# "Math is Cool" Championships -- 2023-24

## High School

## Mental Math Solutions

| | Answer | Solution |
|---|---|---|
| **1** | 46 | What is the value of ten times four and three-fifths?<br>3/5 = 0.6<br>4 3/5 = 4.6<br>10x4.6 = 46 |
| **2** | 16 | The number one two one base three is equal to what number in base ten?<br>$121_3 = 1x3^2 + 2x3^1 + 1x3^0 = 9 + 6 + 1 = 16$ |
| **3** | 4 [= x] | Solve for x in the following equation:<br>Two plus x minus one plus 3x equals seventeen<br>2 + x – 1 + 3x = 17<br>4x + 1 = 17<br>4x = 16<br>x = 4 |
| **4** | 20 [in] | A circle has an area of twelve-hundred square inches. Approximating the value of pi as three, find the radius to the nearest inch.<br>$1200 = 3r^2$<br>$400 = r^2$<br>20 = r |
| **5** | 12 [units] | Triangle ABC and triangle DEF are similar. If AB equals twelve units, DE equals eighteen units and BC equals eight units, what is the length of EF in units?<br>12/8 = 18/EF<br>EF = (18)(8)/12 = 12 |
| **6** | 60 [ways] | In how many different ways can three people be seated in a row of five chairs?<br>5C3 = 5!/(3!2!) = 10 ways to choose 3 chairs. 3! = 6 ways to arrange the people.<br>10x6 = 60 |
| **7** | 9 [times] | The nine digit number nine eight seven six five four three two one is multiplied by nine. How many times does the digit eight appear in the product?<br>987,654,321x9 = 8,888,888,889<br>If you start multiplying mentally, notice the pattern that each number to the left of the 9 will be an 8. |

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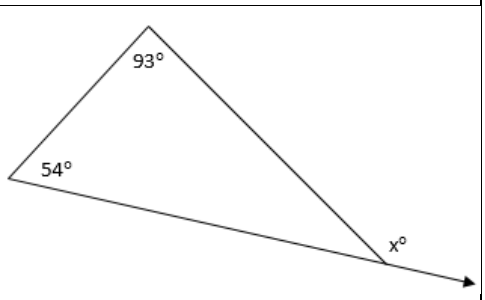
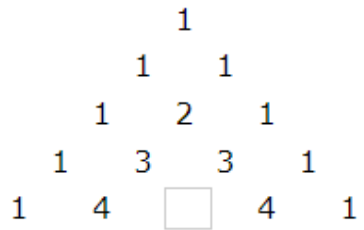
| **8** | 192 | The first two terms of a sequence are one and two. Each of the following terms in the sequence is equal to the sum of all the terms that come before it in the sequence. What is the ninth term in the sequence?<br>1, 2, 3, 6, 12, 24, 48, 96, 192<br>Starting with the $4^{th}$ term, each term is just two times the previous term. |

# "Math is Cool" Championships -- 2023-24
# High School
## Individual Test Solutions

| | Answer | Solution |
|---|---|---|
| **1** | -18 [x = ] | The expression $(b^3)^{-6}$ can be simplified as $b^x$. What is the value of x?<br>$(b^3)^{-6} = b^{-18}$ |
| **2** | 147 [°] | For the following triangle, what is the measure in degrees of exterior angle x?<br>  <br>180 – (93 + 54) = 33<br>180 – 33 = 147 |
| **3** | 12 [outcomes] | Packard flips a fair quarter and rolls a single 6-sided standard die. How many total outcomes are possible?<br>$2x6 = 12$ |
| **4** | 6 [x = ] | Solve for x in the following equation:<br>-19x – 2x + 16x + 12 = -18<br>30 = 5x<br>x = 6 |
| **5** | 6 | What missing number goes in the box in Row 4 of Pascal's Triangle?<br>  <br>Row 4 is: 1 4 6 4 1<br>For each position, add the two numbers above it. |
| **6** | 60 | Find the sum x + y of the numbers in the following sequence:<br>57, 48, x, 30, y, 12, 3, …<br>The pattern is -9.<br>57, 48, 39, 30, 21, 12, 3…<br>39 + 21 = 60 |

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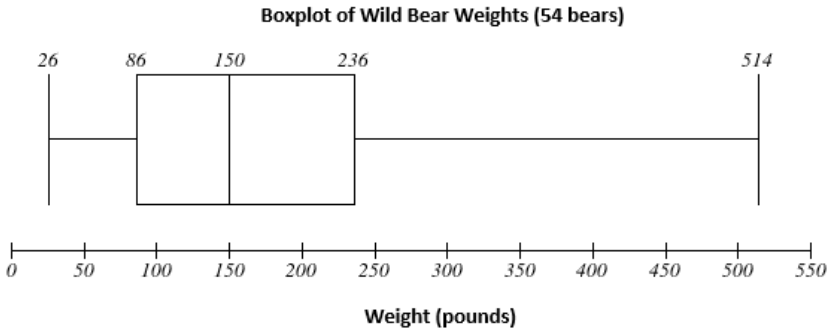
| **7** | 25 [%] | There are four cards numbered 1, 2, 3 and 4. Two cards are selected randomly, one at a time, without replacement. As a percentage, what is the probability that the first card is a '4' and the second card is a factor of 48?<br>P(4 & factor of 48) = (1/4)(1) = 25% |
| **8** | 5 [in] | Two right circular cones are similar. The smaller cone has a slant height of 6 inches and a base radius of 3 inches. The larger cone has a slant height of 10 inches. What is the base radius of the larger cone, in inches?<br>6/3 = 10/5 |
| **9** | 16 [= A + B] | The repeating decimal 0.777… can be written as a reduced common fraction A/B. What is A + B?<br>0.777… = 7/9<br>7 + 9 = 16 |
| **10** | 150 [pounds] |  <br>The following boxplot shows the weight measurements (in pounds) of 54 wild bears. What is the median weight of the bears, in pounds?<br>The median is indicated by the center line in the box. |
| **11** | 10 [%] | A baseball mitt with a retail price of $190 costs the customer $209 once the sales tax is added. As a percentage, what is the sales tax rate?<br>209 – 190 = 19<br>19 is 10% of 190 |
| **12** | 4 [numbers] | How many of the numbers listed below provide a counterexample for the following conditional statement? "If a fraction can be reduced by removing a common factor, then its numerator and denominator are both even."<br>$\frac{1}{2}, \frac{20}{60}, \frac{3}{5}, \frac{3}{21}, \frac{17}{51}, \frac{5}{20}, \frac{9}{36}, \frac{24}{120}$<br>½ can't be reduced.<br>20/60 both are even.<br>3/5 can't be reduced.<br>3/21 – counterexample.<br>17/51 – counterexample.<br>5/20 – counterexample.<br>9/36 – counterexample.<br>24/120 – both are even. |

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| **13** | 11 | Evaluate f(15) for the given function: $f(x) = \frac{120}{x} + 3$<br>120/15 + 3 = 11 |
| **14** | 32 [cents] | A 2-gallon jug of apple cider costs $10.24. In cents, what is the price per cup?<br>10.24/2 = $5.12/gallon<br>5.12/16 cups = $0.32 = 32 cents per cup |
| **15** | 162 | A sequence is defined by the following rule, for n = 1, 2, 3, …<br>What is the value of the 4th term in the sequence, $a_4$?<br>$a_n = 2 \cdot (3)^n$<br>$a_4 = 2 \cdot (3)^4$ = 2(81) = 162 |
| **16** | 29 | Two positive integers have a product of 100, and neither of the two integers contains the digit '0'. What is the sum of the two integers?<br>25x4 = 100<br>25 + 4 = 29 |
| **17** | 28 [ways] | Given the arrangement of letters, in how many different ways can you spell out the name BIFF? You must start with the B in the middle, and can move from one letter to an adjacent letter in a horizontal or vertical direction only (not diagonal).<br><br>From any one of the four 'I's, there are 7 ways to add on an FF. Therefore, 4x7 = 28.<br><br>F<br>F F F<br>F F I F F<br>F F I B I F F<br>F F I F F<br>F F F<br>F |
| **18** | -6 | What is the maximum function value of the following parabola?<br>$f(x) = -x^2 - 4x - 10$<br>The x-coordinate of the vertex is: x = -b/2a = 4/-2 = -2. The y-coordinate and maximum function value is:<br>$y = -(-2)^2 - 4(-2) - 10$<br>= -4 + 8 – 10 = -6 |
| **19** | 738 [n =] | The following data set, which consists of all integers, has a unique mode of 734 and a median of 736. What is the smallest possible value of n?<br><br>734 744 n 737 734 729 736<br>736 is in the middle, therefore 'n' is greater than 736. It can't be 737 because there is only one mode, therefore the smallest possible value is 738.<br>729 734 734 736 737 **n** 744 |

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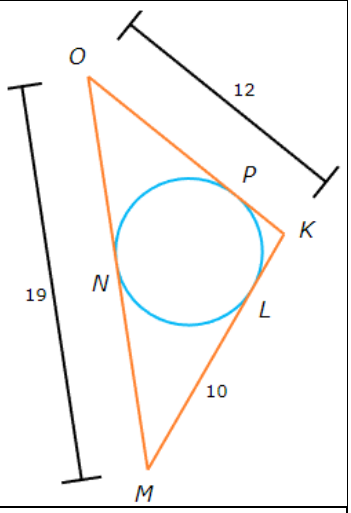
| **20** | 3 [units] | A circle is inscribed in a triangle, with points of tangency at L, N and P. As shown, the length of OK = 12 units, OM = 19 units and LM = 10 units. In units, what is the length of KL?<br><br>MN and ML are congruent, so MN = 10, therefore ON = 9. ON and OP are congruent, so OP = 9, therefore KP = 3. KP and KL and congruent, so KL = 3.<br>  <br> |
| **21** | 2 | Fill in the boxes with the integers 1 through 5, using each one exactly once, so that the following equation is true. The proper order of operations must be followed. Which number is in the right-most box?<br><br>□ + □ − □ × □ ÷ □ = 6<br>(5 + 3) − (4 × 1 ÷ **2**) = 6 |
| **22** | -20 | Find the sum of the first three partial sums of the following series.<br>-4 + (-3) + (-2) + (-1) + 0 + 1 + 2 + …<br>$S_1 = -4$<br>$S_2 = -4 + (-3) = -7$<br>$S_3 = -4 + (-3) + (-2) = -9$<br>$S_1 + S_2 + S_3 = -4 + (-7) + (-9) = -20$ |
| **23** | -35 | Given the functions f(x) and g(x), find:<br>(f − g)( −3)<br><br>$f(x) = -x^2 + 2x + 6$<br>$g(x) = 2x^2 - 3x - 1$<br>$f - g = -x^2 + 2x + 6 - (2x^2 - 3x - 1) = -3x^2 + 5x + 7$<br>$(f - g)(-3) = -3(-3)^2 + 5(-3) + 7$<br>$= -27 - 15 + 7 = -35$ |
| **24** | 9409 | What is the largest 4-digit number that has exactly three positive integer factors?<br>It must be the square of a prime number, and the prime must be < 100. The largest prime number < 100 is 97, and $97^2 = 9409$. |
| **25** | 832 [x = ] | Solve for x: $\log_8 x - \log_8 13 = 2$<br>$\log_8(x/13) = 2$<br>x/13 = 64<br>x = 832 |

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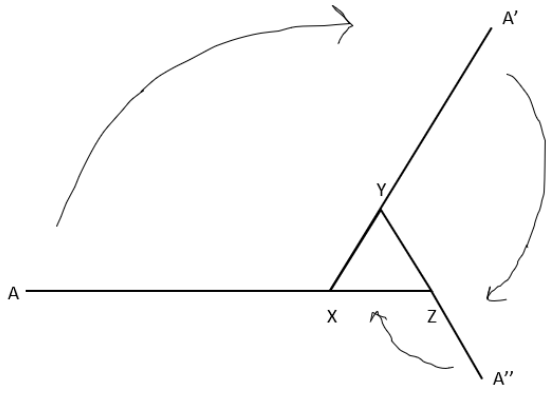
| **26** | 6 [insect legs] | The following sample data represents the number of insect legs that were found in each of five chocolate bars that were sampled at a manufacturing plant. Find the standard deviation of the sample data, in units of insect legs.<br><br>Number of insect legs: 20, 13, 4, 8, 10<br>Mean = 11 legs<br>x | x - mean | (x - mean)^2<br>20 | 9 | 81<br>13 | 2 | 4<br>4 | -7 | 49<br>8 | -3 | 9<br>10 | -1 | 1<br><br>sum = 144<br>Variance = sum/(n – 1) = 144/4 = 36<br>Standard deviation = $36^{1/2}$ = 6 |
| **27** | 42 [= B] | A 9-foot long string, $\overline{AX}$, is attached to vertex X of equilateral triangle XYZ, which has a perimeter of 9 feet. The string is kept taut (pulled straight), and wrapped around the triangle by moving in a circular clockwise direction. The area swept out by the string as it moves around the triangle can be written as Bπ square feet, where B is a positive integer. What is B?<br>The first sweep takes the rope to A'. It has traced out 1/3 of a circle with r = 9. The second sweep takes the rope to A''. It has traced out 1/3 of a circle with r = 6. The last sweep takes the remaining rope to X, and traces out 1/3 of a circle with r = 3.<br><br>The total area swept out is:<br><br>$(1/3)\pi(9^2 + 6^2 + 3^2) = 42\pi$<br>  |

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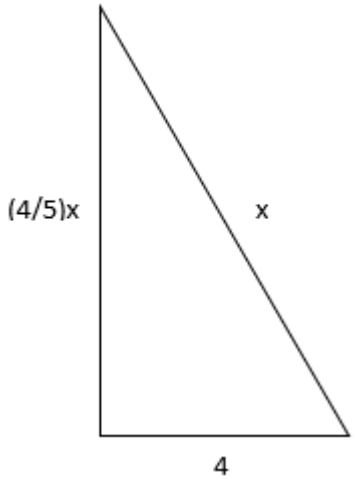
| **28** | 23 | A ladder is resting flat against a wall. The base of the ladder is pulled out 4 feet away from the wall, causing the top of the ladder to slide down the wall a distance equal to one-fifth the length of the ladder. In feet, the length of the ladder can be written as a reduced common fraction A/B. What is A + B?<br>x = ladder length<br>(4/5)x = height on the wall after the ladder has slid down by (1/5)x.<br>$4^2 + [(4/5)x]^2 = x^2$<br>$16 + (16/25)x^2 = x^2$<br>$16 = (9/25)x^2$<br>$x^2 = (25)(16)/9$<br>$x = (5)(4)/3 = 20/3$<br>  |
| **29** | 25 [integers] | The product of the digits in the number 34 is 3x4 = 12. How many positive two-digit integers have an odd product of their digits? Note that a product of 0 is even.<br>e x e = e<br>o x o = o<br>e x o = e<br>Both digits must be odd, i.e. 1, 3, 5, 7 or 9.<br>Therefore, 5x5 = 25 |
| **30** | 21200 | A positive 5-digit integer has the sum of all five of its digits equal to five. The first digit is equal to the number of zeros in the number, the second digit is equal to the number of ones in the number, and so on. What is the number?<br>2 zeros<br>1 one<br>2 twos |

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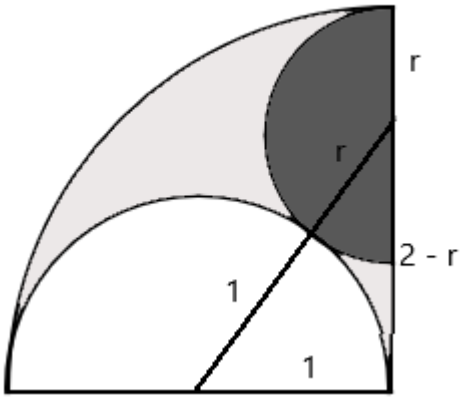
| **31** | 5 [= A + B] | A quarter-circle has two smaller inscribed semi-circles, which are tangent to each other at one point. The radius of the quarter-circle and the diameter of the larger semi-circle (white) equal 2 units. The radius of the smaller semi-circle (dark shaded) can be written as a reduced common fraction A/B. What is A + B?<br>Draw a line connecting the centers of the two semi-circles, which goes through the point of tangency. Let r = radius of smaller semi-circle.<br>$1^2 + (2 - r)^2 = (r + 1)^2$<br>$r^2 - 4r + 4 + 1 = r^2 + 2r + 1$<br>$6r = 4$<br>$r = 2/3$<br>$2 + 3 = 5$<br>  |
| **32** | 4 [black marbles] | If two marbles are removed at random from an urn containing black and white marbles, the probability that they are both white is 1/3. Instead, if three marbles are removed at random, the probability that they are all white is 1/6. How many black marbles are in the urn?<br>W = no. of white, T = total number<br>$P(W\&W) = 1/3 = \frac{W}{T} \cdot \frac{W-1}{T-1}$<br>$P(W\&W\&W) = 1/6 = \frac{W}{T} \cdot \frac{W-1}{T-1} \cdot \frac{W-1}{T-2}$<br>From the problem, we can infer that the probability of getting a 3rd white marble after two white marbles had already been selected is $\frac{1}{2}$, therefore there are 2 more white marbles than black marbles.<br>From this, can use some guess and check to determine that W = 6 and B = 4. |

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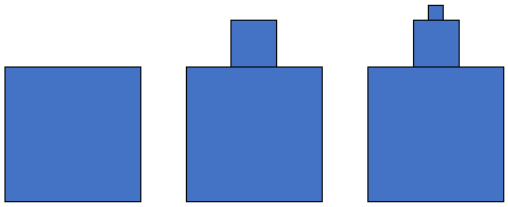
| **33** | 49 [= P + A] | A fractal is created by starting with a square of side length 1 unit. In each iteration, another square with side length equal to 1/3 the previous square's side length is placed on top of the previous square. After infinitely many iterations, the fractal's outer perimeter is P units and its area is A square units. The ratio of the two values can be written as a reduced common fraction P/A. What is P + A?<br>  <br>Iteration 0 Iteration 1 Iteration 2<br>P = 3 + 4(1/3) + 4(1/9) + 4(1/27) + …<br>= 3 + 4(1/3 + 1/9 + 1/27 + …)<br>= 3 + 4(3/2) = 5<br><br>A = 1 + (1/9) + (1/9)^2 + … = 9/8<br><br>P/A = 5/(9/8) = 40/9 |
| **34** | -4 [= $\alpha$] | For the parabola defined by the following equation, the directrix is defined by the line:<br>y = $\alpha$. What is the value of $\alpha$?<br><br>$x^2 - 4x - 12y - 8 = 0$<br><br>Put the equation into standard form of:<br>$(x - h)^2 = 4p(y - k)$<br>$x^2 - 4x - 8 + 12 = 12y + 12$<br>$(x - 2)^2 = 12(y + 1)$<br>Therefore, k = -1 and p = 3<br>The equation for the directrix is: y = k – p<br>y = -1 – 3 = -4 |
| **35** | 19683 | In the following equation, each letter represents a different digit from 0 to 9. What is the value of the number MAGIC?<br>$(M + A + G + I + C)^3 = MAGIC$<br><br>$27^3 = 19683$<br>1+9+6+8+3 = 27<br>As an upper limit, 9+8+7+6+5 = 35<br>As an approximate lower limit, $20^3 = 8000$, which is not big enough. |

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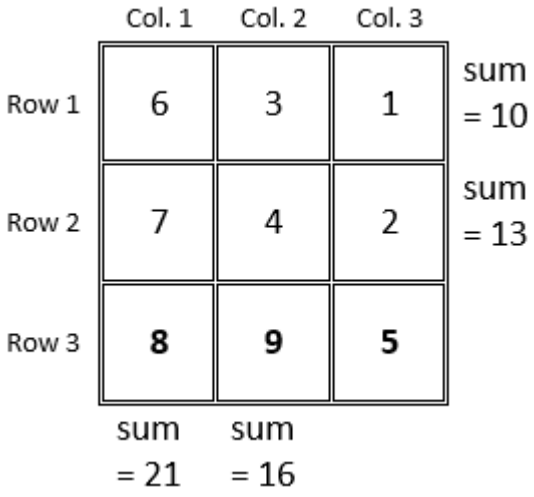
| **36** | 1054 [sq in] | Each dimension of a rectangular prism is an integer number of inches. The volume of the prism is 2023 cubic inches. In square inches, what is the minimum possible surface area of the prism?<br><br>2023 = 7x17x17<br>Various rectangular prisms are possible with integral side lengths, but the one closest to a cube will minimize the surface area. Therefore, a prism with side lengths 7, 17 and 17 is closest to a cube. SA = 2(7x17) + 2(7x17) + 2(17x17) = 1054. |
| **37** | 895 | In the following grid, place each of the digits 1 through 9 into a box, using each digit exactly once, according to the following rules. What 3-digit integer is in Row 3, reading from left to right?<br><br>1. The three digits in the first row sum to 10.<br>2. The three digits in the second row sum to 13.<br>3. The three digits in the first column sum to 21.<br>4. The three digits in the second column sum to 16.<br><br>  |

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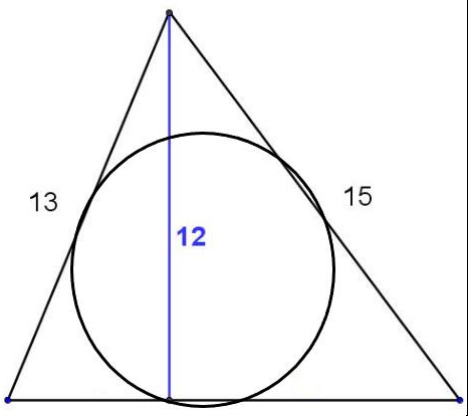
| **38** | 81 [=A + B] | A triangle has sides of length 13 and 15 units, and an altitude of length 12 units, as shown. A circle is constructed that passes through the following 9 points on the triangle:<br>1. The foot of each altitude of the triangle.<br>2. The midpoint of each side of the triangle.<br>3. The midpoint of the line segment from each vertex to the orthocenter of the triangle.<br><br>The radius of this circle can be written as a reduced common fraction A/B. What is A + B?<br><br>  <br><br><br>This "nine-point circle", which is also known as Euler's circle, has a radius equal to ½ of the circumradius of the triangle.<br><br>The circumradius of the triangle is: R = (abc)/(4A), where a, b, and c are the three side lengths, and A is the area.<br><br>Use the Pythagorean Theorem to find that the third side length is 14, because there is a 5-12-13 right triangle on the left, and a 9-12-15 right triangle on the right.<br><br>R = $\frac{(13)(14)(15)}{4\left(\frac{1}{2}\right)(14)(12)} = \frac{65}{8}$<br><br>Therefore, the radius of the nine-point circle is half of that, or 65/16.<br><br>65 + 16 = 81 |

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| **39** | 407 [= A + B] | Three real numbers between 0 and 10 inclusive are randomly selected. As a reduced common fraction, the probability that the sum of the three numbers is 8 or less can be written as A/B. What is A + B?<br><br>0, 10]. The sample space forms a 10x10x10 cube in the 3D coordinate system, with a total volume of 1000. The sums that are 8 or less will be bounded by the points: (8, 0, 0), (0, 8, 0), (0, 0, 8) and (0, 0, 0). These four points, when joined, form a tetrahedron. The volume of the tetrahedron is 1/6 times the determinant:<br>$\begin{vmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{vmatrix}$<br>V = (1/6)(8)(64) = 512/6 = 256/3<br>Therefore, using geometric probability:<br>P = (256/3)/1000 = 256/3000 = 32/375<br>32 + 375 = 407 |
| **40** | -223 | The following polynomial function P(x) has two roots, r and s. Find the value of $r^{10} + s^{10}$.<br><br>$P(x) = x^2 - x + 4$<br><br>From Vieta's Formulas, we have A = sum of roots = -b/a = 1 and B = product of roots = c/a = 4. Use Newton's Identities for a Quadratic to find $P_i = r^i + s^i$.<br><br>$P_1 = A = 1$<br>$P_2 = AP_1 - 2B = 1 - 2(4) = -7$<br>$P_3 = AP_2 - BP_1 = -7 - 4(1) = -11$<br>$P_4 = AP_3 - BP_2 = -11 - 4(-7) = 17$<br>$P_5 = AP_4 - BP_3 = 17 - 4(-11) = 61$<br>$P_6 = AP_5 - BP_4 = 61 - 4(17) = -7$<br>$P_7 = AP_6 - BP_5 = -7 - 4(61) = -251$<br>$P_8 = AP_7 - BP_6 = -251 - 4(-7) = -223$ |

# "Math is Cool" Championships -- 2023-24

## High School

## Multiple Choice Solutions

| 9/ 10th | 11/ 12th | Answer | Solution |
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| **1** | **1** | *C* | A regular pentagon and a regular hexagon can intersect in which of the following?<br><br>A) A ray B) A line C) A line segment D) A rectangle<br>E) None of these.<br><br>**One side from each of the pentagon and hexagon can intersect in a line segment.** |
| **2** | **2** | E<br>(0.857) | What is the next number in the series that begins as follows, where numbers have been rounded off to three decimal places of accuracy:<br><br>0.667, 0.750, 0.800, 0.833, …<br><br>A) 0.841 B) 0.861 C) 0.865 D) 0.873<br>E) Answer not given<br><br>**Pattern is: 2/3, 3/4, 4/5, 5/6, next number is 6/7 = 0.857** |
| **3** | **3** | *A* | Given that x and y are both integers, which of the statements best describes the result of the following expression?<br><br>$(x^2 + x + 9)(2y + 1)$<br><br>A) Odd integer always. B) Even integer always.<br>C) Even integer if x is even.<br>D) Even integer if y is even. E) Answer not given.<br><br>**(2y + 1) will always be odd, because 2y is even.**<br>**$(x^2 + x + 9)$ =**<br>**x(x + 1) + 9**<br>**x(x + 1) will be even, because one of the two numbers must be even. Even + odd = odd.**<br>**odd x odd = odd** |

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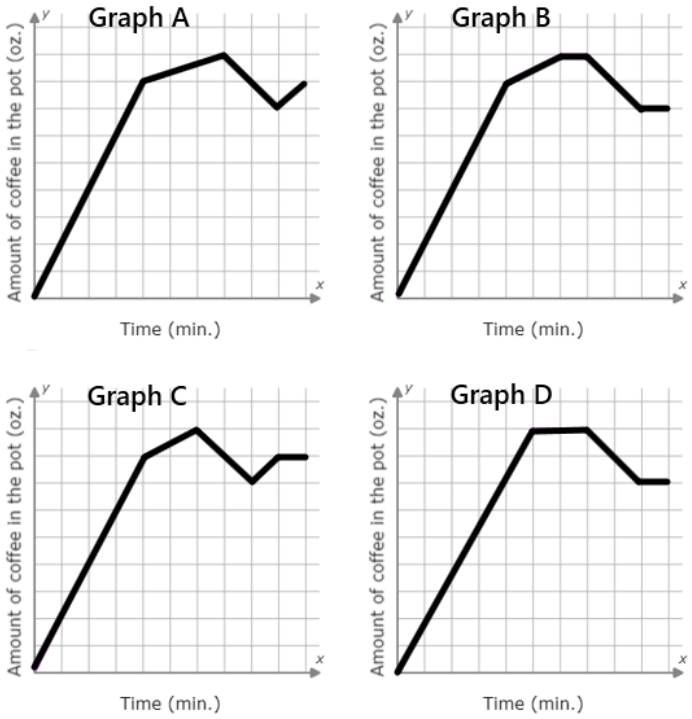
| 4 | | B | Sheldon brews a pot of coffee for his roommate Leonard, who is sad and needs a hot beverage. The pot fills quickly at a constant rate until it is almost full, when it drops to a slower constant rate, then stops. After some time, Sheldon pours a cup of coffee for Leonard, and leaves the rest of the coffee in the pot. Which of the following graphs could show the amount of coffee in the pot as a function of time?<br>  <br><br>A) Graph A B) Graph B C) Graph C<br>D) Graph D E) Answer not given.<br><br>Graph B shows the volume increasing at a steady rapid rate, then increasing at a slower steady rate. At a point, when the coffee stops brewing, the volume remains constant. It then decreases when Sheldon pours a cup, and then remains constant again. |

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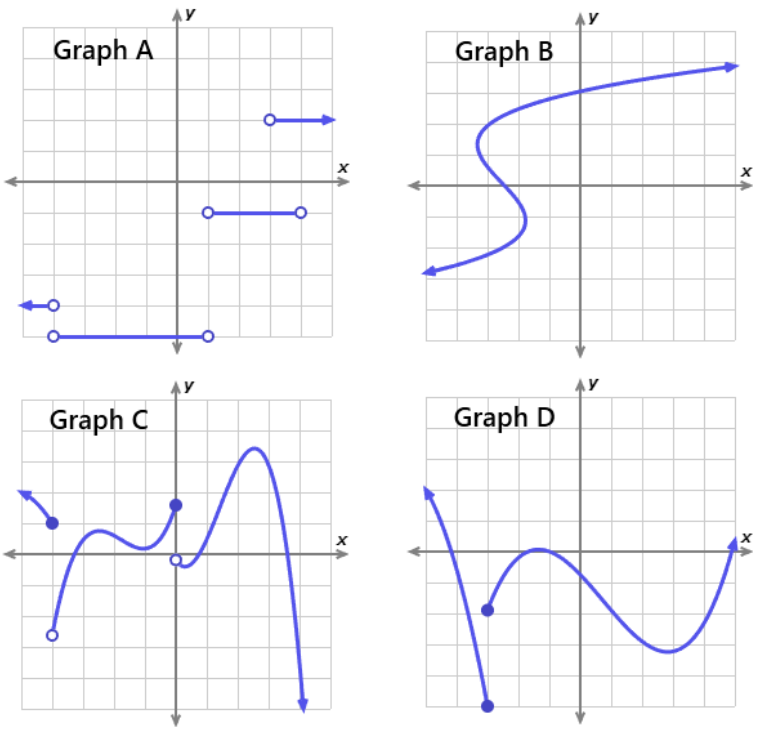
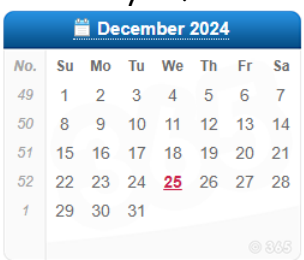
| | 4 | D | Which of the following graphs is the graph of a function?<br>  <br><br>A) A, C and D<br>B) A and C<br>C) B only<br>D) C only<br>E) Answer not given.<br><br>C is the only graph that passes the vertical line test, A, B and D do not. |
| 5 | 5 | B | February 29, 2024 will fall on a Thursday. What day of the week will it be three days after the day before Christmas in 2024, which is celebrated on December 25th?<br><br>A) Thursday B) Friday C) Saturday D) Monday<br>E) Answer not given.<br>Three days after the day before Xmas is Friday 12/27.<br>  <br> |

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| 6 | 6 | D | An urn contains 5 blue marbles and 5 red marbles. Three marbles are randomly selected one at a time, without replacement. What is the probability that the third marble selected is the same color as the first marble selected?<br><br>A) $\frac{1}{2}$ B) 1/3 C) 2/3 D) 4/9 E) Answer not given.<br><br>In other words:<br>P(BBB or BRB) x 2 (if Red was first).<br>P(BBB or BRB) =<br>$\left(\frac{5}{10}\cdot\frac{4}{9}\cdot\frac{3}{8}+\frac{5}{10}\cdot\frac{5}{9}\cdot\frac{4}{8}\right)$<br>= (1/12) + (5/36)<br>= 8/36 = 2/9<br>Multiply by 2 to account for Red being first: 2/9 x 2 = 4/9 |
| 7 | | C | Two numbers sum to 200. What is the minimum possible value of the sum of one of the numbers and the square of the second number?<br><br>A) 199.25 B) 199.5 C) 199.75 D) 200<br>E) Answer not given.<br>$x + y = 200$<br>$y = 200 - x$<br>minimize $x^2 + y = x^2 - x + 200$<br>That is a parabola, and the minimum function value occurs at $x = -b/2a = \frac{1}{2}$<br>$(1/2)^2 - 1/2 + 200 = 199.75$. |

| | | | |
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| | **7** | *C* | Let $f(x) = \frac{2x+4}{5x+2}$<br><br>What are the equations of the horizontal and vertical asymptotes of the function<br>g(x) = f(1 – 2x)?<br><br>A) $x = -\frac{2}{5},\ y = \frac{7}{10}$ B) $x = -\frac{7}{5}, y = \frac{9}{10}$ C) $x = \frac{7}{10}, y = \frac{2}{5}$ D) $x = \frac{9}{10}, y = \frac{7}{5}$<br><br>E) Answer not given.<br>$$g(x) = \frac{2(1-2x)+4}{5(1-2x)+2}$$<br>$$g(x) = \frac{-4x+6}{-10x+7}$$<br><br>The x-asymptote will occur where the denominator = 0, or:<br>-10x + 7 = 0, x = 7/10<br>The y-asymptote will equal the ratio of the leading terms which are both degree 1:<br>y = -4/-10 = 2/5 |
| **8** | **8** | *A* | There are 306 prime numbers less than 2023. What is the sum of the prime numbers less than 2023?<br>A) 283,081 B) 45,865 C) 24,299 D) 327,912 E) 545,634<br><br>All but one of the primes are odd, therefore are adding 305 odd numbers (odd sum) plus 2, so the total sum is odd. Eliminate the even choices. Compute the sum of 1 through 306: (306)(307)/2 = 46971. The sum of the first 306 primes will be greater than this. The only option is (A). |

| | | | |
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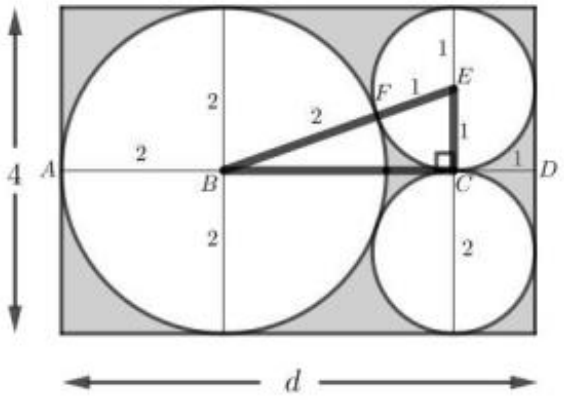
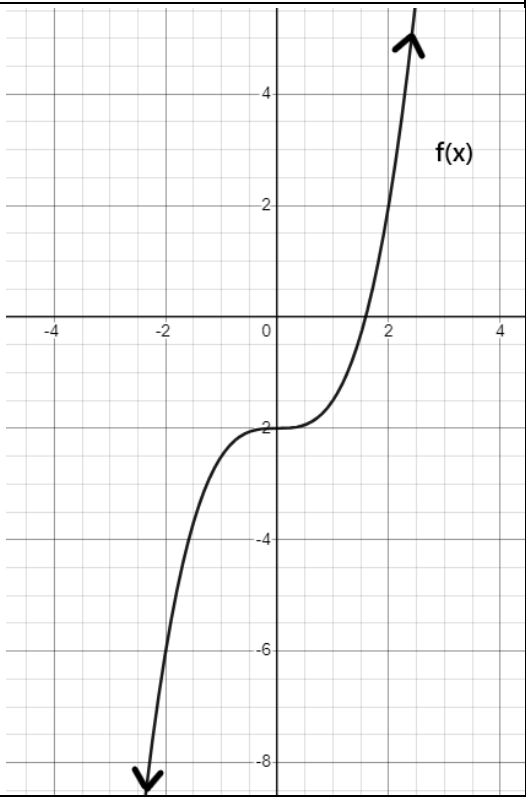
| **9** | | *C* | Three circles are inscribed in a rectangle as shown. The smaller circles each have a radius of 1 unit and are tangent to the large circle, to each other, and to two sides of the rectangle each. The large circle has a radius of 2 units and is tangent to three sides of the rectangle. What is the area, in square units, of the shaded portion of the rectangle?<br><br>A) $12 - 8\sqrt{2}$ B) $8 + 8\sqrt{2} - 6\pi$ C) $12 + 8\sqrt{2} - 6\pi$ D) $16 - 8\sqrt{2} - 6\pi$<br>E) Answer not given.<br>  <br><br><br>The length of BC can be found by creating a right triangle BCE.<br>$BC^2 + CE^2 = BE^2$<br>$BC^2 = 3^2 - 1^1 = 8$<br>$BC = \sqrt{8}$<br>Therefore, the length of the rectangle = $2 + \sqrt{8} + 1 = 3 + \sqrt{8}$<br>The area of the entire rectangle = $4(3 + \sqrt{8}) = 12 + 4\sqrt{8}$<br>Shaded area = Rectangle area – Circles area = $12 + 4\sqrt{8} - \pi(2)^2 - 2\pi(1)^2 = 12 + 4\sqrt{8} - 6\pi$ |

<table>
<tr>
<td></td>
<td>9</td>
<td>A</td>
<td>Which expression is equivalent to the following:<br>$$sin^2\left(\theta - \frac{\pi}{2}\right) + sin^2(-\theta)$$<br>A) 1 B) $2\sin^2(\theta)$ C) $2\sin^2(-\theta)$ D) $2\cos^2(\theta)$<br>E) Answer not given.<br><br>First term:<br>$$sin^2\left(\theta - \frac{\pi}{2}\right) = \left(-sin\left(\frac{\pi}{2} - \theta\right)\right)^2$$<br>$$= sin^2\left(\frac{\pi}{2} - \theta\right) = cos^2(\theta)$$<br>Second term:<br>$$sin^2(-\theta) = \big(-sin(\theta)\big)^2 = sin^2(\theta)$$<br>Substitute:<br>$$cos^2(\theta) + sin^2(\theta) = 1$$</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>A</td>
<td>Mrs. Stephenson is running a carnival game at the Kamiakin High School senior party. There are ten cards, each with a different integer 1 through 10, placed face down on the table. A player randomly turns over three different cards. They win if the smallest of the three numbers is odd, and the next smallest number is even. What is the probability of winning the game?<br><br>A) $\frac{1}{3}$ B) $\frac{29}{120}$ C) $\frac{29}{720}$ D) $\frac{13}{720}$ E) Answer not given.<br><br>The order doesn't matter, so there are 10C3 = 120 total ways to select three cards. Out of these, list the possible ways to win:<br>
<table>
<tr><th>Smallest Number</th><th>Next Smallest Number</th><th>Possible Value(s) for Largest Number</th><th>Number of Selections of Three Cards</th></tr>
<tr><td rowspan="4">1</td><td>2</td><td>3, 4, 5, 6, 7, 8, 9, 10</td><td>8</td></tr>
<tr><td>4</td><td>5, 6, 7, 8, 9, 10</td><td>6</td></tr>
<tr><td>6</td><td>7, 8, 9, 10</td><td>4</td></tr>
<tr><td>8</td><td>9, 10</td><td>2</td></tr>
<tr><td rowspan="3">3</td><td>4</td><td>5, 6, 7, 8, 9, 10</td><td>6</td></tr>
<tr><td>6</td><td>7, 8, 9, 10</td><td>4</td></tr>
<tr><td>8</td><td>9, 10</td><td>2</td></tr>
<tr><td rowspan="2">5</td><td>6</td><td>7, 8, 9, 10</td><td>4</td></tr>
<tr><td>8</td><td>9, 10</td><td>2</td></tr>
<tr><td>7</td><td>8</td><td>9, 10</td><td>2</td></tr>
</table>
There are 40 total ways to win, so: P(win) = 40/120 = 1/3</td>
</tr>
</table>

# "Math is Cool" Championships -- 2023-24

## High School

## Team Test Solutions

| 9/ 10th | 11/ 12th | Answer | Solution |
|---|---|---|---|
| **1** | **1** | 7 | Find the x-coordinate of the x-intercept of the following line:<br><br>$y = \frac{2}{7}x - 2$<br>Set y = 0:<br>$0 = \frac{2}{7}x - 2$<br>$2 = \frac{2}{7}x$<br>x = 7 |
| **2** | | 5 [units] | On the coordinate plane, find the distance in units between the points (6, 5) and (10, 2).<br>It forms a 3-4-5 triangle, or can use the distance formula. |
| | **2** | -2 | The function f(x) shown here has an inverse function $f^{-1}(x)$. What is the value of $f^{-1}(-6)$?<br><br>f(x) contains the point (-2, -6).<br>Therefore, $f^{-1}(x)$ contains the point (-6, -2), therefore $f^{-1}(-6) = -2$.<br>  |

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| **3** | **3** | 15 [minutes] | How many minutes before 10 AM is it now, if 90 minutes later it will be the same number of minutes after 11 AM?<br><br>If it is 9:45 AM, then it is 15 minutes before 10 AM. Also, 90 minutes later after 9:45 AM is 11:15 AM, which is the same number of minutes (15) after 11 AM. |
| **4** | **4** | 75 [%] | Two cards will be selected from a standard deck of 52 playing cards, with replacement. As a percentage, what is the probability that the two cards are from different suits?<br><br>The first card can be any suit. The second card must be a different suit, or $\frac{3}{4}$, which equals 75%. |
| **5** | | 4045 | What is the 2023rd odd natural number?<br><br>Odd numbers = 2n – 1, therefore 2(2023) – 1 = 4045 |
| | **5** | -5 | Find the limit: $\lim_{x\to -3} \frac{x^2+x-6}{x+3}$<br>Simplify by factoring:<br>$\frac{x^2+x-6}{x+3} = x - 2$ for x ≠ -3<br>The graph is a line with a hole at x = -3.<br>f(-3) = -3 – 2 = -5 |
| **6** | **6** | 257 | Certain prime numbers can be written in the form: $N^2 + 1$, where N = 1, 2, 3, … For example, the two smallest prime numbers that can be written like this are 2 (= $1^2 + 1$) and 5 (= $2^2 + 1$). What is the value of the seventh smallest prime number that can be written in the form $N^2 + 1$?<br><br>$1^2 + 1 = 2$<br>$2^2 + 1 = 5$<br>$4^2 + 1 = 17$<br>$6^2 + 1 = 37$<br>$10^2 + 1 = 101$<br>$14^2 + 1 = 197$<br>$16^2 + 1 = 257$ |

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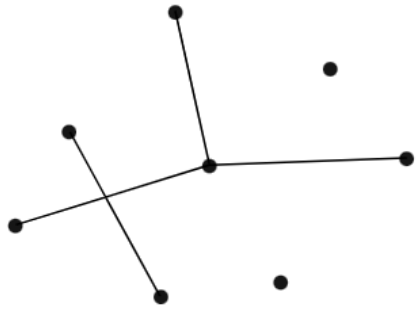
| **7** | **7** | 24 [lines] | In the following diagram, assume that no three points are collinear. How many additional lines are needed to connect each point to every other point in the diagram?<br>  <br>There are 8 points, so a total of 8C2 = 28 lines are needed. 4 of them have already been drawn. 28 – 4 = 24. |
| **8** | | 53 [cm] | A right triangle has legs a and b, with a < b, and hypotenuse c. The area of the triangle is 630 $cm^2$ and the perimeter is 126 cm. In centimeters, what is the length of the hypotenuse, c?<br><br>$\frac{1}{2}$ ab = 630 ➔ ab = 1260<br>a + b + c = 126<br>$a^2 + b^2 = c^2$<br>a + b = 126 – c<br>$(a + b)^2 = (126 - c)^2$<br>$a^2 + 2ab + b^2 = 15876 - 252c + c^2$<br>252c = 13356<br>c = 53 |

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|---|---|---|---|
| | **8** | 28 [cm] | A right triangle has legs a and b, with a < b, and hypotenuse c. The area of the triangle is 630 $cm^2$ and the perimeter is 126 cm. In centimeters, what is the length of the shorter leg, a?<br><br>$\frac{1}{2}$ ab = 630 ➔ ab = 1260<br>a + b + c = 126<br>$a^2 + b^2 = c^2$<br>a + b = 126 – c<br>$(a + b)^2 = (126 - c)^2$<br>$a^2 + 2ab + b^2 = 15876 - 252c + c^2$<br>252c = 13356<br>c = 53<br>a + b + 53 = 126<br>a + b = 73<br>a + 1260/a = 73<br>$a^2 + 1260 = 73a$<br>$a^2 - 73a + 1260 = 0$<br>(a – 28)(a – 45) = 0<br>a = 28, a = 45<br>a is the shortest leg, so 28 is the correct solution. |

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| **9** | **9** | 1011 | The positive integers are written in an array according to the following pattern. The first number in each row (in Column 1) indicates how many numbers are in that row. Each entry can be identified by its unique Row and Column number. Find the sum of the Row and Column number for the location of 2023.<br><br>Notice that all entries in Column 1 are powers of two, therefore, 2023 will be in Row 11 which has a first number of 1024, and a last number of 2047 which is in column 1024. Count backwards from 2047 to 2023 to discover that 2023 is in column 1000.<br><br>1000 + 11 = 1011 |

| | Col. 1 | Col. 2 | Col. 3 | Col. 4 | Col. 5 | Col. 6 | Col. 7 | Col. 8 | … |
|---|---|---|---|---|---|---|---|---|---|
| Row 1: | 1 | | | | | | | | |
| Row 2: | 2 | 3 | | | | | | | |
| Row 3: | 4 | 5 | 6 | 7 | | | | | |
| Row 4: | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | |
| … | … | | | | | | | | |

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|---|---|---|---|
| **10** | | 113 | A 10th positive integer, A, when added to the following set, results in the median of the set equaling the mean of the set. What is the sum of all possible values of A?<br>{13, 15, 21, 28, 33, 38, 42, 48, 51}<br><br>The only number from 28 to 38, when inserted into the set, that makes the mean equal the median is 31.<br>13,15,21,28,28,33,38,42,48,51, mean = 31.7 & median = 30.5<br>13,15,21,28,29,33,38,42,48,51, mean = 31.8 & median = 31<br>13,15,21,28,30,33,38,42,48,51, mean = 31.9 & median = 31.5<br>13,15,21,28,31,33,38,42,48,51, mean = 32 & median = 32<br>13,15,21,28,32,33,38,42,48,51, mean = 32.1 & median = 32.5<br>13,15,21,28,33,33,38,42,48,51, mean = 32.2 & median = 33<br>13,15,21,28,33,34,38,42,48,51, mean = 32.3 & median = 33.5<br>13,15,21,28,33,35,38,42,48,51, mean = 32.4 & median = 34<br>13,15,21,28,33,36,38,42,48,51, mean = 32.5 & median = 34.5<br>13,15,21,28,33,37,38,42,48,51, mean = 32.6 & median = 35<br>13,15,21,28,33,38,38,42,48,51, mean = 32.7 & median = 35.5<br>There is one integer less than 28 and one integer greater than 38 that will cause the median and the mean to be equal. When the 10th number is less than 28, the median would be 30.5, which requires the sum of the 10 numbers to be 305. The tenth number would be 305 – (sum of original 9 numbers) = 305 – 289 = 16<br>When the 10th number is greater than 38, then median would be 35.5, which requires the sum of the 10 numbers to be 355. The tenth number in this case is 355 – 289 = 66.<br>A = 16 + 31 + 66 = 113 |
| | **10** | 100 [%] | Fifteen distinct whole numbers are randomly selected from 1 to 100, inclusive. What is the probability in percent that at least two pairs of these numbers will have the same positive difference?<br><br>There are 15C2 = 105 ways to form pairs with the 15 numbers. However, since the numbers are from 1 to 100, there are only 99 possible differences. Therefore, it is certain that at least 2 pairs of the numbers will have the same difference. |

# "Math is Cool" Championships -- 2023-24

## High School

## Pressure Round Solutions

| 9/ 10th | 11/ 12th | Answer | Solution |
|---|---|---|---|
| 1 | 1 | 790 | The following terms are the first four 3-digit positive integers in an arithmetic sequence, where A, B, and C each represent a single digit from 1 to 9. As a 3-digit integer, what is the fifth term of the sequence?<br><br>AB4, B03, B3C, BA1<br><br>Can conclude that either d must end in 9, i.e. d = 9, 19, 29, … The common difference d = 29. A = 6, B = 7, C = 2. The terms are:<br>674, 703, 732, 761, therefore the $5^{th}$ term is 761 + 29 = 790. |
| 2 | | 700 | Evaluate: $\frac{1}{4}(64^2 - 36^2)$<br>(64 – 36)(64 + 36) = (28)(100) = 2800<br>2800/4 = 700 |
| | 2 | 280 | Find the sum of the positive integral factors of 108.<br>$108 = 2^2 \times 3^3$<br>Sum = $(2^0+2^1+2^2)(3^0+3^1+3^2+3^3)$<br>= $(7)(40) = 280$ |
| 3 | 3 | 217 [integers] | How many integers between 0 and 1000 contain at least one digit '4' but do not contain the digit '5'?<br>1 digit: only 1 (4)<br>2 digits: __4, or 4__, or 44<br>7 + 8 + 1 = 16<br>3 digits: _ _ 4, or _ 4 _, or<br>4 _ _, or 444<br>56 + 56 + 64 + 1 = 200<br>Total = 200 + 16 + 1 = 217 |

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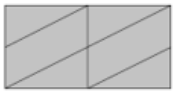
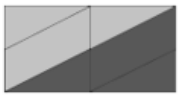
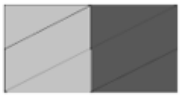
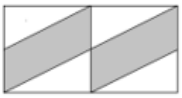
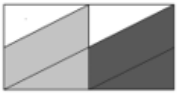
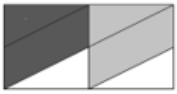
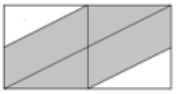
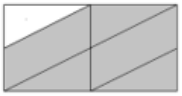
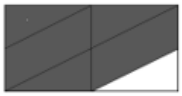
|---|---|---|---|
| **4** | **4** | 20 [convex polygons] | How many total convex polygons are included in the following figure?<br>1 large:  <br>4 small triangles:  <br>2 big triangles:  <br>2 squares:  <br>2 parallelograms:  <br>2 trapezoids:  <br>2 trapezoids:  <br>1 hexagon:  <br>2 trapezoids:  <br>1 pentagon:  <br>1 pentagon:  |
| **5** | | 5 | Two integers x and y are defined as follows. What is the greatest common factor of x and y?<br>$x = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$<br>$y = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$<br>x = 55 = 5x11<br>'y' does not have a factor of 11, so the GCF = 5. |
| | **5** | 14 | Solve for x:<br>$\log_x 2744 = 3$<br>Equivalent to: $2744 = x^3$<br>$10^3 = 1000$, too small<br>$20^3 = 8000$, too big<br>$14^3 = 2744$ |

# "Math is Cool" Championships -- 2023-24

## High School

## College Bowl Round #1 Solutions

| | Answer | Solution |
|---|---|---|
| **1** | 11978 | What is the product of one hundred thirteen and one hundred six?<br>113x106 = 11978 |
| **2** | 64 [k = ] | The surface area of a sphere with radius 4 units is equal to k times pi square units. What is the value of k?<br>SA = $4\pi r^2 = 4\pi(4)^2 = 64\pi$ |
| **3** | 5 [= median] | What is the median of the positive integral factors of twenty-four?<br>Factors are:<br>1, 2, 3, 4, 6, 8, 12, 24<br>An even number, so the median is the mean of the middle 2. |
| **4** | 1 [palindrome] | When the integers from 10 to 20 inclusive are squared, how many of the resulting square numbers are palindromes, where a palindrome is a number that reads the same forwards and backwards.<br>$11^2 = 121$ is the only resulting palindrome. |
| **5** | 4 [combinations] | Juanita paid for a fifteen cent stamp with a one dollar bill, and received nine coins in change, consisting of quarters, dime, nickels and or pennies. How many different combinations of coins could she have received?<br>QQQNPPPPP<br>QQNNNNNNN<br>QDDDDNNNN<br>DDDDDDDDN |
| **6** | 125 | The first three terms of a sequence are one, two and three. Each subsequent term is the sum of the three previous terms. What is the ninth term in the sequence?<br>1, 2, 3, 6, 11, 20, 37, 68, 125 |
| **7** | 20 [%] | What is the probability in percent that a randomly selected positive multiple of three is also a multiple of five?<br>Multiples of 3:<br>3, 6, 9, 12, 15,<br>18, 21, 24, 27, 30<br>Every $5^{th}$ multiple of 3 is also a multiple of 5. |

| | | |
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| **8** | 17 [puppies] | A zebra and five puppies together weigh the same as two tigers. A zebra weighs the same as twenty-nine puppies. How many puppies weigh the same as one tiger?<br>Z + 5P = 2T<br>Z = 29P<br>34P = 2T<br>T = 17P |
| **9** | 13 | The midpoint of line segment JK is at eight comma six. Endpoint J is at seven comma eight. What is the sum of the coordinates x comma y at point K?<br>J (7, 8)<br>Midpoint (8, 6)<br>K(x = 8*2 – 7, y = 6*2 – 8)<br>K(9, 4) 9+4 = 13 |
| **10** | -9 | Evaluate the following expression when x equals negative six: The quantity negative x plus three divided by the quantity negative x minus seven.<br>$\frac{-x+3}{-x-7} = \frac{6+3}{6-7} = \frac{9}{-1}$ |

# "Math is Cool" Championships -- 2023-24

## High School

## College Bowl Round #2 Solutions

| | Answer | Solution |
|---|---|---|
| **1** | -10 | A line contains the points negative four comma three and three comma negative two. What is the value of the slope of the line times fourteen?<br>Points (-4, 3) and (3, -2)<br>Slope = (-2 – 3)/(3 - -4) = -5/7<br>14*(-5/7) = -10 |
| **2** | 2116 | What is forty-six squared?<br>46x46 = 2116 |
| **3** | 73 [m&ms] | There are two hundred m&ms in a bag. Parker eats one m&m the first day, two m&ms the second day, four m&ms the third day, and continues to double how many he eats each day. After he has eaten the m&ms on the seventh day, how many m&ms remain in the bag?<br>He eats: 1+2+4+8+16+32+64=127<br>200 – 127 = 73 |
| **4** | 6 [integers] | How many integers are there between the square root of eight and the square root of eighty?<br>$\sqrt{8} < \sqrt{9}$ which equals 3<br>$\sqrt{80} < \sqrt{81}$ which equals 9<br>Therefore, the integers between are 3, 4, 5, 6, 7 and 8 |
| **5** | 11 [= A + B] | A quarter, a dime and a nickel are flipped. The probability that at least two coins show heads and one of them is the quarter can be written as a reduced common fraction A over B. What is A plus B?<br>There are 2x2x2 = 8 total outcomes.<br>There are 3 outcomes with at least 2 heads where one of them is the quarter: QD, QN, QDN<br>P = 3/8 |
| **6** | 6 | What is the last digit of two raised to the two thousand twenty-three times three raised to the two thousand twenty-three?<br>Powers of 2 starting with $2^1$ end in 2, 4, 8, 6, then repeat.<br>Powers of 3 starting with $3^1$ end in 3, 9, 7, 1, then repeat.<br>Since the cycle is 4, find that 2023/4 has a remainder of 3. Therefore, $2^{2023}$ will end in 8, and $3^{2023}$ will end in 7. 8x7 = 56, so the last digit is 6. |

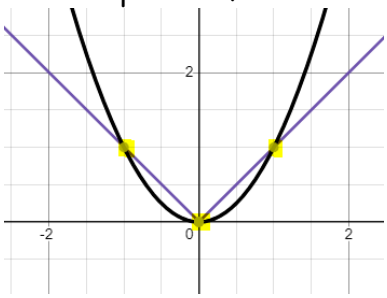
| | | |
|---|---|---|
| **7** | 9 [ways] | In how many ways can two standard six-sided dice be rolled such that the sum of the numbers showing on the dice is divisible by four?<br>Sums of 4, 8 and 12 are divisible by 4:<br>(1, 3), (2, 2), (3, 1)<br>(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)<br>(6, 6) |
| **8** | 60 [=A] | In simplified radical form, the value of the square root of the quantity 2 times three times four times five times six times seven times 10 can be written as A times the square root of B. What is A?<br>$\sqrt{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 10}$<br>=<br>$\sqrt{2 \cdot 3 \cdot 2 \cdot 2 \cdot 5 \cdot 2 \cdot 3 \cdot 7 \cdot 2 \cdot 5}$<br>=<br>$2 \cdot 2 \cdot 3 \cdot 5\sqrt{2 \cdot 7}$<br>=<br>$60\sqrt{14}$ |
| **9** | 96 [°] | An angle measures forty-two degrees. In degrees, what is the measure of its complementary angle times two?<br>Complementary angle = 90 – 42 = 48.<br>2x48 = 96 |
| **10** | 8 | The expression eight y times y to the fifth divided by y to the negative two can be written as eight times y raised to what power?<br>$\frac{8y \cdot y^5}{y^{-2}} = 8y^1 \cdot y^5 \cdot y^2 = 8y^8$ |

# "Math is Cool" Championships -- 2023-24

## High School

## College Bowl Round #3 Solutions

| | Answer | Solution |
|---|---|---|
| **1** | -1 [= x] | What is the largest integer such that the following statement is true: three x minus four is less than negative five.<br>$3x - 4 < -5$<br>$3x < -1$<br>$x < -1/3$<br>Therefore x = -1 is the largest integer that makes it true. |
| **2** | 22 [%] | Four point four is what percent of twenty?<br>$4.4 = 20x$<br>$x = 4.4/20 = 0.22 = 22\%$ |
| **3** | 57 | Find the value of one hundred times the range of the following set of numbers:<br>zero point two, zero point six, zero point zero eight, zero point five, zero point zero three.<br>In order: 0.03, 0.08, 0.2, 0.5, 0.6<br>Range = 0.6 – 0.03 = 0.57<br>100x0.57 = 57 |
| **4** | 2520 | What is the smallest positive integer that is divisible by each of the integers one through nine?<br>1x2x3x4x5x6x7x8x9<br>It needs to have factors of each of these numbers, but can get rid of the repeats. 2x3 = 6, so get rid of that. 2x4 = 8, so get rid of that. Get rid of one factor of 3. Left with:<br>1x2x3x4x5x7x3 = 2520 |
| **5** | 1967 | The first Super Bowl game was played on January fifteenth of a particular year. No digit in the year is an eight. The hundreds digit of the year is three more than the tens digit. The sum of the digits in the year is twenty-three. What year was the first Super Bowl played?<br>Probably already know that it was 19__ __.<br>9 is three more than 6, so: 196__<br>1+9+6+7 = 23 |
| **6** | 1276 | What is the sum of the terms in the finite arithmetic series: two plus five plus eight plus eleven plus dot dot dot plus eighty-three plus eighty-six?<br>$S_n = n(a1+an)/2$<br>$= 29(2+86)/2 = 1276$ |

| | | |
|---|---|---|
| **7** | 25 [%] | There are eight identical urns. Each one contains a different combination of quarters, dimes and nickels that total exactly forty-five cents. If an urn is randomly selected, what is the probability in percent that it contains at least three dimes?<br>There are only two ways to get 45¢ using 3 or more dimes:<br>DDDDN<br>DDDNNN<br>2/8 = ¼ = 25% |
| **8** | 2112 | The current year is two thousand twenty-three. What is the next year that is a palindrome?<br>2112 is the next palindrome. |
| **9** | 360 [°] | In degrees, what is the sum of the exterior angles of a regular pentagon?<br>A property of polygons is that the sum of the exterior angles of any polygon is 360°. |
| **10** | 3 [points] | When the graphs of y equals the absolute value of x and y equals x squared are plotted on the same coordinate plane, how many distinct points of intersection exist?<br>  |

# "Math is Cool" Championships -- 2023-24

## High School

## College Bowl Round #4 Solutions

| | Answer | Solution |
|---|---|---|
| **1** | 80 [= LCM] | What is the least common multiple of sixteen and twenty? |
| **2** | 31 [=A + B] | The reciprocal of the number zero point two four can be written as a reduced fraction A over B. What is A plus B? |
| **3** | -23 | Find the next term in the sequence that begins:<br>negative forty-three, negative forty-one, negative thirty-seven, negative thirty-one, and so on. |
| **4** | 72 | One-half of a number plus two-thirds of the same number equals 42. What is two times the number? |
| **5** | 20 [posts] | A fence is being built around a triangular garden that is twenty feet by twenty feet by ten feet. One post is placed at each vertex, and the posts are placed so that their centers are thirty inches apart. How many total posts are needed? |
| **6** | 25 [%] | A four-digit positive integer is made by randomly ordering the digits one, two, three and four, using each one exactly once. What is the probability in percent that a randomly selected integer from this list is divisible by four? |
| **7** | 720 [tiles] | How many square tiles that are four inches on a side will be needed to completely cover a rectangular patio that measures eight feet by ten feet? |
| **8** | 2 [minutes per month] | A doctor's office tracked the average waiting time each month. In March, the average waiting time was twenty minutes, and in July the average waiting time was twenty-eight minutes. What was the average rate of change in the waiting time between March and July in minutes per month? |
| **9** | 60 [°] | What is the measure of acute angle x, in degrees, if cosine of x equals one-half? |
| **10** | 15 | Find the greatest common factor of the following monomials:<br>fifteen x, forty-five y squared, and 30 z cubed |

# "Math is Cool" Championships -- 2023-24

## High School

## College Bowl Round #5 Solutions

| | Answer | Solution |
|---|---|---|
| **1** | 6600 | What is the product of eleven, twenty-four and twenty-five?<br>11x24x25 = 6600 |
| **2** | 10101 [base 4] | Four to the fourth power plus four squared plus four to the zero power equals what number in base four?<br>There's no need to calculate the number in base 10, because this is given as the definition of a base 4 number. |
| **3** | 1 [= x + y] | On the coordinate plane, the point negative five comma two is reflected over the x-axis, then shifted two units to the right, then reflected over the y-axis to the point with coordinates x comma y. What is x plus y?<br>Start: (-5, 2)<br>Reflect over x-axis: (-5, -2)<br>Shift 2 units right: (-3, -2)<br>Reflect over y-axis: (3, -2)<br>3 + (-2) = 1 |
| **4** | 8 | Fifty percent more than what number is twenty-five percent less than sixty percent more than ten?<br>60% more than 10 = 16<br>25% less than 16 = 12<br>50% more than 8 = 12 |
| **5** | 640 [acres] | Two miles of fence is needed to enclose a square plot of one hundred sixty acres. How many acres can be enclosed in a square plot using four miles of fence?<br>The area scale factor is 4 when the sides of a square are doubled. |
| **6** | 1365 | What is the sum of the terms in the finite geometric series:<br>one plus four plus sixteen plus dot dot dot plus one thousand twenty-four?<br>1+4+16+64+256+1024 = 1365<br>or: $S_6 = (1)(1-4^6)/(1-4)$<br>= -4095/-3 = 1365 |
| **7** | 40 [%] | A sock drawer contains two black socks and three white socks. If two socks are randomly removed from the drawer without replacement, what is the probability in percent that they are the same color?<br>P(BB or WW)<br>= (2/5)(1/4)+(3/5)(2/4) = 8/20<br>= 40/100 = 40% |

| | | |
|---|---|---|
| **8** | 128 [sq units] | What is the maximum area in square units of a rectangle with a diagonal length of sixteen units?<br>A square will always give the greatest area. If the side of the square = x, then<br>$x^2 + x^2 = 16^2$<br>$2x^2 = 256$<br>$x^2$ = area = 128. |
| **9** | 7 [units] | On the coordinate plane, how many units away from the origin is the center of the circle described by the following equation?<br>x-squared plus y-squared minus 14x plus forty-five equals zero.<br>$x^2 + y^2 – 14x + 45 = 0$<br>$x^2 – 14x + 49 + y^2 = 49 – 45$<br>$(x – 7)^2 + y^2 = 4$<br>Center (7, 0) |
| **10** | -6 | The range of the function y equals x-squared minus two x minus five can be written as y is greater than or equal to q. What is the value of q?<br>$y = x^2 – 2x – 5$<br>Vertex is at x = -b/2a = 2/2 = 1<br>$y = 1^2 – 2(1) – 5 = -6$ |

# "Math is Cool" Championships -- 2023-24

## High School

## College Bowl Round #6 Solutions

| | Answer | Solution |
|---|---|---|
| **1** | 210 [grams] | If one gram equals zero point zero four ounces, then how many grams are in eight point four ounces?<br>1 g / 0.04 oz = ? g / 8.4 oz<br>? = 8.4/0.04 = 210 |
| **2** | 98 | If x equals five and y equals three, find the value of the quantity x minus y times the quantity x-squared plus xy plus y-squared.<br>$(x - y)(x^2 + xy + y^2) = x^3 - y^3$<br>$5^3 - 3^3 = 125 - 27 = 98$ |
| **3** | 7 [units] | An acute triangle has integer side lengths of 2 units, 7 units and x units. In units, what is the largest possible value of x?<br>Sides of 7-7-2 will give an acute isosceles triangle. Sides of 8-7-2 will give an obtuse triangle. By the Pythagorean Inequality Theorem, $7^2 < 7^2 + 2^2$ which makes it acute and $8^2 > 7^2 + 2^2$ which makes it obtuse. |
| **4** | 115 [cents] | Letters are worth the following amounts: A is one cent, B is two cents, C is three cents, all the way up to Z is twenty-six cents. In cents, what would be the value of the phrase 'MATH IS COOL'?<br>M 13<br>A 1<br>T 20<br>H 8<br>I 9<br>S 19<br>C 3<br>O 15<br>O 15<br>L 12 |
| **5** | 41 | How many cards need to be drawn from a standard fifty-two card deck without replacement to make sure that at least two of them are hearts?<br>There are 13 cards of each suit. Worst case scenario is if you draw all 13 diamonds, all 13 spades, and all 13 clubs. Then you need 2 more to make sure there are 2 hearts. 13+13+13+2 = 41 |

| | | |
|---|---|---|
| **6** | 36 | The first three terms in an arithmetic sequence are x, 2x plus eleven, and 4x minus three. What is the common difference between consecutive terms in this sequence?<br>$x + d = 2x + 11$<br>$2x + 11 + d = 4x - 3$<br>Solve for d = 36 |
| **7** | 20 [%] | If one integer from one to twenty inclusive is randomly selected, what is the probability in percent that it's spelling in the English language begins with a vowel?<br>20 total integers<br>4 of them begin with a vowel: one, eight, eleven, eighteen<br>4/20 = 20/100 = 20% |
| **8** | 409 [base 10] | As a base 10 number, what is the value of:<br>three one one four base 5?<br>$3x5^3 + 1x5^2 + 1x5^1 + 4x5^0 = 409$ |
| **9** | 42 [square units] | What is the area, in square units, of triangle ABC, with vertices at negative two comma seven, five comma two, and negative two comma negative five?<br>A = (1/2)bh = (1/2)(12)(7) = 42 |
| **10** | -28 | If x-squared equals the quantity x plus fifty-six squared, what is the value of x?<br>$x^2 = (x+56)^2$<br>$x = \pm(x + 56)$<br>(1) $x = x + 56$, not valid<br>(2) $x = -x - 56$<br>$2x = -56$<br>$x = -28$ |

# "Math is Cool" Championships -- 2023-24

## High School

## College Bowl Extra Questions Solutions

| | Answer | Solution |
|---|---|---|
| **1** | 7400 | What is the value of seven point four one two times ten to the third power as an integer rounded to the nearest hundred?<br>$7.412 \times 10^3 = 7412$<br>Rounded to the nearest hundred = 7400. |
| **2** | 4 [points] | If the point eight comma nine is the center of a circle of radius ten units, at how many points does the circle intersect the coordinate axes?<br>Since the radius is greater than the distance from either axis, the circle will intersect each axis in the points. |
| **3** | -2 [= x] | Solve for x in the following equation:<br>three times the quantity seven plus 2x equals thirty plus seven times the quantity x minus 1.<br>$3(7 + 2x) = 30 + 7(x - 1)$<br>$21 + 6x = 30 + 7x - 7$<br>$-2 = x$ |
| **4** | 10,000 | What number is one percent of one million?<br>$1000000(0.01) = 10,000$ |
| **5** | 22 | Find the mean of the following data set:<br>thirteen, thirty-two, twenty-five, twenty-seven, and thirteen<br>$13+32+25+27+13 = 110$<br>$110/5 = 22$ |
| **6** | 3 [=x] | Solve the following equation for x:<br>log base three of the quantity 2x plus three equals two<br>$\log_3(2x + 3) = 2$<br>$2x + 3 = 3^2 = 9$<br>$2x = 6$<br>$x = 3$ |