“Math is Cool” Masters -- 2021-22

8th Grade

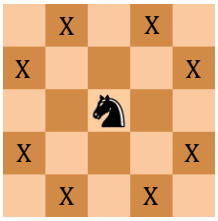
Mental Math Solutions

8th	Answer	Solution
1	21 [inches]	The area of a rectangle is sixty-three square inches. If the length of the rectangle is three inches, how many inches long is the width? 63/3 = 21
2	31 [quarters]	A stack of quarters has a value of seven dollars and seventy-five cents. How many quarters are in the stack? 775/25 = 31
3	3000	What is seventy-five times forty? 75*40 = 75*4*10 = 3000
4	[x =] 24	If you multiply X by two and then divide the result by six, you get eight. What is the value of X? 8*6/2 = 24
5	11 [days]	Dana can complete one painting every five hours and she paints for ten hours a day. How many days will it take her to complete twenty-two paintings? She completes 2 per day. 22/2 = 11
6	[A + B =] 27	As a reduced common fraction, the probability that a red seven is selected when one card is drawn from a standard deck is A over B. What is the value of A plus B? P(red 7) = 2/52 = 1/26, and 1 + 26 = 27
7	320	What is twenty-four squared minus sixteen squared? $24^2 - 16^2 = (24 + 16)(24 - 16) = 40*8 = 320$

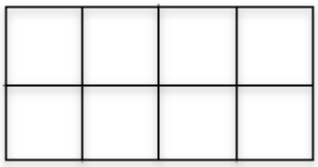
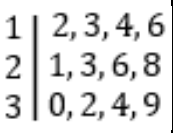
8	263	What is the largest positive three-digit integer in which the ones digit is half of the tens digit, and the tens digit is three times the hundreds digit? The only three-digit integer that matches this description is 263

"Math is Cool" Masters -- 2021-22
8th Grade
Individual Test Solutions

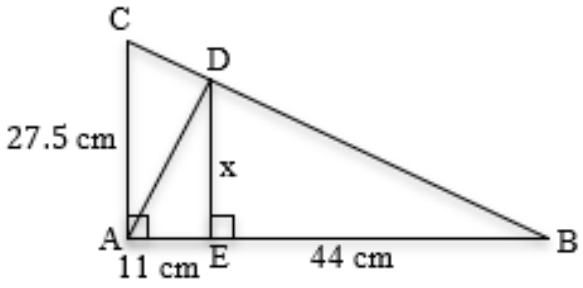
8th	Answer	Solution
1	240	Evaluate: $1 * 2 * 3 * 4 * 10$ 1*2*3*4*10 = 240
2	[x =] 23	What is the value of x in the following equation? 7x – 35 = 126 7x – 35 = 126 → 7x = 161 → x = 23
3	[A + B =] 6	The number of cups of water that a 12-oz glass can hold is A.B, where A and B are each single-digit integers. What is the value of A + B? A cup has 8 ounces, so 12/8 = 1.5, and 1 + 5 = 6.
4	47	What is the 8th number in the arithmetic sequence whose first three terms are: 5, 11, 17, . . . ? 5, 11, 17, 23, 29, 35, 41, 47
5	[A + B =] 17	As a reduced common fraction, the probability that a randomly chosen letter from the letters in the words EENSIE WEENSIE is an N or an S is A/B. What is the value of A + B? There are 13 total letters and 2 of them are Ns and 2 of them are Ss, so the probability is 4/13, and 4 + 13 = 17.
6	15 [palindromes]	A palindrome is a number that reads the same forwards and backwards, for example 1221. How many palindromes are there between 50 and 200? The list includes 55, 66, 77, 88, 99, 101, 111, 121, 131, 141, 151, 161, 171, 181, and 191 and there are 15 numbers in the list, so the answer is 15.
7	-15	Evaluate: 15 + 30 – 60 15 + 30 – 60 = -15

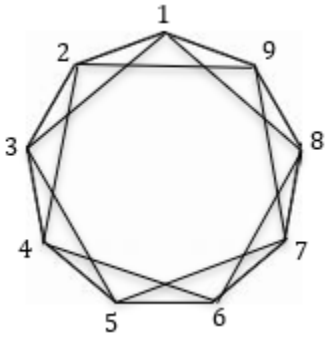
8	18	What is the median of the set of data given below: 13, 18, 12, 8, 23, 100, 19 The list in order is 8, 12, 13, 18, 19, 23, 100, so18 is the median
9	25 [%]	In a school with 120 students there are thirty 6^{th} graders. What percentage of the students in the school are 6^{th} graders? 30/120 = 25/100 = 25%
10	9 [people]	Boris is waiting in line. There are three people in front of him and five people behind him. How many total people are in the line? 1 + 3 + 5 = 9
11	[x =] 20	If x > 19.875, what is the smallest possible integer value of x? The smallest integer larger than 19.875 is 20
12	[A + B =] 28	The product of $\frac{2}{5} * \frac{5}{16} * \frac{8}{27}$ as a reduced common fraction is A/B. What is the value of A + B? 2/5*5/16*8/27 = 1/27, and 1 + 27 = 28.
13	8 [values]	How many integer values of N are there such that the expression $\frac{104}{N}$ represents a positive integer? The factors of 104 are 1, 2, 4, 8, 13, 26, 52, and 104 and there are 8 of them so the answer is 8.
14	8 [squares]	In the game of chess, a knight can move either one space right or left in combination with two spaces up or down, or two spaces right or left in combination with one space up or down. On the board shown, how many squares are possible landing spots for the knight? The 8 marked squares are the possible landing spots for the knight. 

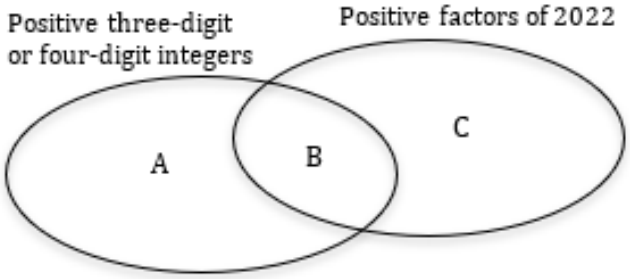
15	81 [in^2]	Two rectangles have a perimeter of 40 inches, but different areas. In square inches, what is the largest possible difference in the areas of the two rectangles, if side lengths must be integers? The largest possible area would be 100 in^2, if it were a 10x10 square. The smallest possible area would be 19 in^2, if it were a 1x19 square. 100 – 19 = 81.
16	17	Evaluate: $(11 * 23 - 15) \div 14$ 11*23 – 15 = 238 and 238/14 = 17
17	520 [pages]	Alonzo has read 35% of his book and he's on page 182. How many pages long is the book? 35% = 182, divide both by 7 and 5% = 26, multiply both by 20 and 100% = 520.
18	7 [minutes]	It takes Anna 35 minutes to walk home from school. Her sister Jane rides her bike at an average rate that is 5 times Anna's average walking rate. In minutes, how long does it take Jane to ride home from school along the same route that Anna walks? A rate that is 5 times as fast means the time needed to travel the same route is 1/5 as much, so 1/5 of 35 is 7.
19	52	Rows 0 through 4 of Pascal's Triangle are shown here. What is the sum of all the numbers that are not 1s in rows 0 through 5? 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 The sum of the numbers in row n is 2^n and there is one 1 in row 0, and there are two 1s in each of the other rows, so $2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 - 1 - 5(2) = 1 + 2 + 4 + 8 + 16 + 32 - 11 = 52$.
20	[A + B =] 47	The sum of the four fractions $\frac{7}{6}, \frac{7}{5}, \frac{7}{3}$ and $\frac{7}{2}$, as a reduced common fraction is A/B. What is the value of A + B? 7/6 + 7/5 + 7/3 + 7/2 = 35/30 + 42/30 + 70/30 + 105/30 = 252/30 = 84/10 = 42/5, and 42 + 5 = 47
21	1375 [km]	How many kilometers are equivalent to 1.375 x 109 millimeters? Give your answer as an integer. There are 1000 millimeters in a meter and 1000 meters in a kilometer, so there are 1,000,000 or 10^6 millimeters in a kilometer. $(1.375 \times 10^9)/(1 \times 10^6) = 1.375 \times 10^3 = 1375$

22	[x =] -25	If a ¥ b = a – 2b, and (x ¥ 3) ¥ x = 19, what is the value of x? (x ¥ 3) ¥ x = 19 → (x – 6) ¥ x = 19 → x – 6 – 2x = 19 → -x – 6 = 19 → -x = 25 → x = -25
23	19 [rectangles]	The figure shown here consists of 8 congruent squares. How many non-square rectangles of any size are there in the figure?  1x2 horizontal – 6 1x2 vertical – 4 1x3 – 4 1x4 – 2 2x3 – 2 2x4 - 1 1 + 2 + 2 + 4 + 4 + 6 = 19
24	1	Ayanna is looking at Raul and Raul is looking at Clarice. Ayanna owns a dog and Clarice does not, and we don't know whether Raul owns a dog. Is a dog-owner looking at someone who does not own a dog? Answer 1 for yes, 2 for no, and 3 for "cannot determine". If Raul owns a dog, he is looking at Clarice, who does not. If Raul does not own a dog, Ayanna the dog-owner is looking at him, so either way, the answer is yes.
25	1365	What is the sum of the first 6 numbers in the geometric series that begins with: 1, 4, 16, . . . ? 1 + 4 + 16 + 64 + 256 + 1024 = 1365
26	24	What is the mean of the numbers represented in the stem and leaf plot shown here? Note: the numbers represented in row 1 are 12, 13, 14, and 16.  12 + 13 + 14 + 16 + 21 + 23 + 26 + 28 + 30 + 32 + 34 + 39 = 40 + 80 + 120 + 48 = 288 and 288/12 = 24.
27	56 [ways]	Eight fair coins are flipped at the same time. What is the number of ways that heads could be showing on three of the coins and tails showing on the rest? 8!/(3!*5!) = 56

28	[A =] 1331	The base-10 number 512 equals the base-7 number A_7, where A has four digits. What is the value of A? $512 = 1*7^3 + 3*7^2 + 3*7^1 + 1*7^0 = 343 + 147 + 21 + 1$, so the answer is 1331.
29	[A =] 3960	The wheels on a bicycle have a diameter of 16 inches and on one occasion it is ridden for one mile, or 5280 feet. In terms of π and as a reduced fraction, the number of complete revolutions that each wheel makes during the ride is $\frac{A}{\pi}$. What is the value of A? One revolution is 16π inches or 4π/3 ft, so in 5280 feet the wheels each complete 5280/(4π/3) = 15840/4π = 7920/2π = 3960/π, so A = 3960.
30	[P + Q =] 10	Consider the set {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}. Let A be a number selected at random from the set and let B be a different number selected at random from the set. As a reduced common fraction, the probability that A + B is a prime number is P/Q. What is the value of P + Q? Any odd number plus another odd number will result in an even number, and therefore will not be prime. The only way the sum can be prime is if 2 is one of the numbers added together. The ordered pairs of numbers that include a 2 are (2, 3), (2, 5), (2, 7), (2, 11), (2, 13), (2, 17), (2, 19), (2, 23), (2, 29), (3, 2), (5, 2), (7, 2), (11, 2), (13, 2), (17, 2), (19, 2), (23, 2), and (29, 2). The ten ordered pairs (2, 3), (2, 5), (2, 11), (2, 17), (2, 29), (3, 2), (5, 2), (11, 2), (17, 2), and (29, 2) represent sums that are prime numbers, so the numerator of the probability will be 10. The denominator is the total possible number of ordered pairs, which is 10*9 = 90, so P/Q = 10/90 = 1/9, and 1 + 9 = 10.

31	[x =] 22 [cm]	In the figure, ΔABC ~ ΔEDA. The '~' symbol means 'similar to', and this means that the vertices are named in corresponding order. In other words, vertex A from ΔABC corresponds with vertex E from ΔEDA, and so on. If AE = 11 cm, BE = 44 cm, and AC = 27.5 cm, then DE = x cm. What is the value of x in centimeters?  $\overline{AE}$ corresponds with $\overline{AC}$ and $\overline{AE}$ corresponds with $\overline{AE}$ so the proportion is $\frac{x}{55} = \frac{11}{27.5} \rightarrow 27.5x = 605 \rightarrow x = 605/27.5 = 1210/55 = 242/11 = 22$. Or ΔABC ~ ΔEDA ~ ΔEBD ~ ΔDBA so the geometric mean equation would be $\frac{x}{44} = \frac{11}{x} \rightarrow x^2 = 484 \rightarrow x = 22$.
32	20 [series]	Consider an infinite series of digits represented by the expression …abcabcabc…, where a, b, and c, each represent distinct single-digit positive integers. How many such series are possible if a, b, and c must all be odd? Note: the infinite series …123123123…, …231231231…, and …312312312… would all be subsections of the same series and therefore would all count as one series. Any set of three distinct single-digit integers can be arranged in six ways, but these six ways could be divided into two groups of three, in which the 3 arrangements in one particular group would all be subsets of the same series, as noted at the end of the problem. So, the task is to count the number of distinct sets of three odd single-digit integers and then multiply by 2. Since there are 5 single-digit odd integers, the number of distinct sets of three of them is ${}_5C_3 = 10$. The brute force list is: 135, 137, 139, 157, 159, 179, 357, 359, 379, 579. Therefore, the answer is 2 * 10 = 20.
33	[A + B =] 94	The complex fraction shown here expressed as a reduced common fraction is A/B. What is the value of A + B? $3 - \cfrac{2}{3 - \cfrac{2}{3 - \cfrac{2}{3 - \cfrac{2}{3}}}}$ 3 – 2/(3 – 2/(3 – 2/(3 – 2/3))) = 3 – 2/(3 – 2/(3 – 2/(7/3))) = 3 – 2/(3 – 2/(3 – 6/7)) = 3 – 2/(3 – 2/(15/7)) = 3 – 2/(3 – 14/15) = 3 – 2/(31/15) = 3 – 30/31 = 63/31, and 63 + 31 = 94.

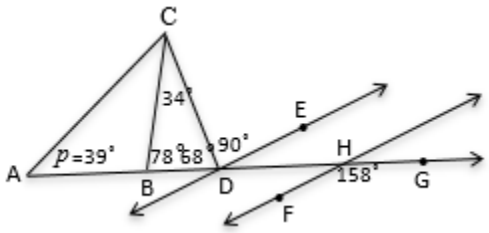
34	[A + B =] 8126	A standard deck of cards is divided into two piles, one with 26 red cards and one with 26 black cards. If two cards are randomly drawn from each pile, the probability as a reduced common fraction that the cards form four of a kind, for example, four 7s, four kings, four aces, etc., is A/B. What is the value of A + B? P(two of a kind from red pile) = 26/26*1/25 = 1/25 P(two of a kind from black pile that match the two from the red pile) = 2/26*1/25 = 1/325 P(four of a kind) = 1/25*1/325 = 1/8125, so 1 + 8125 = 8126.
35	18 [triangles]	A regular nonagon has 9 congruent sides and 9 congruent interior angles as shown. Bart draws a star by drawing a line segment from vertex 1 to vertex 3, then another line segment from vertex 3 to vertex 5, and so on, skipping one vertex each time until ending up back at vertex 1. This process creates two sets of congruent non-overlapping triangles. What is the total number of triangles combined in these two sets of non-overlapping triangles?  There are 9 triangles in each set for a total of 18.
36	-5500	What is the coefficient of the x^3y term when the expression $(5x - 11y)^4$ is written in expanded form? $(5x - 11y)^4 = 1(5x)^4(-11y)^0 + 4(5x)^3(-11y)^1 + 6(5x)^2(-11y)^2 + 4(5x)^1(-11y)^3 + 1(5x)^0(-11y)^4 = 625x^4 - 5500x^3y + 18150x^2y^2 - 26620xy^3 + 14641y^4$, so the coefficient of the x^3 term is -5500.

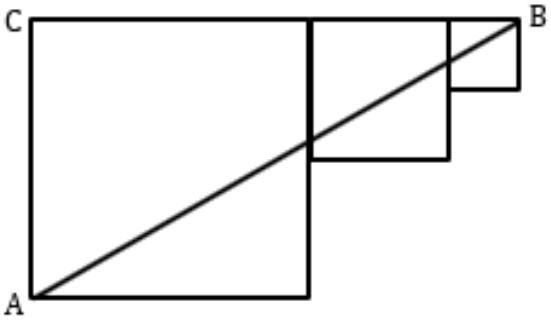
37	[A + B = C =] 9904	In the diagram below, let B represent the number of numbers that are both positive factors of 2022 and three-digit or four-digit integers. Let A represent the number of positive three-digit or four-digit integers that are not also factors of 2022 and let C represent the number of positive factors of 2022 that are not also three-digit or four-digit integers. What is the value of A + B + C?  2022 = 2*1011 = 2*3*337, so there are 8 factors: 1, 2, 3, 6, 337, 674, 1011, 2022. There are 9900 positive three- and four-digit integers, and four of them are factors of 2022, so section A has 9896 numbers, section B has 4 numbers, and section C has 4 numbers. The answer is 9896 + 4 + 4 = 9904.
38	[D + N =] 33	A box contains only dimes and nickels. If there were 25% more nickels, there would be 8% more value to the money in the box. As a reduced common fraction, the ratio of the original number of dimes to the original number of nickels in the box is D/N. What is the value of D + N? Let d = original number of dimes, n = original number of nickels, and v = value of the coins in the box in cents, then: 10d + 5n = v After increasing the number of nickels by 25% the equation becomes 10d + 6.25n = 1.08v Subtracting the two equations results in 1.25n = 0.08v → 5n = 0.32v, which means the original number of nickels in the box is 32% of the value, so the original number of dimes in the box is 68% of the value. If there were a total of \$10 or 1000 cents in the box, \$6.80 would be dimes and \$3.20 would be nickels, so 68 dimes and 64 nickels. Therefore D/N = 68/64 = 17/16, and 17 + 16 = 33.

<table>
<tr><td>39</td><td>9 [values]</td><td>A data set consists of only positive integers and has a mean of 40, a median of 65 and a unique mode of 75. What is the fewest possible number of values in the data set?

The fewest number of data values will happen when 65 is the middle number, so that there are an odd number of values. To minimize the top half of the set, the number of 75s must be as low as possible, so there must be two 75s and since 75 is a unique mode, there can be at most one of every other number.
1 + 2 + 65 + 75 + 75 = 218, which is more than 5(40), so a set with 5 numbers is not possible.
1 + 2 + 3 + 65 + 75 + 75 + 66 = 287, which is more than 7(40), so a set with 7 numbers is not possible.
1 + 2 + 3 + 4 + 65 + 75 + 75 + 66 + 67 = 358, which is less than 9(40), so a set with 9 number is possible and the answer is 9.</td></tr>
<tr><td>40</td><td>6 [marbles]</td><td>A jar contains 11 marbles that are either red, green, or blue. If two marbles are randomly selected from the jar without replacement, the probability that they will be the same color is $\frac{21}{55}$. What is the largest number of marbles of any one color in the jar?

Let (a, b, c) represent the number of marbles distributed by color, with each variable representing the number of one of the three colors. It doesn't matter what color each variable represents, since P(9,1,1) = P(1,9,1) = P(1,1,9). Here is a table of probabilities.
<table>
<tr><td>(9,1,1)</td><td>9/11*8/10 = 36/55</td></tr>
<tr><td>(8,2,1)</td><td>8/11*7/10 + 2/11*1/10 = 29/55</td></tr>
<tr><td>(7,3,1)</td><td>7/11*6/10 + 3/11*2/10 = 24/55</td></tr>
<tr><td>(7,2,2)</td><td>7/11*6/10 + 2/11*1/10 + 2/11*1/10 = 23/55</td></tr>
<tr><td>(6,4,1)</td><td>6/11*5/10 + 4/11*3/10 = 21/55</td></tr>
<tr><td>(6,3,2)</td><td>6/11*5/10 + 3/11*2/10 + 2/11*1/10 = 19/55</td></tr>
<tr><td>(5,4,2)</td><td>5/11*4/10 + 4/11*3/10 + 2/11*1/10 = 17/55</td></tr>
<tr><td>(5,3,3)</td><td>5/11*4/10 + 3/11*2/10 + 3/11*2/10 = 16/55</td></tr>
<tr><td>(4,4,3)</td><td>4/11*3/10 + 4/11*3/10 + 3/11*2/10 = 3/11</td></tr>
</table>
When the probability is 39/110, the largest number of marbles of one color is 7.</td></tr>
<tr><td colspan="3">IF taking Algebra or Geometry, continue to questions 41 – 42.</td></tr>
</table>

41	[A + B =] 14	The sum of the solutions to the equation (2x – 5)(x – 7) = 26 is A.B, where A and B are each single-digit integers. What is the value of A + B? $(2x - 5)(x - 7) = 26$ $2x^2 - 19x + 35 = 26$ $2x^2 - 19x + 9 = 0$ $(2x - 1)(x - 9) = 0$ x = ½ and x = 9, 1/2 + 9 = 9.5, so the answer is 9 + 5 = 14.
42	[A + B =] 8	A kayaker can row 4 miles upstream against a 1.5-mph current in one hour more than she can row 5 miles downstream. Her average speed in still water is A.B miles per hour, where A and B are each single-digit integers. What is the value of A + B? Let r be her average speed in still water and t be the time it takes her to row 5 miles downstream, then solve the system of equations $5 = (r + 1.5)t$ and $4 = (r - 1.5)(t + 1) \rightarrow r + 1.5 = 5/t$ and $r - 1.5 = 4/(t + 1) \rightarrow r = 5/t - 1.5$ and $r = 4/(t + 1) + 1.5 \rightarrow 5/t - 1.5 = 4/(t + 1) + 1.5 \rightarrow 5/t = 4/(t + 1) + 3 \rightarrow 5(t + 1) = 4t + 3t(t + 1) \rightarrow 5t + 5 = 4t + 3t^2 + 3t \rightarrow 3t^2 + 2t - 5 = 0 \rightarrow (3t + 5)(t - 1) = 0 \rightarrow$ t = 1 hour $\rightarrow 5 = (r + 1.5)*1 \rightarrow$ r = 3.5 mph, and 3 + 5 = 8.
IF taking Geometry, continue to questions 43 – 45.		
43	39 [°]	In the figure below AB = BC, and $\overleftrightarrow{DE}$ is parallel to $\overleftrightarrow{FG}$. Also, $\overleftrightarrow{DE}$, $\overline{CD}$, and $\overline{AB}$ all intersect at point D. Finally, m∠BCD = 34°, m∠CDE = 90°, and m∠FGH = 158°. What is the number of degrees in the measure of the angle labeled p? Since $\overleftrightarrow{DE} \parallel \overleftrightarrow{FG}$, 158° = m∠BDC + 90°, because they are alternate exterior angles, so m∠BDC = 68°. Then m∠DBC = 180° – 34° – 68° = 78°, which is an exterior angle of ΔABC. The measure of an exterior angle is equal to the sum of the measures of the two remote interior angles, which in this case are congruent, since ABC is isosceles. So, the angle labeled p would be half of 78°, or 39°. 

44	[A =] 12	A construction worker drills eight holes through a concrete wall for drainpipes. The wall is 8 inches thick and each of the holes has a diameter of 3 inches. As a reduced fraction in and in terms of π, the number of cubic feet of combined space in the eight holes is $\frac{\pi}{A}$. What is the value of A? $(3/2*1/12)^2\pi*2/3*8 = (1/8)^2\pi*16/3 = \pi/64*16/3 = \pi/12$, so the answer is 12.
45	[P + Q =] 68	In the figure shown below, there are three squares placed so that their top edges are aligned in a straight line. The medium-sized square has half the perimeter of the large square and the small square has half the perimeter of the medium-sized square. The area of the three squares combined is 189 cm^2. In simplest radical form, the length of $\overline{AB}$ is $P\sqrt{Q}$ cm. What is the value of P + Q? Let x be the side length of the small square, then the area of the three squares is x^2, $(2x)^2$, and $(4x)^2$. Then $x^2 + (2x)^2 + (4x)^2 = x^2 + 4x^2 + 16x^2 = 21x^2 = 189 \rightarrow x^2 = 9 \rightarrow x = 3$, so BC = 3 + 6 + 12 = 21 and AC = 12. Then $AB^2 = 12^2 + 21^2 \rightarrow AB^2 = 144 + 441 = 585 \rightarrow AB = \sqrt{585} = 3\sqrt{65}$, and 3 + 65 = 68. 

"Math is Cool" Masters -- 2021-22

8th Grade

Multiple Choice Solutions

8th	Answer	Solution
REFER TO THE FOLLOWING INFORMATION FOR PROBLEMS #1 THROUGH #4. The pattern of numbers in the following table continues infinitely. All rows have 9 numbers in them, and all columns continue infinitely. For example, row 1 has numbers 1 – 9 in it and column 1 has numbers 1, 10, 19, 28, 37, 46, 55, 64, . . . in it. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69		
1	*C*	What is the median of the numbers in the first three rows? A) 9.5 B) 13.5 C) 14 D) 14.5 E) 18.5 Since there are an odd number of numbers in each row, the middle number of the 2nd row will be the median of the first three rows, which is 14.
2	*A*	What is the sum of the numbers in the fourth row? A) 288 B) 298 C) 320 D) 352 E) 576 28 + 29 + … + 35 + 36 = 9(64)/2 = 288
3	*B*	How many rows are made up entirely of two-digit integers? A) 9 B) 10 C) 11 D) 12 E) 13 Row 2 through row 11 have all 2-digit integers in them, so this is 10 rows.

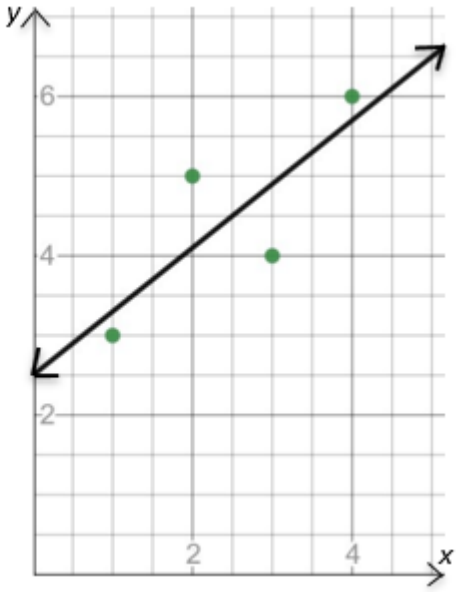
4	*C*	Starting from and including row 1, how many consecutive rows of numbers would need to be added together such that the sum of all the numbers in those rows is more than 45,000? A) 26 B) 33 C) 34 D) 39 E) 43 Let n be the number of numbers that must be added together such that the sum is equal than 45000. Then n(n + 1)/2 = 45000 and n(n + 1) = 90000. Since $\sqrt{90000}$ = 300, this means n and n + 1 would be either 299*300 = 89700 or 300*301 = 90300, which is greater than 90000, so the row that has 301 in it would be the last row in the target group of rows. Each row ends with a multiple of 9 and the closest multiple of 9 to 301 is 297 = 9(33), which is less than 301, so 301 is in the 34rd row and the answer is 34.

REFER TO THE FOLLOWING INFORMATION FOR PROBLEMS #5 THROUGH #7.

The four points shown in the graph have their coordinates recorded in the table shown here. The line on the graph is the best-fit line and its equation is:
$\hat{y} = \frac{4}{5}x + \frac{5}{2}$.

The symbol $\hat{y}$ is called "y-hat" and is notation used in the equation of the best-fit line, also known as a linear regression equation.

x	*y*
1	3
2	5
3	4
4	6



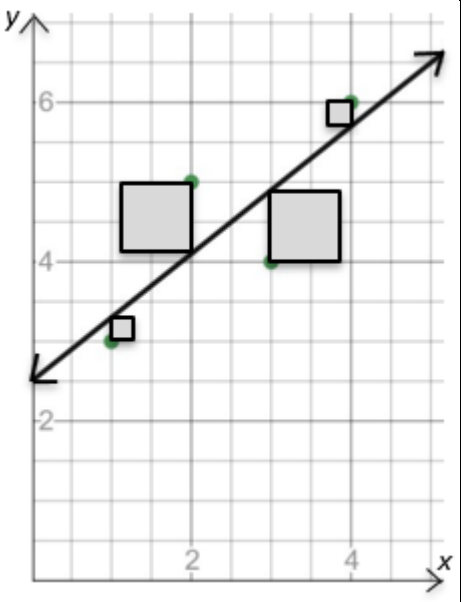
5	*B*	What are the coordinates of the two points that are in the half-plane below the best-fit line? A) (1, 3) & (2, 5) B) (1, 3) & (3, 4) C) (1, 3) & (4, 6) D) (2, 5) & (3, 4) E) (2, 5) & (4, 6) (1, 3) & (3, 4) are below the line.

<table>
<tr>
<td>6</td>
<td>E</td>
<td>According to the equation for the best-fit line, if x = 1 then $\hat{y}$ = 3.3, if x = 2 then $\hat{y}$ = 4.1, and so on. What is the value of $\hat{y}$ if x = 17?

A) 27/2 B) 14.5 C) 15.1 D) 15.3 E) 16.1

Using the slope of 4/5, the y-values increase by 0.8 each time x increases by 1, so when x = 17, $\hat{y}$ = 4.1 + 0.8(15) = 4.1 + 12 = 16.1.
-or-
Plug in 17 for x in the best-fit line's equation and solve for $\hat{y}$.</td>
</tr>
<tr>
<td>7</td>
<td>C</td>
<td>On the graph shown here, four squares have been added, such that each square has one vertex at one of the four points and a second vertex directly below or above the point and on the best-fit line. What is the combined area of the four squares?

A) 1.6 units^2 B) 1.625 units^2
C) 1.8 units^2 D) 2.1 units^2
E) 7.4 units^2

In the table, $|y - \hat{y}|$ is the side length of each square, so the sum of the areas of the four squares will be $0.3^2 + 0.9^2 + 0.9^2 + 0.3^2$ = 0.09 + 0.81 + 0.81 + 0.09 = 1.8.

<table>
<tr><th>x</th><th>y</th><th>$\hat{y}$</th><th>y - $\hat{y}$</th></tr>
<tr><td>1</td><td>3</td><td>3.3</td><td>-0.3</td></tr>
<tr><td>2</td><td>5</td><td>4.1</td><td>0.9</td></tr>
<tr><td>3</td><td>4</td><td>4.9</td><td>-0.9</td></tr>
<tr><td>4</td><td>6</td><td>5.7</td><td>0.3</td></tr>
</table>
</td>
</tr>
</table>

USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #8 THROUGH #10.

If A Ω B = the sum of the common factors of A and B, then 27 Ω 36 = 13, because the factors of 27 are 1, 3, 9, 27, the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36, and the sum of the common factors is 1 + 3 + 9 = 13.

If A Ɔ B = the least common multiple of A and B, then 27 Ɔ 36 = 108, because 108 is the smallest number that is evenly divisible by both 27 and 36.

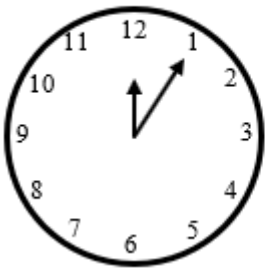
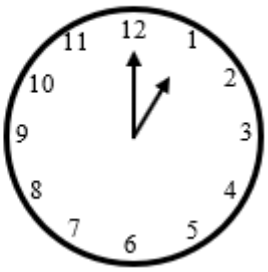
8	D	What is the value of 20 Ω 30? A) 12 B) 13 C) 17 D) 18 E) 21 Factors of 20: 1, 2, 4, 5, 10, 20 Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30 The sum of the common factors is 1 + 2 + 5 + 10 = 18
9	B	What is the value of (40 Ω 50) Ɔ 60? A) 120 B) 180 C) 240 D) 300 E) 360 Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40 Factors of 50: 1, 2, 5, 10, 25, 50 The sum of the common factors is 1 + 2 + 5 + 10 = 18. The LCM of 18 and 60 is 180.
10	B	If (x Ω y) Ɔ z = 77, and x, y, and z are distinct positive integers, then what is the smallest possible value of x + y + z? A) 22 B) 23 C) 25 D) 28 E) 31 Either x Ω y = 7 and z = 11, or x Ω y = 11 and z = 7. If x Ω y = 7, since the factors of 4 add up to 7, one possibility is 4 Ω 8 = 7. In this case z would be 11, so x + y + z = 4 + 8 + 11 = 23. The other possibility is that x Ω y = 11 and z = 7. For the sum of x + y + z to be less than 23, x + y needs to be less than 16. Numbers less than 16 whose factors add up to at least 11 include: 6, 8, 9, 10, 11, 12, 13, 14, and 15. Only two pairs of these numbers add up to less than 16 and they are (6, 8) and (6, 9). Checking these two results in neither satisfying x Ω y = 11. 6 Ω 8 = 3 6 Ω 9 = 4 So, 23 is the smallest sum of x, y, and z.

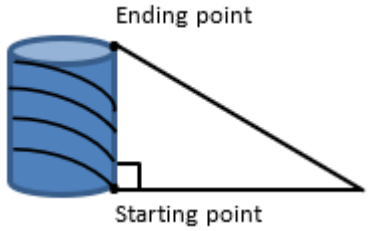
"Math is Cool" Masters -- 2021-22

8th Grade

Team Test Solutions

8th	Answer	Solution
1	736 [grizzly bears]	Twenty-three grizzly bears weigh the same as one African elephant and 32 African elephants weigh the same as one blue whale. How many grizzly bears weigh the same as one blue whale? 23 * 32 = 736
2	5	Evaluate: 10000 + 1000 + 100 + 10 + 1 – 9999 – 999 – 99 – 9 Regroup as follows: 10000 – 9999 + 1000 – 999 + 100 – 99 + 10 – 9 + 1 = 1 + 1 + 1 + 1 + 1 = 5
3	71 [handfuls]	A carton of cheesy crackers holds about 1633 crackers. How many handfuls would it take Tiny Tim to empty the carton, assuming he gets an average of 23 crackers per handful? 23*40 = 920 and 23*30 = 690, so 23*70 = 1610 and 23 * 71 = 1633, so the answer is 71.
4	11 [cm]	A square has an area less than 140 cm^2. In centimeters, what is the largest possible integer length of one of the sides of the square? $11^2 = 121 < 140$, and $12^2 = 144 > 140$, so the answer is 11
5	[x =] 17	What is the value of x in the following equation? -50 + 3x = 35 – 2x $-50 + 3x = 35 - 2x \rightarrow 5x = 85 \rightarrow x = 17$
6	[A + B =] 5	Consider the infinite string of digits . . . 02220222022202220 . . . The probability as a reduced common fraction that a randomly selected set of four adjacent digits is the number 2022 is A/B. What is the value of A + B? There are four distinct sets of four adjacent digits that can be selected, 0222, 2220, 2202, and 2022, and one of them is 2022, so the probability is 1/4, and 1 + 4 = 5.

7	10 [times]	On a normal clock the hour (short) hand rotates 30° in one hour, while the minute (long) hand rotates 360° in one hour. On my clock, the hands are reversed, so the short hand is the minute hand, which takes one hour to rotate 360°, and the long hand is the hour hand, which takes one hour to rotate 30°. Between one PM and eleven PM on a single day, how many times does my clock show the correct time? My clock at 1:00  Normal clock at 1:00  When the hour and minute hands are not swapped, during each hour, the minute hand will pass the hour hand exactly once, and at each of these instances the two hands will be in the exact same place. If the hands are swapped, the hands will still meet once per hour, and at these instances the clock will be correct. From 1 PM to 11 PM is a total of 10 hours, so the answer is 10.
8	42 [minutes]	Zeke can wash a boat in 91 minutes. Yael can wash the same boat in 78 minutes. How many minutes would it take Zeke and Yael to wash the boat if they worked together? Zeke can wash 1/91 of the boat in one minute and Yael can wash 1/78 of the boat in one minute and working together they can wash 1/x of the boat in one minute, so $1/91 + 1/78 = 1/x \rightarrow 6x + 7x = 546$ from multiplying both sides of the equation by 546x, so $13x = 546$ and $x = 42$. Working together they can wash 1/42 of the boat in one minute, which means it will take them a total of 42 minutes.

9	[A =] 50	A cylinder has a radius of 6 millimeters and a height of 14π millimeters. A continuous spiral begins at a point on the edge of the bottom base of the cylinder and ends at a point on the edge of the top base, directly above the point where it begins. The spiral is evenly spaced as it wraps around the cylinder exactly 4 complete times. The overall length of the spiral is Aπ millimeters. What is the value of A? Imagine a string wrapping around the cylinder directly on top of the entirety of the spiral. If you unwrap the string, pull it tight, it would be the hypotenuse of a right triangle as shown below. One leg would be 14π and the other leg would be 4 times the circumference of the cylinder, or $4*12\pi = 48\pi$. Using the Pythagorean Theorem, $(14\pi)^2 + (48\pi)^2 = c^2$, the hypotenuse would be 50π, so A = 50. 
10	6	The expression $(2^{21} + 2^{20})(3^{21} - 3^{20})$, is equal to a 17-digit integer. What is the units digit of this integer? Using the distributive property, $(2^{21} + 2^{20})(3^{21} - 3^{20}) = 2^{21}*3^{21} - 2^{21}*3^{20} + 2^{20}*3^{21} - 2^{20}*3^{20} = 6^{21} - 2(2^{20}*3^{20}) + 3(2^{20}*3^{20}) - 1(2^{20}*3^{20}) = 6^{21}$, and all powers of 6 have a 6 as a units digit.

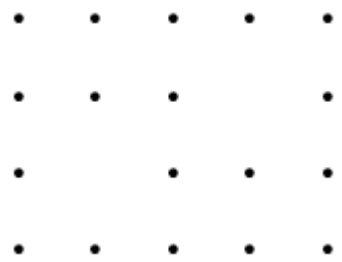
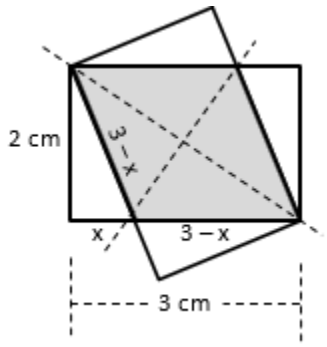
"Math is Cool" Masters -- 2021-22

8th Grade

Triple Jump Solutions

8th	Answer	Solution
1	[A + B =] 23	A bag has 9 red, 12 orange, 11 yellow, 13 green, and 7 purple candies in it. As a reduced common fraction, the probability that a randomly selected candy is not orange is A/B. What is the value of A + B? There are 9 + 12 + 11 + 13 + 7 = 52 candies in total and all but 12 are not orange, so the probability is $40/52 = 10/13$, and 10 + 13 = 23.
2	[X =] 1156	The area of a circle with a radius of 34 cm is Xπ cm^2. What is the value of X? Since the radius is 34, the Area = $34^2\pi = 1156\pi$, so X =1156.
3	4 [mph]	It takes Mably 12 minutes to walk home from school. Her school is 4/5 of a mile from her house. What is her average rate in miles per hour during her walk from school to home? $60/12 = 5$ and $5 * 4/5 = 4$
4	[x =] 5	What is the positive solution for x in the following equation? $x^2 = \sqrt{625}$ $x^2 = \sqrt{625} \rightarrow x^2 = \pm 25 \rightarrow x = \pm 5$, and the positive value of x is 5.
5	11 [days]	It takes 3 crews 20 days to paint 12 trucks. How many days would it take 5 crews to paint 11 trucks? 3 crews: 20 days: 12 trucks 1 crew:20 days:4 trucks 5 crews:4 days:4 trucks 5 crews:1 day:1 truck 5 crews:11 days:11 trucks

6	157	In the image of the month of June below, there are 35 individual rectangles making up the grid and five of the rectangles are empty. There are 10 distinct 3-by-3 arrays of 9 rectangles in this calendar month in which every rectangle contains a number. See the example below in which one of 3-by-3 arrays is outlined by a dashed rectangle. What is the sum of the numbers in the center rectangle in all of the 3-by-3 arrays?  **June 2022** Sun Mon Tue Wed Thu Fri Sat 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 The 3-by-3 arrays include the ones with the following numbers in the center rectangle: 9, 10, 13, 14, 15, 16, 17, 20, 21, 22. 9 + 10 + 13 + 14 + 15 + 16 + 17 + 20 + 21 + 22 = 157
7	[A + B =] 379	The average number of distinct positive integer factors for the numbers from 25 to 32 inclusive is A.B, where A is a single-digit integer and B is a three-digit integer. What is the value of A + B? 25 – 5^2 – 3 factors 26 – 2 * 13 – 4 factors 27 – 3^3 – 4 factors 28 – 2^2 * 7 – 6 factors 29 – prime – 2 factors 30 – 2 * 3 * 5 – 8 factors 31 – prime – 2 factors 32 – 2^5 – 6 factors 3 + 4 + 4 + 6 + 2 + 8 + 2 + 6 = 35, 35/8 = 4.375, and 4 + 375 = 379.
8	330	In base-10, what is the sum of the 2-digit base-6 numbers that are greater than the largest 2-digit base-5 number? Do not include the base 10 in your answer. The largest 2-digit base-5 number is $44_5 = 24_{10} = 40_6$, so the rest of the 2-digit base-6 numbers are 41, 42, 43, 44, 45, 50, 51, 52, 53, 54, and 55, which in base-10 are 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, and 35. 25 + 26 + 27 + 28 + 29 + 30 + 31 + 32 + 33 + 34 + 35 = 330.

9	[A + B =] 1537	In the set of dots shown, the 4 horizontal and 5 vertical lines that could be drawn to contain the dots are equally spaced. If four different dots are selected at random, the probability that they are the vertices of a square as a reduced common fraction is A/B. What is the value of A + B?  There are $18C_4 = (18*17*16*15)/(4 * 3* 2* 1) = 3*17*4*15 = 51*60 = 3060$ ways to select four dots, so that will be the denominator of the probability fraction. Assuming each horizontal and vertical space between adjacent dots is 1 unit, you can make the following number of squares with four dots as the vertices: $1x1 - 4$, $\sqrt{2}x\sqrt{2} - 2$, $2x2 - 4$, $\sqrt{5}x\sqrt{5} - 2$, $3x3 - 2$, and $4 + 2 + 4 + 2 + 2 = 14$, so 14 is the top number. $14/3060 = 7/1530$, and $7 + 1530 = 1537$.
10	[A + B =] 16	Two 2-cm-by-3-cm rectangles overlap as shown. As a reduced common fraction, the area of the shaded region is A/B cm^2. What is the value of A + B?  One line of reflection can be drawn through the two pairs of coinciding vertices of the rectangles and another through the other two vertices of the shaded region, so the four right triangles are congruent. Let the short leg of each triangle be x, then the hypotenuse would be 3 – x and the long leg is 2. By the Pythagorean Theorem, $2^2 + x^2 = (3 - x)^2 \rightarrow 4 + x^2 = 9 - 6x + x^2 \rightarrow 4 = 9 - 6x \rightarrow -5 = -6x \rightarrow x = 5/6$ and $3 - x = 3 - 5/6 = 13/6$. The area of the shaded region is $13/6*2 = 26/6 = 13/3$, and $13 + 3 = 16$.

"Math is Cool" Masters -- 2021-22

8th Grade

College Bowl Round #1 Solutions

8th	Answer	Solution
1	26	If the sum of two numbers is sixty, and their difference is eight, what is the smaller of the two numbers? a + b = 60 a – b = 8 2a = 68 a = 34 60 – 34 = 26
2	4 [castles]	Six wizards can conjure twenty-four castles in eighteen minutes. How many castles can three wizards conjure in six minutes? 6 wizards:24 castles:18 min = 3 wizards:12 castles:18 min = 3 wizards:4 castles:6 min
3	48	Evaluate six factorial divided by fifteen. 6*5*4*3*2*1 = 720/15 = 48
4	[A + B =] 11	As a reduced common fraction, the probability of getting exactly two heads when flipping a coin four times is A over B. What is the value of A plus B? The probability is ${}_4C_2/2^4 = 6/16 = 3/8$, and 3 + 8 = 11.
5	[A =] 8	Four ounces of salad dressing consist of forty percent vinegar and sixty percent oil. To make the dressing fifty percent vinegar, zero-point A ounces of vinegar need to be added, where A is a single-digit integer. What is the value of A? There are 0.8 oz of vinegar to start with, because 4*0.4 = 1.6, and 2.4 oz of oil. So, 2.4 – 1.6 = 0.8 oz of vinegar need to be added, and A = 8.

6	16 [years]	Audrey is three-quarters of Sean's age. If Audrey was half of Sean's age eight years ago, how old is Sean now, in years? $a = 0.75s$ $a - 8 = 0.5(s - 8)$ $a = 0.5s + 4$ $0.75s = 0.5s + 4$ $0.25s = 4 \rightarrow s = 16$
7	[E + F =] 13	Let A over B and C over D represent two fractions. A, B, C, and D are each replaced with a distinct integer from one through four. As a common fraction, the largest possible value of A over B minus C over D is E over F. What is the value of E plus F? $4/1 - 2/3 = 12/3 - 2/3 = 10/3$, so $10 + 3 = 13$
8	29 [minutes]	Rosa completes three-fourths of an assignment in eighty-seven minutes. How many minutes will it take her to complete what remains of the assignment, assuming she continues working at the same pace? $87/3 = 29$
9	4	One million, two hundred nine thousand, three hundred forty-eight is multiplied by sixty-two million, three hundred eighty-seven thousand, four hundred fifty-three. What is the units digit of the result? In the product $1209348*62387453$, look at just $8*3$, which is 24, to see that the units digit will be a 4.
10	68 [students]	Four hundred students attend a certain university. Twenty percent of them are in their final year and eighty-five percent of those in their final year will graduate this year. How many students will graduate this year? $400*0.2*0.85 = 68$

"Math is Cool" Masters -- 2021-22

8th Grade

College Bowl Round #2 Solutions

8th	Answer	Solution
1	2420	What is twenty-two squared plus forty-four squared? 22*22+44*44= 2420
2	12 [units^2]	The vertices of a triangle are located at one comma two, five comma two, and one comma eight on a coordinate plane. In square units, what is the area of the triangle? Base is 4, Height is 6, 6*4/2=12
3	0	If seven N plus seventeen equals twelve N plus seventeen, what is N? 7x + 17 = 12x + 17 5x = 0, x = 0
4	[$] 5 [dollars]	Biff buys two lattes and one muffin for five dollars and fifty cents. Eho buys two hot chocolates and one muffin for four dollars and fifty cents. In dollars, what is the total cost of one latte, one muffin and one hot chocolate? 2L + M = 5.50 2H + M = 4.50 2H + 2M + 2L = 10, 10/ 2 = 5
5	8 [donuts]	One donut is equal to five bear claws. Two bear claws are equal to three apple tarts. How many donuts are equal to sixty apple tarts? 1D = 5B, 2B = 3A, 40B = 60A = 8D
6	168 [miles]	The scale of Kirby's map is such that one-fourth of an inch equals three miles. If the route Kirby plans to take is fourteen inches on the map, how far will Kirby actually travel, in miles? 14 / (1/4) = 56 56* 3 = 168

7	4 [marbles]	A bag contains twelve marbles, all of which are either black or white. As a reduced common fraction, the probability of pulling out two black marbles in a row without replacement is one over eleven. How many black marbles are in the bag? Let b = the number of black marbles in the bag, then b/12*(b – 1)/11 = 1/11, so b(b – 1) must equal 12 and two consecutive integers that multiply to make 12 are 3 and 4, so the answer is 4.
8	64 [cups]	How many cups are there in sixteen quarts? 4*16=64
9	9 [inches]	In inches, what is the height of a trapezoid with an area of twenty-seven square inches and bases of two and four inches? $(2 + 4)h/2 = 27 \rightarrow 3h = 27 \rightarrow h = 9$
10	42 [dollars]	The cost of gasoline at the local gas station is three dollars and fifty cents per gallon. Naveen's car has a fifteen-gallon gas tank that is twenty percent full. How much will it cost Naveen to fill up the gas tank, in dollars? He needs 80% of 15 gallons which is 12 gallons. 12 * 3.5 = 42

"Math is Cool" Masters -- 2021-22

8th Grade

College Bowl Round #3 Solutions

8th	Answer	Solution
1	3125	Evaluate five to the fifth power. 5^4= 625 625*5= 3125
2	57 [days]	Summer vacation lasts from June twenty-eighth to August twenty-third inclusive. How many days is this? 6/28 to 6/30 = 3 days July has 31 days 8/1 to 8/23 = 23 days 3 + 31 + 23 = 57
3	8	When the fraction 1/7 is converted into a decimal number, what digit is in the 64th place to the right of the decimal point? 1/7 = the repeating decimal 0.142857… It repeats every 6 digits, therefore after 60 digits it will start a new cycle, and the 64th digit will be '8'.
4	44	Seventy-five percent of the number N is eighty-eight. What is three-eighths of N? 75% = 3/4 = 6/8 3/8 is half of 6/8, so the answer is half of 88 = 44
5	27 [diagonals]	How many diagonals can be drawn in a convex nonagon? $n(n-3)/2$ 9(6)/2 = 27
6	[A + B =] 14	Rohan rolls three fair six-sided dice. As a reduced common fraction, the probability that there is a different number showing on each of the three dice is A over B. What is the value of A plus B? 1/6*5/6*4/6*6 = 120/216 = 5/9 The extra *6 is because there are 6 ways any set of three different numbers can be arranged. So, 5 + 9 = 14.

7	4 [laps]	Ana and Bea run in the same direction around the same track, and they start at the same place. To complete a lap, Ana takes one minute, and Bea takes forty-five seconds. After they start running, the first time that Ana and Bea are at the same starting point again at the same time is after Bea has completed N laps. What is the value of N? 60*3=180 45*4=180 LCM
8	10 [units]	On the coordinate plane, what is the distance in units between the points located at (1, -3) and (-5, 5)? The distance is the hypotenuse of a right triangles with legs of distance 6 and 8.
9	310 [minutes]	Sonya's favorite book is nine hundred and thirty pages long and she reads at an average rate of one page every twenty seconds. How long does it take Sonya to read the book in minutes? 60/20 = 3 pages per minute 930/3 = 310 minutes
10	50 [gradians]	One full rotation around a circle is equal to an angular measure of four hundred gradians. How many gradians are equivalent to forty-five degrees? Since 45 = 360/8, the number of gradians would be 400/8 = 50.