"Math is Cool" Masters -- 2022-23

5th grade

Mental Math Solutions

	Answer	Solution
1	20 [nights]	Gustavo reads twelve pages of his book every night. If his book has two hundred forty pages, how many nights will it take him to read his book? 240 / 12 = 240
2	18	What is the next term in the sequence that begins: Three, four, six, nine, thirteen, and so on? Pattern is +1, +2, +3, … 13 + 5 = 18
3	56 [cents]	If Olivia has one dollar and twenty-nine cents, and Arjun has seventy-three cents, how much more money, in cents does Olivia have than Arjun? 129 – 73 = 56
4	121	What is the smallest perfect square greater than one hundred? $11^2 = 121$
5	16 [inches]	What is the perimeter in inches of a rectangle with side lengths of three point two inches and four point eight inches? 3.2 + 4.8 = 8 8x2 = 16
6	60 [%]	Parth has twenty marbles. Five of the marbles are red, three of the marbles are green, and the rest are blue. What percentage of the marbles are blue? 20 – 5 – 3 = 12 blue 12/20 * 100% = 60%
7	12 [ways]	How many ways are there to rearrange the letters A, B, C and D from left to right, if A and B refuse to be next to each other? There are 4! = 24 total ways, but 12 of them have A and B next to each other.

8	84 [points]	Kylie's average score on three tests is ninety points. What is her new average score in points if on her fourth test she scores a sixty-six? 90 * 3 = 270 (270 + 66)/4 = 84

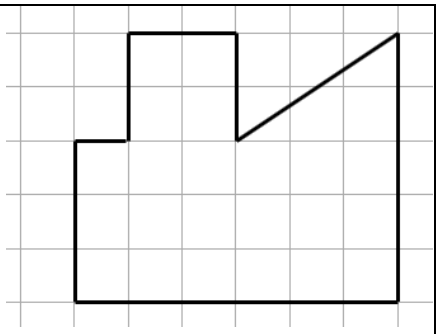
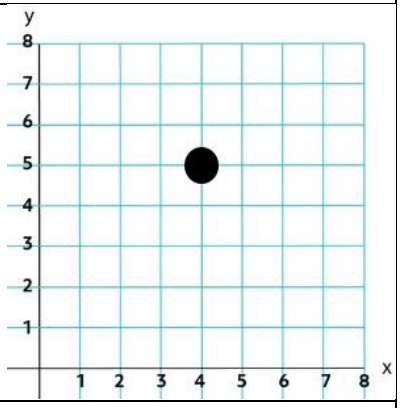
"Math is Cool" Masters -- 2022-23

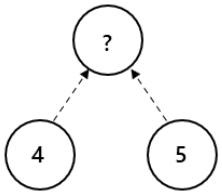
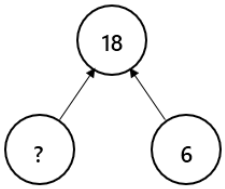
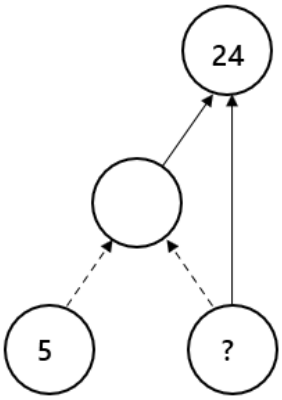
5th grade

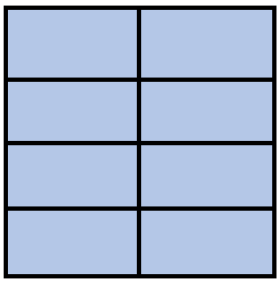
Individual Test Solutions

	Answer	Solution
1	21	Evaluate: 18 – 7 + 2 × 5 11+2x5 11 + 10 = 21
2	4	Find the value of the expression x ÷ 3 when x = 12. 12 ÷ 3 = 4
3	26 [cups]	Dheeraj has made 6 cups of lemonade. How many more cups of lemonade does he need to make to have two gallons of lemonade? There are 16 cups in a gallon, so he needs 10 more cups to get to one gallon, plus another 16 cups to make 2 gallons.
4	6 [candies]	Aurora has twice as many candies as Brooklyn, and one-third as many candies as Cierra. If Brooklyn has one candy, how many candies does Cierra have? Aurora has 1 x 2 = 2 candies and Cierra has 2 x 3 = 6 candies.
5	27 [sixths]	How many sixths are there in $4\frac{1}{2}$? 4.5 ÷ 1/6 = 27
6	75 [%]	The following table shows the number of students in each 5th grade class at Vitsa Elementary School. If one student is chosen at random, what is the probability, in percent, that they are not in Ms. Devine's class? Total students = 80 60/80 = 3/4 = 75%
7	16	What is the next number in the sequence that begins with: 1, 1, 2, 4, 7, 11, … The pattern is +0, +1, +2, +3, … 11 + 5 = 16

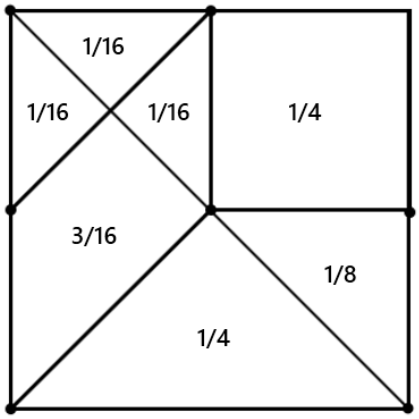
Teacher	Number of Students
Ms. Devine	20
Mrs. Maxson	21
Mr. Melville	18
Mr. Medrano	21

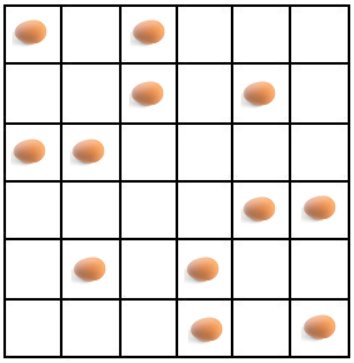
8	25 [square units]	Find the area of this shape in square units. Each square on the grid is 1 unit by 1 unit.  Break it up into smaller pieces. The little square on the top is 4 units^2. The big rectangle is 3x6 = 18 units^2. The triangle is (3)(2)/2 = 3 units^2. Total = 4 + 18 + 3 = 25.
9	2960	What is 2,956.21 rounded to the nearest ten? The ones digit is greater than 4, so the tens digit increases: 2,960.
10	5	What is the remainder when 479 is divided by 6? 479 ÷ 6 = 79 R5
11	1 [= mode]	Find the mode of the following data set: {7, 6, 2, 9, 1, 1, 5} 1 is the only repeated value.
12	350 [cents]	Ziquan is buying tickets for the fair. He needs to buy two tickets and each ticket costs $8.25. If he pays with a twenty-dollar bill, how much change, in cents, will he get back? 825 x 2 = 1650 2000 – 1650 = 350 cents
13	[y =] 5	What is the y-coordinate of the grid point on the coordinate plane that is covered by the circle?  The circle is located at (4, 5), so the y-coordinate is 5.
14	32 [units]	A rectangle with integer side lengths has an area of 28 square units. If both the length and width of the rectangle are an even number of units, what is the perimeter of the rectangle, in units? Possible side lengths are: 1 x 28, 2 x 14, 4 x 7. Only 2 and 14 are both even so the perimeter is 2 + 14 + 2 + 14 = 32

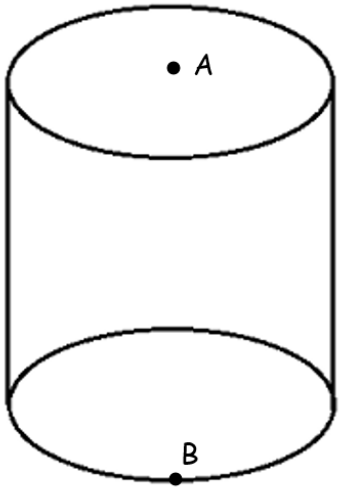
15	192 [cents]	Mei has twelve coins (pennies, nickels, dimes, or quarters) and has at least one of each type of coin. What is the positive difference, in cents, between the most amount of money she could have and the least amount of money she could have? The least amount is 9 pennies, 1 nickel, 1 dime, and 1 quarter, which is 49 cents. The most is 1 penny, 1 nickel, 1 dime, and 9 quarters which is 241 cents. 241 – 49 = 192 cents.
16	2500 [meters]	Bogdan is running a 10 kilometer race. He has already completed $\frac{3}{4}$ of the race course. How many more meters does he have to run? 1 km = 1000 meters 10 km = 10000 meters Therefore, $\frac{1}{4}$ remaining = 10000/4 = 2500 m
17	3 [points]	On his past five tests, Eric has earned 87, 99, 82, 79 and 91 points. On her past five tests, Samantha has earned 90, 88, 91, 97, and 72 points. What is the positive difference between Eric's and Samantha's median scores? The median of Eric's set, {79, 82, 87, 91, 99}, is 87. The median of Samantha's set, {72, 88, 90, 91, 97} is 90. 90 – 87 = 3.
18	3	In a circle puzzle like the ones shown here, dashed arrows mean to add and solid arrows mean to multiply. For example, the unknown value in the first puzzle is 9, because 4 + 5 = 9. The unknown value in the second puzzle is 3, because 3 x 6 = 18.   In this circle puzzle to the right, what is the unknown number indicated with a question mark? All numbers in the puzzle are positive integers.  3+5 = 8 8 x 3 = 24
19	19 [numbers]	How many of the first 100 counting numbers (starting with 1, 2, 3, …) contain the digit 7? 7, 17, 27, 37, 47, 57, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 87, 97

20	20 [%]	The Taterade beverage company recently downsized their bottled energy drink from 25 ounces to 20 ounces. What was the percent decrease in the drink size? Answer as a positive integer. % change = (new–old)/old= (20–25)/25 = -5/25 =-1/5 = 20% decrease
21	5 [students]	On a spelling test, students are asked to spell the word BUG. Each student uses all the correct letters exactly one time, but all of the students spell the word wrong, and each student has a different spelling! What is the maximum number of students that could be in the class? There are 3! or 6 possible arrangements or 6 possible spellings using the letters B-U-G. Only 5 of them are incorrect, so there can be at most 5 students.
22	20 [years]	Trinity is 3 years older than Juho. Six years ago, the sum of their ages was 25 years. How many years old is Trinity currently? Their ages each increased by 6, so the sum of their ages is now 37. Two numbers that add up to 37 and are 3 apart are 20 and 17. Trinity is older so Trinity is 20.
23	37	Julia and Maxim are each thinking of a number. Julia's number is prime and has two digits. Maxim's number is one greater than a perfect square and does not contain the digit 1. What number could they both be thinking of? Two-digit possibilities for Maxim's number are 26, 37, 50, 65, and 82. Only 37 is prime from that list.
24	425 [minutes]	It is currently 9:54 PM, where 9+5+4 = 18. How many minutes will it be until the sum of the digits of the time is 18 again? The next time the digits sum to 18 is 4:59 which is in 7 hours and 5 minutes, which is 7 x 60 + 5 = 425 minutes.
25	18 [cm]	A square with an area of 144 cm^2 is divided into 8 congruent (equal) rectangles, as shown. What is the perimeter of one of these rectangles, in centimeters? Each rectangle is 6x3 cm. Perimeter = 2(6) + 2(3) = 18. 
26	5 [%]	Two fair ten-sided dice with sides numbered 1-10 are rolled. What is the probability, as a percent, that the sum of the numbers rolled is 16? Possible rolls are (6, 10), (7, 9), (8, 8), (9, 7), and (10, 6). There are 100 total possibilities. 5/100 = 5%

27	495	What is the sum of the first nine positive multiples of 11? 11 + 22 + … + 99 = 11(1 + 2 + … + 9) = 11(45) = 495
28	29	What is the smallest positive integer that has a remainder of 1 when divided by 7, a remainder of 5 when divided by 8, and a remainder of 2 when divided by 9? The first few numbers that have a remainder of 1 when divided by 7 are 1, 8, 15, 22, 29. 29 is the first number to have a remainder of 5 when divided by 8 and also has a remainder of 2 when divided by 9.
29	55 [minutes]	Alex can ride her bike at a constant speed of 12 miles per hour. She bikes for ten minutes then stops and rests for five minutes. If this pattern continues, how long in minutes will it take her to bike 8 miles? 8 miles ÷ 12 mph = 2/3 hours or 40 minutes Alex will need to take three breaks in this time. 40 + 3 x 5 = 55 minutes
30	6 [points]	Jackson and Josue play a game where the winner gains 2 points, the loser loses 1 point, and there are no ties. If Jackson won exactly 15 games, and Josue had a final score of 33 points, what was Jackson's final score? Josue must have earned 15 + 33 or 48 points from winning, which means he won 24 games. Jackson would then have 2 x 15 – 24 or 6 points.
31	50	Find the next term in the following sequence: 0, 2, 7, 16, 30, … 0, 2, 7, 16, 30… 50 +2 +5 +9 +14… +20 +3 +4 +5… +6
32	[x =] -3	If the mean of the following set is 5, what is the value of x? {4, x, 10, 10, 6, 7, 9, x} (4+x+10+10+6+7+9+x)/8 = 5 2x + 46 = 40 2x = -6 x = -3
33	600 [ways]	Six books of different heights are to be arranged on a bookshelf from left to right. How many ways can the books be arranged if the tallest book cannot be in the last (furthest right) spot? 5 choices for the last spot, leaving 5 choices for the 1st, 4 for the 2nd, 3 for the 3rd, 2 for the 4th, and 1 for the 5th. 5 * 5 * 4 * 3 * 2 * 1 = 600

34	4	The sum of four two-digit numbers is 221. None of the digits are 0 and no two of the digits are the same. Which of the digits 1 through 9 does not appear in one of the two-digit numbers? The ones digits must add up to 11 or 21 and the tens digits must add up to 21 or 20 respectively. The ones digits are 2, 3, 7, 9 and the tens digits are 8, 6, 5, 1. The missing digit is 4.
35	8 [mph]	Biff and Eho are running around a track that is a quarter of a mile long at a constant speed. They start at the same spot but run in opposite directions. If Biff runs twice as fast as Eho, and they meet after 1 minute and 15 seconds, what is Biff's speed, in miles per hour? It takes them 1 minute and 15 seconds to combined run a fourth of a mile, so it would take them 5 minutes or 1/12 hours to run a full mile. Thus, there combined speed is 12 mph. Biff runs twice as fast, so Biff's speed is 8 mph and Eho's is 4 mph.
36	12 [ways]	The following square has been split into seven distinct regions. All lines go through the square's vertices, side midpoints, or the center of the square. How many different ways are there to color in two or more distinct regions of the square such that 3/8 of the square will be colored? The combinations that work: $\frac{1}{4}$ + 1/8 (2 ways) $\frac{1}{4}$ + 1/16 + 1/16 (6 ways) 3/16 + 1/16 + 1/16 + 1/16 (1 way) 3/16 + 1/8 + 1/16 (3 ways) 

37	12 [eggs]	A farmer is placing eggs in a 6 x 6 crate such that each row, column and the two diagonals has no more than two eggs and such that each space has only one egg. Two eggs have already been placed, as shown below. What is the maximum number of eggs that can be placed in the crate, including the two that are already there?  **One possible solution is shown.**
38	12 [cubic inches]	Itzel has 2023 cubes that have a side length of one inch. She then makes the largest possible rectangular prism with a height of one inch and a square base. With the remaining squares, she makes the largest possible rectangular prism with a height of two inches and a square base. She continues making rectangular prisms with square bases and increasing the height by one each time until she does not have enough cubes to make the next prism. What is the volume of the last rectangular prism she makes? $1 * 44^2 < 2023 < 1 * 45^2$ $2023 - 1 * 44^2 = 87$ $2 * 6^2 < 87 < 2 * 7^2$ $87 - 2 * 6^2 = 15$ $3 * 2^2 < 15 < 3 * 3^2$ $15 - 3 * 2^2 = 3$ 3 blocks are not enough to make a prism with a height of four inches, so the last prism made is 2 x 2 x 3, which has a volume of 12 cubic inches

39	26 [units]	The cylinder shown here has a volume of 2400π cubic units, and the area of the base is 100π square units. Point A is the center of one of the bases and point B is on the circumference of the opposite base. In units, what is the shortest straight-line distance between A and B, which may go through the interior of the cylinder?  The cylinder has a height of 24 and a radius of 10. The right triangle that has legs that are the line from A to the center of the opposite base and the center of the opposite base to B has hypotenuse equal to AB. The legs are 24 and 10 units respectively. 24^2 + 10^2 = 576 + 100 = 676 = 26^2, so AB has a length of 26 units.
40	11 [%]	A factor of 10! is chosen at random. What is the probability, as a percent, that the factor chosen is odd? Round your answer to the nearest whole number. 10! = 10 * 9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1 = 2^8 * 3 ^4 * 5^2 * 7, so 10! has (8 + 1) * (4 + 1) * (2 + 1) * (1 + 1) or 270 factors and (4 + 1) * (2 + 1) * (1 + 1) or 30 odd factors. 30 / 270 ≈ 0.11 = 11%

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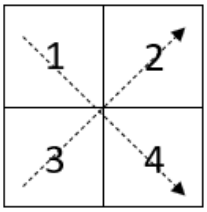
5th grade

Multiple Choice Solutions

	Answer	Solution
USE THE FOLLOWING MAP AND KEY TO SOLVE PROBLEMS #1 THROUGH #4. Consider the following two-by-two arrays of consecutive integers. For example, Array #1 consists of the numbers 1, 2, 3, 4 starting at the upper-left corner and moving across the two rows. Each following array has an upper-left number one higher than the previous array. The pattern continues with additional arrays. **Array #1** 1 2 3 4 **Array #2** 2 3 4 5 **Array #3** 3 4 5 6		
1	C	What is the product of all four numbers contained in Array #5? A) 26 B) 840 C) 1680 D) 3024 E) Answer not given. Array #5 contains: 5, 6, 7, 8. 5x6x7x8 = 1680
2	E	What is the positive difference between the sum of the four numbers contained in Array #6 and the sum of the four numbers contained in Array #3? A) 10 B) 16 C) 20 D) 26 E) Answer not given. Array #6 contains: 6, 7, 8, 9. 6+7+8+9 = 30 Array #3: 3+4+5+6 = 18 30 – 18 = 12

3	*C*	The sum of each diagonal in Array #1 is equal to 5, since 1 + 4 = 5 and 2 + 3 = 5. What is the sum of each diagonal in Array #10? A) 21 B) 22 C) 23 D) 24 E) Answer not given. Array 10 will contain: 10, 11, 12, 13. The sum of the diagonals will be 10 + 13 = 11 + 12 = 23.
4	D	Consider a different set of two-by-two arrays, each of which contains the first four positive consecutive multiples of the integers, starting with 2. Array #1 contains the multiples of 2: 2, 4, 6, 8, starting at upper-left and moving across the two rows. Array #2 contains the multiples of 3: 3, 6, 9, 12. The pattern continues infinitely. In this pattern, what is the largest product that can be made by multiplying the two numbers on either diagonal of Array #5? A) 30 B) 150 C) 144 D) 216 E) Answer not given. Array 5 will contain the multiples of 6: 6, 12 18, 24 6*24 = 144 12*18 = 216

Array #1



USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #5 THROUGH #7.

The table shown here displays the approximate number of pandas that live in the wild in China from the years 1970 through 2020. The panda estimates are found by researchers conducting a "panda census" every few years.

Year	No. of wild pandas
1970	2670
1975	2170
1980	1640
1985	1100
1990	?
1995	1360
2000	1400
2005	1630
2010	1770
2015	1780
2020	1860

5	*C*	In 1990, there were 310 fewer pandas than in 1980. How many pandas were there in 1990? A) 790 B) 940 C) 1330 D) 1410 E) Answer not given. 1640 – 310 = 1330
6	*C*	The panda population is projected to increase by 6.5% between 2020 and 2025. What is the projected number of pandas in 2025? Round your answer to the nearest whole panda. A) 1962 B) 1972 C) 1981 D) 1990 E) 1992 2020 = 1860 pandas. 1860*1.065 = 1980.9, rounds to 1981.
7	*B*	The panda population decreased at a fairly constant rate from 1970 to 1985. On average, the panda population was decreased by how many pandas per year over that time period? Round your answer to the nearest whole panda. A) 100 B) 105 C) 115 D) 117 E) 118 Difference in pandas = 2670 – 1100 = 1570. 1570/15 years = 104.666… Rounds to 105 pandas per year.
Use the following information to answer Questions 8-10 There are 14 identical objects that are going to be placed into different groups (sets). Unless otherwise specified, assume that the order of the groups does not make a difference. For example, placing 12 objects into Set 1 and two objects into Set 2 is the same as placing two objects into Set 1 and 12 objects into Set 2. It is possible to have an "empty set" which contains zero objects.		
8	*B*	In how many different ways can the 14 identical objects be placed into two different groups (sets)? A) 5 B) 8 C) 10 D) 14 E) Answer not given. They can be divided as follows: 14, 0 13, 1 12, 2 11, 3 10, 4 9, 5 8, 6 7, 7 Eight ways total.

9	*C*	Suppose that the 14 identical objects are Oreo cookies that are being divided between two friends, Zahir and Parth. In this case, the order of the groups matters, because Parth will be very unhappy for example if Zahir gets all 14 Oreos. In how many different ways can the 14 Oreos be distributed between the two friends? A) 8 B) 12 C) 15 D) 28 E) Answer not given. They can be divided as follows: 14, 0 or 0, 14 13, 1 or 1, 13 12, 2 or 2, 12 11, 3 or 3, 11 10, 4 or 4, 10 9, 5 or 5, 9 8, 6 or 6, 8 7, 7 15 ways total.

10	*A*	In how many different ways can 14 identical objects be placed into three different groups (sets)? Assume that order does not matter. A) 24 B) 28 C) 30 D) 48 E) Answer not given. They can be divided as follows: 14, 0, 0 13, 1, 0 12, 2, 0 12, 1, 1 11, 3, 0 11, 2, 1 10, 4, 0 10, 3, 1 10, 2, 2 9, 5, 0 9, 5, 1 9, 3, 2 8, 6, 0 8, 5, 1 8, 4, 2 8, 3, 3 7, 7, 0 7, 6, 1 7, 5, 2 7, 4, 3 6, 6, 2 6, 5, 3 6, 4, 4 5, 5, 4

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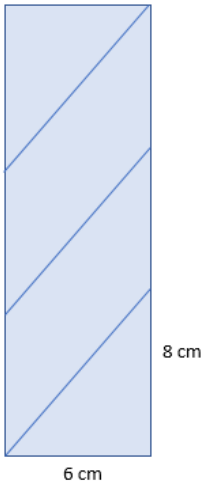
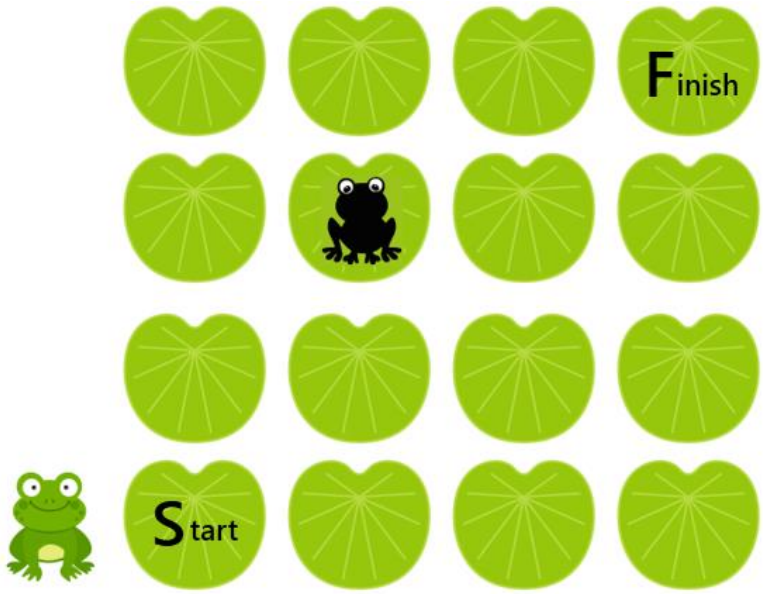
5th grade

Team Test Solutions

	Answer	Solution
1	45 [integers]	How many integers are there between -5 and 39 inclusive (including -5 and 39)? 1 through 39 = 39 -5 through 0 = 6 45 total
2	3 [= quotient]	Make the largest quotient possible by choosing from the numbers shown below. One number goes in each box, and you can use each number at most one time. What is the quotient? 7, 8, 10, 2, 5, 3 $\frac{\square + \square}{\square \times \square}$ (10 + 8)/(2x3) = 3
3	9 [hours]	A plane travels at 550 miles per hour. If the distance between Seattle and Tokyo is 4950 miles, how many hours will it take for the plane to travel between the two locations? 4950 mi/550 mph = 9 hours
4	12 [positive integers]	How many positive integers less than 60 are divisible by 4 but not divisible by 5? There are 14 total, the largest is 56: 56 / 4 = 14 20 and 40 are divisible by 5. 14 – 2 = 12

5	145	What is the sum of the first 10 numbers in the following arithmetic sequence? 1, 4, 7, 10, … 1 + 3(10 – 1) = 28 10 * (1 + 28) / 2 = 145 Or just add them up: 1+4+7+10+13+16+19+22+25+28 = 145
6	[$] 18	At Parker's Pie Shop, a slice of pie is one eighth of a full pie and costs $3. If a full pie costs 25% less than eight slices of pie, how much, in dollars, does a full pie cost? 3 * 8 = 24 24 – 24 * 0.25 = 18
7	30 [centimeters]	A cylinder has a circumference of 6 centimeters and a height of 24 centimeters. A string is wrapped around the cylinder beginning at a point on the edge of the bottom base of the cylinder and ending at a point on the edge of the top base directly above the point where it begins. The string is evenly spaced as it wraps around the cylinder exactly three times. In centimeters, what is the length of the string? "Unwrap" the cylinder and look at a net of the lateral surface. Since the string goes around three times, it divides the cylinder into three sections of height 8 cm. The base is 6 cm which means that the hypotenuse is 10 cm. 10x3 = 30 cm. 

8	11 [ways]	Lily the frog loves leaping across lily pads. Her pond has sixteen lily pads as shown below. From any lily pad, she can jump up or to the right to any adjacent lily pad (not diagonally). Except she cannot jump to the one lily pad currently occupied by Florence, because Florence refuses to move. How many ways are there for Lily to get to the lily pad labelled **Finish** from the lily pad labelled **Start** using the fewest leaps possible?  U = up R = right In total, need to make three Up moves and three Right moves, or: UUURRR in any order. Without Florence in the way, that can be calculated as: 6!/(3!3!) = 20. However, Florence is blocking 9 of the pathways, so there are only 20 – 9 = 11 remaining.

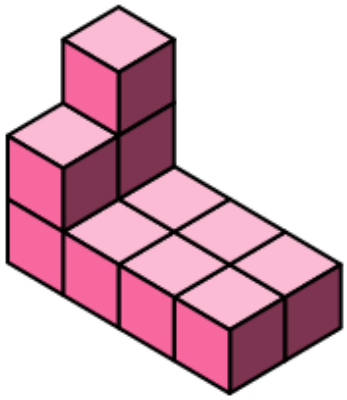
9	35 [*eggs*]	Andrei, Bo, Claire and Devon went on an Easter egg hunt. They found red, purple, yellow and green colored eggs. The number of purple eggs that they found was three more than the number of green eggs. The number of red eggs was twice the number of green eggs. The number of yellow eggs was two more than the number of red eggs. Andrei found as many eggs as Bo did. Claire found three eggs more than Andrei did. Devon found four eggs more than Bo did. Claire, whose favorite color is red, gathered only red eggs. None of the others gathered red eggs. How many total eggs did they find? G = green eggs R = red eggs P = purple eggs Y = yellow eggs A = Andrei eggs B = Bo eggs C = Claire eggs D = Devon eggs $P = G + 3$ $R = 2G$ $Y = R + 2$ $A = B$ $C = A + 3$ $D = B + 4$ $C = R$ $A + B + C + D = G + R + P + Y$ You can express all the variables in terms of G. $R = 2G$ $P = G + 3$ $Y = 2G + 2$ $C = 2G$ $A = 2G - 3$ $B = 2G - 3$ $D = 2G + 1$ Substituting into the 8th equation, we get $8G - 5 = 6G + 5$ Then $2G = 10$ So $G = 5$ Then $R = 10$

		P = 8 Y = 12 A = 7 B = 7 C = 10 D = 11.
10	58 [= A+B]	Two distinct positive integers less than or equal to 10 are chosen at random and multiplied together. The order of the integers does not matter, for example 1×2 is the same as 2×1. The probability that their product ends in a zero can be written as a reduced common fraction A/B. What is the value of A + B? 5 + even number besides 10: 2 * 1/10 * 4/9 = 4/45 10 + any number: 2 * 1/10 = 1/5 4/45 + 1/5 = 13/45 13 + 45 = 58

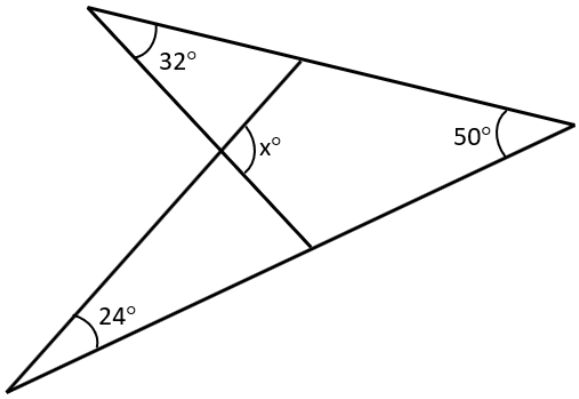
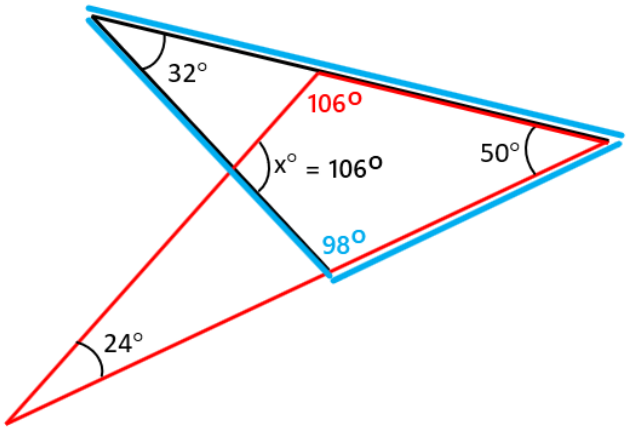
"Math is Cool" Masters -- 2022-23

5th grade

Linda Moore Triple Jump Solutions

	Answer	Solution
1	11 [cubic units]	The following figure is made of unit cubes, some of which are stacked. What is the volume of the entire figure, in cubic units? Two rows of 4 on the bottom, plus 3 on top. 
2	10 [nickels]	If I have ten quarters, seven dimes, five nickels, and four pennies, what is the least number of additional nickels I need to pay for something that costs $3.98? 3.98 – (10 * 0.25 + 7 * 0.10 + 5 * 0.05 + 4 * 0.01) = 0.49 Therefore, need 10 nickels, because 9 will not be enough.
3	9420 [seconds]	If it is currently 3:42 PM, how many seconds will it be until 6:19 PM on the same day? 3:42 to 5:42 = 2 hours 5:42 to 6:12 = ½ hour 6:12 to 6:19 = 7 min Total = 120 + 30 + 7 = 157 minutes * 60 sec/min = 9420 sec
4	[A =] 7	For what digit A is the following number evenly divisible by 9? 42A968 The sum of the digits must be divisible by 9. 4+2+9+6+8 = 29 The only digit 0-9 that will additionally give a sum divisible by 9 is 7. 29 + 7 = 36

5	26 [%]	At a certain math competition, the 5th grade trophy is 5% taller than the 4th grade trophy, and the 6th grade trophy is 20% taller than the 5th grade trophy. What percent taller is the 6th grade trophy than the 4th grade trophy? 100 * 1.05 = 105 105 * 1.20 = 126 126 is 26% more than 100.
6	75 [%]	Biff and Eho each roll a fair six-sided die numbered one through six, and then multiply together the number that each of them got. If the product of the two numbers is even, Biff wins. If the product is odd, Eho wins. As a percentage, what is the probability that Biff will win the game? Out of the 36 total outcomes, 9 of them have an odd product and 27 of them have an even product. 27/36 = $\frac{3}{4}$ = 75%.
7	105 [degrees]	The angles of a triangle are in a 1:3:8 ratio. What is the positive difference, in degrees, between the largest angle and the smallest angle? x+3x+8x = 180 x = 15 8x = 120 8x – x = 105
8	10	A set of seven distinct positive integers has a unique mode of 7, a median of 5, and a mean of 5. What is the largest possible integer in the set? 5 must be in the middle. The sum of the numbers = 7*5 = 35. Assign two 7s to be the unique mode. Fill in the smallest possible integers: 1, 2, 3. Therefore the largest possible number is 10. {1, 2, 3, 5, 7, 7, 10}
9	6043	What is the sum of the first digits of the first 2023 counting numbers? For example, the first (left-most) digit in the number 587 is 5. 1 – 9: sum = 45 For the tens, such as 10-19, 20-29, …, the sum is 10+20+…+90 = 450. For the hundreds, such as 100-199, 200-299, …, 900-999, the sum is 100+200+…+900 = 4500. For 1000-1999, the sum of the 1s is 1000. For 2000-2023, the sum of the 2s is 48. Total = 45+450+4500+1000+48 = 6043

10	106 [°]	In the following figure, what is the measure of angle x, in degrees?  From the red triangle, the upper center angle measures 180 – 24 – 50 = 106. From the blue triangle, the bottom center angle measures 180 – 32 – 50 = 98. 'x' is the only unknown angle in the central quadrilateral, so it equals 360 – 106 – 50 – 98 = 106. 

"Math is Cool" Masters -- 2022-23

5th grade

College Bowl Round #1 Solutions

	Answer	Solution
1	3	In the number four hundred thirty five thousand two hundred and six, what digit from zero to nine is in the ten-thousands place? 435,206 is the number, a '3' is in the ten-thousands place.
2	4 [sides]	The interior angles of a polygon have a sum of 360 degrees. How many sides does this polygon have? A quadrilateral has interior angles summing to 360 degrees.
3	30575 [trees]	A logging company cut down twenty four thousand five hundred and thirty-two trees on the south side of Mount Eho and six thousand forty-three trees on the north side of Mount Eho. How many total trees did they cut down? 24,532 + 6,043 = 30575
4	-36	What is the next term in the following sequence: One, negative four, nine, negative sixteen, twenty-five, and so on. Squares of the positive integers with alternating signs: 1, -4, 9, -16, 25, -36, …
5	28 [= 2x]	If sixteen minus x equals two, what is the value of two times x? 16 – x = 2 14 = x 2x = 28
6	2380 [reviews]	Eighty-five percent out of two thousand eight hundred reviews of the Super Mario Brothers Movie were positive. How many of the reviews were positive? 2800*0.85 = 2380
7	36 [%]	If an integer is randomly chosen from one to twenty-five inclusive, what is the probability in percent that it is prime? There are 9 primes from 1 – 25: 2, 3, 5, 7, 11, 13, 17, 19, 23 9/25 = 36/100 = 36%
8	9 [= x]	Jose has the following data: X, six, ten, twelve, three, seven and ten If the median of the data is nine, what is the value of x? 3, 6, 7, 9, 10, 10, 12
9	9	As an integer, what is the value of three halves plus six fourths plus ten minus four? 3/2 + 6/4 + 10 – 4 = 9

10	8 [pieces of paper]	Lyla is making a banner for her little brother's birthday party. She tapes together pieces of construction paper until the banner is two yards long. Each piece of paper is nine inches long. How many pieces of paper does she need? 2 yards = 6 feet = 72 inches 72/9 = 8 pieces of paper

"Math is Cool" Masters -- 2022-23

5th grade

College Bowl Round #2 Solutions

	Answer	Solution
1	5	Ten raised to what power equals one hundred thousand? 10^5 = 100000
2	49	Find the sum of the eighth and ninth terms of the arithmetic sequence that begins with: two, five, eight, eleven and so on. +3 each term 2, 5, 8, 11, 14, 17, 20, 23, 26 23 + 26 = 49
3	8 [coins]	Using only pennies, nickels and quarters, what is the least amount of coins needed to make eighty-eight cents? QQQNNPPP
4	75 [%]	Two cards are selected one at a time with replacement from a standard 52-card deck. What is the probability in percent that the cards are from different suits, where the suits are hearts, diamonds, clubs and spades? The first card can be anything. The second card needs to be a different suit, which is 39/52 = ¾ = 75%
5	10008	What is the smallest positive five-digit integer that is divisible both by eight and nine? Start at 10000. Sum of digits must = 9, last three digits must be divisible by 8.
6	3 [sets]	How many distinct sets of three positive integers have a mean of six, a median of seven, and no mode? The sum of the three must be 18, and 7 must be in the "middle". The only possibilities are: 1, 7, 10 2, 7, 9 3, 7, 8
7	3	A number begins with the following digits and the pattern continues infinitely: One two three four one two three four one two three four and so on. What is the two thousand and twenty third digit in the number? It repeats in groups of 4. 2023/4 = 505 R3, therefore it will be the 3rd digit in the group which is 3.

8	29 [units]	What is the length of a rectangle in units if the perimeter is 84 units and the width is 13 units? 84 – 13 – 13 = 58 58/2 = 29
9	80	What is the product of one point six and fifty? 1.6 x 50 = 80
10	1 [angle]	In an obtuse triangle, how many of the interior angles measure more than ninety degrees? That is the definition of an obtuse triangle.

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College Bowl Round #3 Solutions

	Answer	Solution
1	317 [degrees]	If one interior angle of a rhombus measures forty-three degrees, what is the sum in degrees of the measures of the other three interior angles? 360 – 43 = 317
2	20 [%]	If an integer is chosen at random from one to ten, including one and ten, what is the probability in percent that the integer contains the digit one? Two of the 10 numbers (1 and 10) contain the digit 1. 2/10 = 20%.
3	600 [spins]	A wheel spins one hundred times in two minutes and thirty seconds. At this rate, how many times does the wheel spin in fifteen minutes? 100/2.5 = x/15 X = 600
4	6 [ways]	Four high school students are sitting in a car. If there are four different seats, including the driver's seat, and Kirby must drive the car, in how many ways can the four students sit in the car? One position is fixed. 3! = 6 ways.
5	10 [= mean]	What is the mean of the first ten positive odd integers? Sum = 10^2 = 100 100/10 = 10
6	6 [dogs]	There are fourteen dogs at Dylan's Doggie Daycare. Twelve of them are females and the rest are males. Eight of them are white. What is the smallest possible number of white female dogs? 12F, 2M 8 white If the two male dogs are white, then the least number of white female dogs is 6.
7	71 [marbles]	Jovanni is sorting marbles into boxes. He put 36 marbles in the first box, 43 marbles in the second box, 50 marbles in the third box, and 57 marbles in the fourth box. If this pattern continues, how many marbles will Jovanni put in the sixth box? Pattern is +7 57+7 = 64 64+7 = 71

8	5045 [g]	Five kilograms and forty-five grams is equal to how many grams? $1\ \text{kg} = 1000\ \text{g}$ $5000+45 = 5045$
9	189 [cubic inches]	How much more volume in cubic inches does a cube with side lengths of six inches have compared to a cube with side lengths of three inches? $6^3 = 216$ $3^3 = 27$ $216 - 27 = 189$
10	62 [%]	The decimal number zero point six two is equal to what percentage? $0.62 = 62\%$

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College Bowl Round #4 Solutions

	Answer	Solution
1	3516	What whole number divided by 100 equals thirty-five point one six? 3516/100 = 35.16
2	16	What is the next number in the sequence that begins: One hundred twenty-eight, sixty-four, thirty-two and so on. Divide by 2 each time.
3	44 [units]	On a number line, how many units apart are the numbers thirty-five and negative nine? 35 + 9 = 44
4	6 [hours]	Working together, two hoses can fill a swimming pool with water in four hours. One hose alone can fill the pool in twelve hours. How many hours would it take the other hose alone to fill the pool with water? ab/(a + b) = 4 12b/(12+b) = 4 B = 6
5	4 [ways]	In how many ways can the number four be written as the sum of two or more not necessarily different positive integers, if the order of the integers does not matter? 1+3 2+2 2+1+1 1+1+1+1
6	100 [units]	A square's side length is the same number of units as the diameter of a circle. If the area of the circle is two thousand five hundred times pi square units, how many units is the side length of the square? Area of circle = pi*r^2 = 2500pi, therefore r = 50 and diameter = 100.
7	17	The range of the following set of positive integers is 15. What is the value of x? Fourteen, two, x, twelve, nine, three Range = max – min The only way to get a range of 15 is if x = 17 17 – 2 = 15

8	12	What is the least common denominator of the fractions one third and three fourths? The LCM of 3 and 4 is 12.
9	12 [edges]	How many edges does a cube have? Four on the top, four on the bottom and four on the sides.
10	13 [students]	Mr. Houser kept data on how many books each of his students read last week. Two students read zero books, eight students read one book, four students read two books and nine students read three books. How many students read more than one book last week? 4+9=13

"Math is Cool" Masters -- 2022-23

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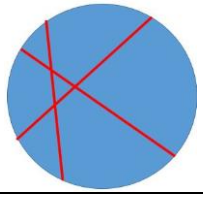
College Bowl Round #5 Solutions

	Answer	Solution
1	47 [minutes]	Two hundred eighty-seven minutes equals four hours and how many minutes? 4 hours = 4*60 = 240 minutes 287 - 240 = 40 minutes
2	8 [interior angles]	How many interior angles does an octagon have? Definition of an octagon.
3	141 [students]	The Math Is Cool students in Wenatchee voted on their favorite prize. A total of one hundred eighty-eight students voted for either fidget spinners or beanies. Three times as many students voted for fidget spinners as voted for beanies. How many students voted for fidget spinners? ¾(188) = 141
4	7 [pieces]	What is the maximum number of pieces of pizza that can be created by making three straight cuts across a round pizza? Pieces may not be moved or stacked, and they do not have to be the same size or shape. 
5	20 [%]	A positive integer from one to one hundred, including one and one hundred, is selected at random. What is the probability in percent that the number is divisible by five? There are 20 multiples of 5, 5*1 through 5*20 between 1 – 100. 20/100 = 20%
6	4 [marbles]	Two friends are playing marbles. Max says to Ruby, if you give me one of your marbles, we would each have the same number of marbles. Ruby says, if you give me one of your marbles, I would have five times as many marbles as you. How many marbles does Ruby currently have? Max has 2. Ruby has 4.

7	26	What two-digit positive integer is one more than a perfect square and one less than a perfect cube? $25 + 1 = 26$ $26 + 1 = 27$
8	2808	What number plus nine hundred and fifteen equals three thousand seven hundred twenty-three? $3723 - 915 = 2808$
9	152 [cubic inches]	What is the volume in cubic inches of a rectangular prism with side lengths of seven point six inches, ten inches and two inches? $7.6*10*2 = 152$
10	4	What is the greatest common factor of sixteen, twelve and twenty? 4 is the largest number that divides evenly into 16, 12 and 20.

"Math is Cool" Masters -- 2022-23

5th grade

College Bowl Round #6 Solutions

	Answer	Solution
1	3 [numbers]	How many numbers in the following set are prime? One, two, seven, twenty-one, twenty-seven, thirty-one, thirty-nine 2, 7 and 31 are prime
2	75	What is the median of the following set of numbers? Seventy-five, seventy-seven, sixty-four, sixty-six, eighty-four Put in order: 64, 66, 75, 77, 84
3	5 [more sides]	How many more sides does an octagon have than a triangle? 8 – 3 = 5
4	18	The sum of three positive integers is seventy-two. The integers are in the ratios of three to four to five. What is the smallest of the three integers? 3:4:5 x 6 = 18:24:30 18+24+30 = 72
5	50 [%]	Out of all of the different arrangements that can be made using the digits one, two three and four, what percentage of them start with a digit that is prime? There are 4! = 24 total arrangements. Half of them start with a prime digit, either 2 or 3.
6	20 [= A + B]	The mean of the following set of numbers is 12: A, B, thirteen, and fifteen What is A plus B? A + B + 13 + 15 = 12x4 A + B = 20
7	58 [degrees]	One of a right triangle's angles measures thirty-two degrees. What is the measure in degrees of the other acute angle of the triangle? 90 – 32 = 58
8	2300	What is two thousand two hundred seventy-one rounded to the nearest hundred? 2271 rounded to the hundreds = 2300

9	18 [minutes]	Ms. Carlson is taking her class to a field trip at the science museum. They have three hours to spend at the museum and want to divide their time equally among the ten exhibits. How many minutes should they spend at each exhibit? 3 hrs * 60 minutes/hour = 180 minutes / 10 = 18
10	352 [students]	If there are twenty-two testing rooms at a math competition, and each room has exactly four teams and each team has exactly four students, how many total students are at the math competition? 22x4x4 = 352

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College Bowl EXTRA

	Answer	Solution
1	3 [comic books]	My brother spent ten dollars on comic books and candy bars. He bought seven items. If comic books cost two dollars each and candy bars cost one dollar each, how many comic books did he buy? 3*$2 + 4*$1 = $10
2	20 [multiples]	How many multiples of one hundred are between one and two thousand twenty-three? 100, 200, ..., 2000
3	15	Find the value of five squared minus four squared plus three squared minus two squared plus one squared. 25 – 16 + 9 – 4 + 1 = 15
4	108 [units]	How many units greater is the perimeter of a square with side length 37 units than the perimeter of a square with area one hundred square units? 4(37 – 10) = 108
5	17 [yards]	Six hundred and twelve inches is equal to how many yards? 612/12 = 51 feet 51/3 = 17 yards
6	3 [boxes]	Ellie is delivering three thousand three hundred boxes of Girl Scout cookies to three neighborhoods. One neighborhood has two hundred fifty houses, the second has five hundred houses, and the third has three hundred fifty houses. If each house gets the same number of boxes of cookies, how many boxes does each house get? 250+500+350 = 1100 3300/1100 = 3