**
	2	11	A square with side length 2 units is centered at the point (1, 2) on the coordinate plane, with one side of the square lying on the y-axis. The square is dilated around the point (1, 2) by a scale factor of 4. After dilation, the vertex of the square lying in Quadrant I is at point (a, b). What is a + b? **The starting upper-right vertex is at (2, 3). After dilation it will be at (5, 6).** 

3		20	The distance around the Earth is approximately 4×10^4 km. The distance from the Earth to the Moon is approximately 4×10^5 km. Approximately how many trips around Earth is equivalent to a round-trip visit to the Moon? **A round trip to the Moon will be 8×10^5 km.** **$\frac{8\times10^5}{4\times10^4} = 2 \times 10^1$ = 20**
	3	3000000 [ladybugs]	If an average human weighs 60 kilograms, and an average ladybug weighs 20 milligrams, how many ladybugs does it take to weigh the same as one human? **20 mg = 0.000002 kg = $2x10^{-5}$ kg** **$6x10^1/2x10^{-5} = 3x10^6 = 3000000$**
4	**4**	47	The set of integers shown below have the same median and mode. What is the value of n? {47, 91, 39, 20, 83, n} **Given: 20, 39, 47, 83, 91** **'n' must be a repeat to form a mode, and the only possible value is 47, which puts 47, 47 in the middle so that the median is also 47.**
5	**5**	9 [inches]	A bin in the shape of a rectangular prism is 144 inches long, 72 inches wide and 45 inches tall, as shown below.  When the bin contains exactly 8 cubic yards of water, how many inches are in the distance between the top edge of the bin and the top surface of the water? Note: the diagram is not to scale. **1 cubic yard = 27 cubic feet, 8 cubic yards = 216 cubic feet The area of the base of the bin is 12* 6 = 72 square feet. 216/72 = 3 feet = 36 inches = height of the water 45 – 36 = 9 inches**

6	**6**	80 [hours]	PumpCo has many identical pumps to remove water after a flood. Multiple pumps are being used to remove water from a flooded basement. With the number of pumps being used, it will take 10 hours to remove the water. If 2 additional pumps had been used, it would only take 8 hours. How many hours would it take one pump to remove the water? **P = no. of pumps** **P pumps : 10 hours : 1 job = 10p pump-hours/job** **P + 2 pumps : 8 hours : 1 job = 8p + 16 pump-hours/job** **8p + 16 = 10p, p = 8, and it takes 10(8) = 80 pump-hours/job. Therefore with one pump, it will take 80 hours.**
7	**7**	13	Nikhar has a one-dollar bill, a five-dollar bill, and a ten-dollar bill and Devanshi has a one-, a five-, a ten-, and a twenty-dollar bill. If the two randomly exchange one bill, the probability as a reduced common fraction that Devanshi has the same amount of money as Nikhar is A/B. What is the value of A + B? **There are 12 exchanges possible shown below, one of which results in Nikhar and Devanshi having the same amount of money, so the probability is 1/12, and 1 + 12 = 13** **(J's bills/sum) (M's bills/sum)** **(1, 5, 10 / 16) (1, 5, 10, 20 / 36)** **(5, 5, 10 / 20) (1, 1, 10, 20 / 32)** **(10, 5, 10 / 25) (1, 5, 1, 20 / 27)** **(20, 5, 10 / 35) (1, 5, 10, 1 / 17)** **(1, 1, 10 / 12) (5, 5, 10, 20 / 40)** **(1, 5, 10 / 16) (1, 5, 10, 20 / 36)** **(1, 10, 10 / 21) (1, 5, 5, 20 / 31)** **(1, 20, 10 / 31) (1, 5, 10, 5 / 21)** **(1, 5, 1 / 7) (10, 5, 10, 20 / 45)** **(1, 5, 5 / 11) (1, 10, 10, 20 / 41)** **(1, 5, 10 / 16) (1, 5, 10, 20 / 36)** **(1, 5, 20 / 26) (1, 5, 10, 10 / 26)**

8		29	The circle shown here has twelve equal sections. Each section will contain a different integer from 1 to 10, and the remaining two sections will contain stars. Use the statements below to determine the layout of the numbers. What is the sum of the numbers in the four sections that are immediately adjacent to the stars? 1. The five even numbers appear in order counterclockwise, though not necessarily in consecutive sections. 2. The numbers 1 and 3 are adjacent to the section that is directly opposite from the number 2. The number 7 is directly between two sections with numbers totaling 7. 3. Two numbers are adjacent to, in a clockwise direction, their respective square roots (if the number is in one section, move clockwise to get to its square root). 4. No two adjacent sections have numbers totaling more than 12. **The sections adjacent to the stars are: 8, 6, 5, 10.** **8+6+5+10 = 29** 

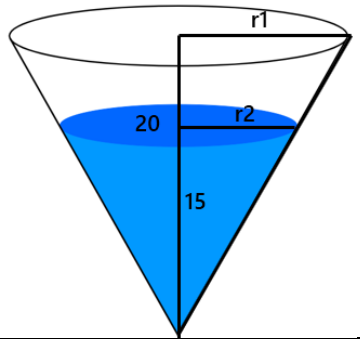
	8	0	In a circle puzzle like the ones shown here, dashed arrows mean to add and solid arrows mean to multiply. For example, the unknown value in the first puzzle is 9, because 4 + 5 = 9. The unknown value in the second puzzle is 4, because 2x3x4 = 24.  In the following circle puzzle, what is the unknown value, indicated by the question mark in the top circle? Assign variables to the unknown circles. Write a system of equations: A + b = -5 2 + c = d 2a = c 2c = b Solve for: a = -1, b = -4, c = -2, d = 0 The question mark is the product of a(-5)b(2)cd which = 0, since d = 0 

9	**9**	-11	Let $i = \sqrt{-1}$, $a + b = 15$, and $ab = 54 - 64i$. If $a^2 + b^2 = c + di$, for real c and d, then what is $c - d$? $\boldsymbol{15^2 = (a+b)^2}$ $\boldsymbol{225 = a^2 + 2ab + b^2}$ $\boldsymbol{225 = a^2 + b^2 + 2(54 - 64i)}$ $\boldsymbol{a^2 + b^2 = 225 - 108 + 128i}$ $\boldsymbol{= 117 + 128i = c + di}$ **Therefore, c = 117, d = 128, c – d = 117 – 128 = -11**
10	**10**	35	Let a, b, and c be three distinct integer bases, such that: $12_a + 54_b = 31_c \ \ and \ \ 23_a + 8_b = 16_c$ where, for example, 12_a indicates 12 base a. What is the minimum possible sum a + b + c? **As setup, we know the following:** **a > 3, b > 8, c > 6 and are integers, and the equations** $\boldsymbol{(a+2) + (5b+4) = 3c+1}$ $\boldsymbol{(2a+3) + 8 = c+6}$ **After simplifying:** $\boldsymbol{a + 5b + 5 = 3c}$ $\boldsymbol{2a + 5 = c}$ **By differently combining these two equations we can derive the following relationships between variable pairs** $\boldsymbol{b = a + 2}$ $\boldsymbol{c = 2a + 5}$ $\boldsymbol{c = 2b + 1}$ **From here we can guess and check reasonably quickly from lower bounds until we find a valid set.** **For example, the minimum a is 4, which leads to an invalid b=6.** **The minimum b=9 gives a=7, c=19, which satisfies all three simplified relations and the original equations.** **7+9+19=35**

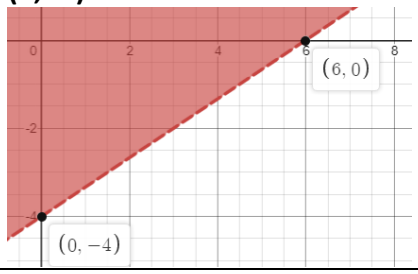
"Math is Cool" Championships -- 2022-23

High School

Pressure Round Solutions

9/ 10th	11/ 12th	Answer	Solution
1	1	91	A 20 foot tall cone that was full of water has now been partially drained. The water level is now 5 feet from the top of the cone in the vertical direction. The ratio of the volume of water remaining in the cone to the original volume of water can be written as a reduced common fraction A/B. What is A + B? **Volume = $\frac{1}{3}\pi r^2 h$** **Let original full volume = V1** **Reduced volume = V2** **Want: V2/V1** **The two triangles are similar, which creates a ratio:** **r2/r1 = 15/20 = ¾** **V2/V1 simplifies to the ratios of r^2h for each volume** **= $(3/4)^2(3/4)$ = 27/64** **27+64 = 91** 
2		-6	Solve for x: $$\frac{3}{2} - \frac{7}{x-1} = \frac{5}{2}$$ $$\frac{-7}{x-1} = 1$$ **-7 = x – 1** **x = -6**

	2	3844	Solve for x: $\sqrt{\left(1+\sqrt{\left(3-\sqrt{\left(1+\sqrt{(2+\sqrt{x})}\right)}\right)}\right)}=1$ **Repeatedly square both sides of the equation to remove the radicals. Next to last step is:** $\sqrt{x}=62$ **Therefore, x = 62^2 = 3844.**
3	**3**	123	A game is played as follows. There are 8 boxes on a table, 4 of which contain a golden coin. The other 4 contain nothing. A contestant wins if they get at least 2 golden coins. The contestant randomly selects 4 of the boxes. The probability that the contestant wins the game can be written as a reduced fraction A/B. What is A + B? **The total number of ways to choose 4 boxes out of 8 is 8C4 = 70.** **The contestant wins if they choose either 2, 3, or 4 boxes with a golden coin.** **No. of ways to choose all 4 = 1** **No. of ways to choose 3 = 4C3 x 4C1 = 16** **No. of ways to choose 2 = 4C2 x 4C2 = 36** **Sum of ways = 53** **P= 53/70**
4	**4**	23 [palindromes]	A palindrome is a number that reads the same forwards and backwards. How many three-digit palindromes are there in which the units digit is a factor of the middle digit? **111, 121, 131, 141, 151, 161, 171, 181, 191, 222, 242, 262, 282, 333, 363, 393, 444, 484, 555, 666, 777, 888, and 999 makes 23 palindromes.**

5		1 [point]	On the graph of the line 2x – 3y = 12, how many grid points (where the x- and y-coordinates are both integers) are on the portion of the line that lies in Quadrant IV? **The equation is y = (2/3)x – 4 in slope intercept form. Starting at the y-intercept of (0, -4), count up 2 and over 3 to get to a grid point of (3, -2). Count up 2 and over 3 again to get to (6, 0), which is not in QIV. Therefore, there is only one point.**
	5	7 [points]	On the graph of the linear inequality: 2x – 3y < 12, how many grid points (where the x- and y-coordinates are both integers) are contained in the portion of the solution region that lies in Quadrant IV? **The equation for the boundary line is 2x – 3y = 12, which is y = (2/3)x – 4 in slope intercept form. Points on the line (like (3, -2)) do not count because the inequality is < and not ≤ and points on the axes do not count. There are 7 points in quadrant 4 within the solution region, (1, -1), (1, -2), (1, -3), (2, -1), (2, -2), and (3, -1), (4, -1).** 

"Math is Cool" Championships -- 2022-23

High School

College Bowl Round #1 Solutions

	Answer	Solution
1	432 [minutes]	How many minutes are there in 30% of one day? **24 hours * 60 minutes/hour * 0.3 = 432**
2	48 [blocks]	Micah has a 12 foot long by 8 foot wide by 8 foot tall box. What is the maximum number of 2 foot by 2 foot by 4 foot blocks that can fit inside the box? **The blocks will fit evenly inside the box and fill it completely. The bottom layer consists of 2 rows of 6 blocks = 12 blocks, times 4 layers = 48 blocks.**
3	16 [posts]	Gregg plans to build a fence to enclose his hamster pasture. The pasture is square with a perimeter of 96 feet, and he sets fence posts at each corner, with additional posts at regular 6-foot intervals. How many posts will he need? **Each side is 24 feet. One post at each corner plus an additional 3 on each side = 16 total.**
4	49	A coin is flipped three times and a fair 6-sided die is rolled once. The probability that the coin turns up heads all three times and the die rolls a 2 can be written as a reduced common fraction A/B (read: A over B). What is A + B? **P(H&H&H&2) = (1/2)(1/2)(1/2)(1/6) = 1/48**
5	36	The Greatest Common Factor for a pair of numbers is 18, and their Least Common Multiple is 180. If one of the numbers is 90, what is the other number? **The product of a pair of numbers = the product of their GCF and LCM.** **(18)(180) = 3240. 3240/90 = 36**
6	145	What is the sum of the first 10 terms of the sequence that starts with index n = 1 and has a general term a_n (read: a-sub-n) equal to 3n – 2? **Sequence is: 1, 4, 7, …, so d = 3. S_{10} = (10/2)[2(1) + (9)(3)] = 145**
7	59	The number 77 is written as the sum of two numbers. One of the numbers is 5 more than three times the other number. What is the larger of the two numbers? **A+B = 77** **A = 3B + 5** **3B + 5 + B = 77** **B = 18, A = 59**

8	1 [triangle]	How many scalene triangles with integral side lengths have a perimeter equal to 12 units? **The only scalene triangle possible has sides 3-4-5.**
9	53	What is the median of the set of integers from 6 through 100, inclusive? **The median will be the 48th number, which is 53.**
10	10 [whole numbers]	The 'digit sum' of a whole number is the sum of its individual digits. For example, the digit sum of 123 is 6, because 1+2+3 = 6. How many positive 3-digit whole numbers have a digit sum of 4? **List:** **400** **301,310** **202,211,220** **103,112,121,130**

"Math is Cool" Championships -- 2022-23

High School

College Bowl Round #2 Solutions

	Answer	Solution
1	107	The mean of 8 numbers is 100. If seven is added to each of the numbers, what is the new mean of the 8 numbers? **Initial sum of the 8 numbers = 800. After adding 7 to each number, the sum = 1056. 1056/8 = 107.**
2	19	Given that a prime number is less than 37, the probability that it is less than 23 can be written as a reduced common fraction A over B (read: A over B). What is A + B? **2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31** **Eleven total, and 8 of them are < 23. P = 8/11**
3	5	How many distinct prime factors does the number 32340 (read: thirty-two thousand three hundred forty) have? **$32340 = 2^2x3x5x7^2x11$**
4	11	The Incredible Hulk can double the distance he jumps with each succeeding jump. If his first jump is 1 meter, the second jump is 2 meters, the third jump is 4 meters, and so on, on which jump number will he first be able to jump more than 1 kilometer in a single jump? **The sequence of terms is 2^0, 2^1, …, 2^n. Since $2^{10} = 1024$, the 11^{th} term will equal 1024 meters, which is the first one to exceed 1 km.**
5	10 [questions]	On a 26-question math test, 5 points were deducted for each wrong answer, and 8 points were awarded for each correct answer. If all questions were answered, and the final score was 0, how many questions were answered correctly? **A + B = 26** **-5A + 8B = 0** **Solve system for A = 16 incorrect, B = 10 correct**
6	10 [orders]	Aidan and Ruby are playing tic-tac-toe. Wins are worth 1 point, ties are worth half a point and losses are worth 0 points. In how many different orders of wins, losses and ties can Ruby get 3 points in the first four games? **She can have 3 wins, 4!/3! = 4 ways. Or she can have 2 wins and 2 ties, 4!/(2!2!) = 6 ways.**

7	4 [points]	How many points in a plane are 1 cm away from a given line segment that is 4 cm long, and are also 2 cm away from the midpoint of the segment? **Points that are 2 cm away from the midpoint will form a circle around the center of the segment. Points that are 1 cm away will form a Twinkie shape along the line segment, with half circles at either end that intersect the circle, for a total of 4 points.**
8	11333 [digits]	Hannah writes down all of the integers from 1 to 3110 (read: three thousand one hundred and ten), inclusive. How many total digits did Hannah write? **1 digit: 1-9 = 9** **2 digit: 10-99 = 90x2 = 180 digits** **3 digit: 100-999 = 900x3 = 2700 digits** **4 digit: 1000 – 3110 = 2111x4 = 8444** **9+180+2700+8444 = 11333**
9	93160	Arrange the digits 1, 3, 9, 0 and 6 to form the greatest 5-digit number that is divisible by 4. **Put 9 first to make it big. The last two digits must be divisible by 4, and the only possibility is 60. Fill in the remaining two digits to get 93160.**
10	90 [ways]	How many ways are there to hang 2 identical red shirts, 2 identical blue shirts and 2 identical white shirts in a closet? **6!/(2!2!2!) = 90**

"Math is Cool" Championships -- 2022-23

High School

College Bowl Round #3 Solutions

	Answer	Solution
1	50	What is the mean of the positive odd integers from 1 through 99, inclusive? **Each pair has a mean of 50, i.e. (1+99)/2.**
2	40 [%]	Jaxon bought 12 mangoes and 12 lemons. He can get 8 ounces of mango juice from 3 mangoes and 8 ounces of lemon juice from 2 lemons. He makes a juice blend from an equal number of mangoes and lemons. What percent of the juice blend is mango juice? **MMM = 8oz** **LL = 8 oz** **Suppose there are 6 of each fruit:** **MMM MMM = 16 oz** **LL LL LL = 24 oz** **Total = 40 oz** **16/40 = 0.40 = 40%**
3	5	Consider the function $f(x) = 3x^2 - 5$ (read: 3 x-squared minus 5). Find the value of f(1) + f(2) (read: f of 1 plus f of 2). **f(1) = 3 – 5 = -2** **f(2) = 12 – 5 = 7**
4	32	LeBron ate 100 cookies in five days. On each day, he ate 6 more than on the previous day. How many cookies did he eat on the fifth day? **No. of cookies each day: a1, a1+6, a1+12, a1+18, a1+24** **Sum = 5a1 + 60 = 100** **a1 = 8, therefore a1+24 = 32**
5	6 [turns]	Biff and Eho play a game involving a circle whose circumference is divided by 12 equally spaced points. The points are numbered clockwise, from 1 to 12. They both start on point 12. In one turn, Biff moves 5 points clockwise and Eho moves 9 points counterclockwise. How many turns will it take until they are on the same point again? **After 6 turns they will both be on point number 6.**
6	40 [cm]	A rhombus has diagonals of lengths 12 centimeters and 16 centimeters. What is the perimeter of the rhombus in centimeters? **The diagonals split the rhombus into 4 right triangles with side lengths 6-8-10. Therefore the perimeter is 4x10 = 40.**

7	76 [workers]	Twenty workers completed one-fourth of a job in 8 days. Working at the same rate, how many additional workers will be needed to complete the job in the next 5 days. **It took 20x8 = 160 worker-days to do ¼ of the job, therefore it will take 3*160 = 480 worker-days to do the remaining ¾ of the job. 480/5 = 96 workers, which is an additional of 76 workers.**
8	10	In a bag with 20 marbles, five of them are blue. How many blue marbles must be added to the bag so that the probability of selecting a blue marble at random is $\frac{1}{2}$? **5 blue, 15 are not** **Add 10 blue to get 15 and 15.**
9	289 [minutes]	How many minutes are there between 9:23 and 0 seconds AM and 2:12 and 0 seconds PM on the same day? **9:23 AM to 2:23 PM = 5 hours = 300 minutes. Subtract off 11 minutes to get back to 2:12 pm.**
10	21	What is the sum of the three consecutive integers whose product is 336? **6x7x8 = 336**

"Math is Cool" Championships -- 2022-23

High School

College Bowl Round #4 Solutions

	Answer	Solution
1	5	Let f(x) = $\sqrt{x}$ (read: f of x equal the square root of x) and g(x) = x^2 + 9 (read: g of x equal x-squared plus 9). Find the value of f(g(4)) (read: f of g of 4). **4^2 + 9 = 25** **Square root of 25 = 5.**
2	65	Hunter ordered a pizza on January 1st. On January 1st he ate half of the pizza. For the next five days, each day he ate half of the remaining pizza. After eating his allotted pizza on January 6th, the remaining fraction of the pizza can be written as a reduced common fraction A/B (read: A over B). What is A + B? **The geometric sequence is: ½, ¼, 1/8, 1/16, 1/32, 1/64 on the 6th day.** **Since he ate 1/64th on the 6th day, which was half, that is also the amount remaining.**
3	25 [%]	When two fair six-sided dice are rolled, what is the probability in percent that the largest number rolled, not necessarily distinct, is a 5? **There are 9 outcomes where there is a 5 + (1, 2, 3, 4 or 5). 9/36 = ¼ = 25%**
4	48 [miles per hour]	Casey travels from point A to point B at a rate of 2 minutes per mile, and returns over the same route at 2 miles per minute. What was their average speed in miles per hour for the whole trip? **2 minutes/mile = 1 mile/2 minutes = 30 miles/hr** **2 miles/minute = 120 miles/hr** **Avg rate = 2AB/(A+B) = 2(30)(120)/(120+30) = 48**
5	8	If x plus y = 6 and x times y = 2, what is the value of 1 over x-squared + 1 over y-squared? $$\frac{1}{x^2}+\frac{1}{y^2}=\frac{x^2+y^2}{x^2y^2}=\frac{(x+y)^2-2xy}{x^2y^2}=\frac{6^2-2(2)}{2^2}=8$$
6	14	There are 24 arrangements of the four letters M, A, T, H. If the arrangements are put into alphabetical order and numbered from 1 to 24, in what position is the arrangement MATH, spelled M-A-T-H? **There are 6 permutations for each leading letters, so 1-6 start with A, 7-12 start with H. M starts in position 13 with MAHT, followed by MATH in position 14.**

7	234 [square units]	ABCD is a quadrilateral with right angles at B and D. The length of segment AB = 7 units, BC = 24 units, CD = 20 units and DA = 15 units. What is the area of the quadrilateral in square units? **Two right triangles are formed: ABC with legs 7 and 24, area = 84. CDA with legs 20 and 15, area = 150. Total area = 234.**
8	85 [points]	On a Statistics exam, 10% of the students got 70 points, 25% got 80 points, 20% got 85 points, 15% got 90 points, and the rest got 95 points. What is the median score for this exam in points? **Assume 100 students, so the median will be half-way between score 50 and 51. Both 50 and 51 are 85 points.**
9	41	What is the thirteenth prime number? **2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41**
10	2	What is the slope of the line 12x/3y = 2 (read: 12x over 3y equals 2)? **3y = 12x/2 = 6x** **y = 6x/3 = 2x**

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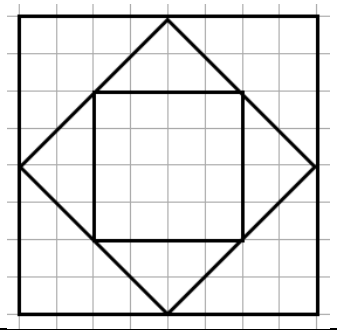
College Bowl Round #5 Solutions

	Answer	Solution
1	153	How many distinct factors does the number 12^8 (read: 12 to the 8^{th}) have? **Rewrite as $3^8 2^{16}$** **Number of factors = 9x17 = 153**
2	5	The sum of the first nine terms of an arithmetic sequence is 45. What is the fifth term of the arithmetic sequence? **Let the first term be a_1 and the common difference = d. The sum of the first 9 terms is $9a_1 + 36d = 9(a_1 + 4d) = 45$. Therefore, $a_1 + 4d = 5$, which is equal to a_5.**
3	9	A regular hexagon has side length 2 inches. The area of the hexagon in square inches can be written as A times the square root of B. What is A + B? **Area of regular hexagon = $\frac{3\sqrt{3}}{2}s^2 = \frac{3\sqrt{3}}{2}2^2 = 6\sqrt{3}$.**
4	-2	The domain of the function f(x) = $\sqrt{-x^2 - 2x + 3}$ (read: f of x equals the square root of the quantity negative x-squared minus 2x + 3) is the set of real numbers x such that A ≤ x ≤ B (read: A is less than or equal to x is less than or equal to B), where A and B are integers. What is the value of A + B? **$-x^2 - 2x + 3 \geq 0$** **$X^2 + 2x - 3 \leq 0$** **$(x + 3)(x - 1) \leq 0$** **Zeroes are at x = -3, 1** **The inequality is true from $-3 \leq x \leq 1$. -3 + 1 = -2**
5	36	Four distinct girls and 3 distinct boys randomly line up in a row. The probability that nobody is standing next to someone of the same gender can be written as a reduced common fraction A/B (read: A over B). What is A + B? **There are 7! total ways to arrange them. To have alternating boys and girls, it must be GBGBGBG. The number of ways to pick the first girl is 4, times the number of ways to pick the first boy is 3, etc., giving 4x3x3x2x2x1x1. That divided by 7! Reduces to 1/35.**
6	25	What is 10 times the log base 4 of 32? **$\log_4 32 = \log_4 2^5 = 5\log_4 2$** **Let $y = \log_4 2$** **$4^y = 2$, $y = ½$** **Therefore, $\log_4 32 = 5(1/2) = 5/2$** **10 times that = 25.**

7	10	If 6 factorial times 7 factorial equals N factorial, what is the value of N? **6! = 720 = 10x9x8** **6!7! = 10x9x8x7! = 10!**
8	3	If x + y = 8, y + z = 3, and x + z = 1, what is the value of x? **Solve the system of equations.**
9	50 [marbles]	In a bag of marbles, two-fifths of the marbles are red, three-tenths of the marbles are white, and one-tenth of the marbles are blue. The remaining 10 marbles are green. How many total marbles are in the bag? **2/5 + 3/10 + 1/10 = 4/5, which means that the remaining 10 marbles = 1/5 of the total. (1/5)x = 10, x = 50**
10	4 [square units]	Square S1 has an area of 16 square units. Each side of square S1 is bisected, and a smaller square S2 is constructed using the bisection points as vertices. Each side of square S2 is bisected, and a smaller square S3 is constructed using the bisection points as vertices. What is the area of square S3 in square units? 

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College Bowl Round #6 Solutions

	Answer	Solution
1	20	A geometric sequence has 2nd term 20 and 10th term 350. What is the product of the first term and the common ratio r? **The product of the first term and the common ratio will equal the second term, which is 20.**
2	1	What value of x will give the maximum function value for $f(x) = -6x^2 + 12x + 7$ (read: negative six x-squared plus 12 x plus 7)? **The maximum will be at the vertex, which is given by x = -b/2a = -12/-12 = 1.**
3	1080 [cm]	A regular polygon has interior angles measuring 179° each, and a side length of 3 cm. What is the perimeter of the polygon in centimeters? **Each exterior angle is therefore 1°, so there are 360 sides. 360*3 = 1080.**
4	8 [divisors]	How many divisors of 1000 are also multiples of 4? **$1000 = 2^3 5^3$, 4x4 = 16 total factors** **Subtract off the ones that do not include 2^2 or $2^3 = 2^0, 2^1, 2^0 5^1, 2^0 5^2, 2^0 5^3, 2^1 5^1, 2^1 5^2, 2^1 5^3$**
5	24	Hershey the dog has to climb a flight of six stairs to get to his favorite sleeping spot. He can take the stairs 1, 2, or 3 at a time. For example, he could climb 2, then 3, then 1. In how many different ways can Hershey climb the 6 stairs? 3+3: 1 way **3+2+1: 6 ways** **3+1+1+1: 4 ways** **2+2+2: 1 way** **2+2+1+1: 6 ways** **2+1+1+1+1: 5 ways** **1+1+1+1+1+1: 1 way**
6	11	Four integers are added to the set {3, 4, 5, 5, 8}, increasing the mean, median and mode each by 1. What is the greatest integer in the new set? **The current mean, median and mode = 5, therefore the new mean, median and mode = 6. The new sum must = 54. If three 6's are added to get a mode of 6, then the 4th number must be 11.**

7	75 [minutes]	Alyssa can mow a lawn in 50 minutes. Working together, Alyssa and Jenny can mow the same lawn in 30 minutes. How many minutes would it take Jenny to mow the lawn alone? **A = 50, J = ?** **30 = (50J)/(50+J)** **1500 + 30J = 50J** **1500 = 20J, J = 75**
8	37	Carlos rolls three 6-sided dice. The probability that all three of the dice show the same number can be written as a reduced common fraction A/B (read: A over B). What is A + B? **6/216 = 1/36**
9	108 [inches]	A 12-foot long piece of lumber is cut into two pieces with lengths in a ratio of 1:3 (read: 1 to 3). How long is the longer piece, in inches? **The 12-foot piece is cut into a 3 foot and 9 foot section. 9x12 = 108.**
10	15	80 times 10 to the 4^{th} divided by 0.4 times 10 to the -9 equals 2 times 10 to the nth power. What is n? **$80 \times 10^4 = 8 \times 10^5$** **$0.4 \times 10^{-9} = 4 \times 10^{-10}$** **$(8 \times 10^5)/(4 \times 10^{-10}) = 2 \times 10^{15}$**