"Math is Cool" Championships -- 2023-24

7th/8th Grade

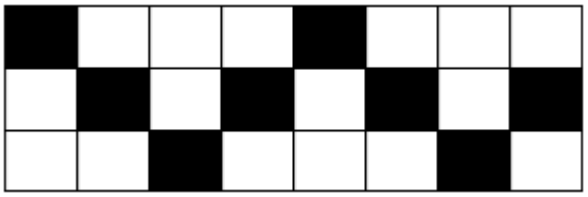
Mental Math Solutions

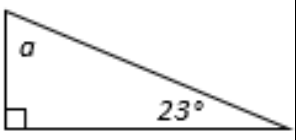
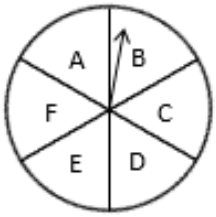
	Answer	Solution
1	40	What is eight plus nine plus twenty-three? $8 + 9 + 23 = 40$
2	3 [integers]	How many of the single-digit integers one through nine are spelled with three letters in English? One, two, and six make 3 single-digit integers
3	360	What is forty percent of nine hundred? $0.4 \cdot 900 = 360$
4	22 [days]	How many days are in three and one-seventh weeks? $3\frac{1}{7} \cdot 7 = \frac{22}{7} \cdot 7 = 22$
5	11	The first three numbers in a geometric series are two hundred ninety-seven, ninety-nine, and thirty-three. What is the fourth number in the series? $297 \cdot 1/3 = 99$, $99 \cdot 1/3 = 33$, $33 \cdot 1/3 = 11$
6	9779	What is the largest four-digit palindrome in which the hundreds and tens digits are prime numbers and the ones and thousands digits are composite numbers? 9779
7	11	A set of three integers has a mean of thirty-five. What integer can be added to the set so that the mean of the four integers is now twenty-nine? $3 \cdot 35 = 105$, $4 \cdot 29 = 116$, and $116 - 105 = 11$
8	152 [cubes]	A six-by-six-by-six cube has two-by-two-by-two cubes removed from each of its corners. How many one-by-one-by-one cubes can the remaining portion be divided into? $6 \cdot 6 \cdot 6 = 216$ and $2 \cdot 2 \cdot 2 = 8$ and there are 8 corners, so $216 - 8 \cdot 8 = 152$

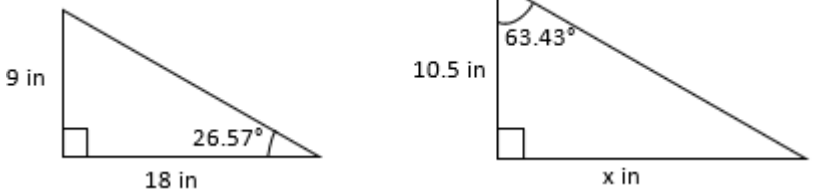
"Math is Cool" Championships -- 2023-24

7th/8th Grade

Individual Test Solutions

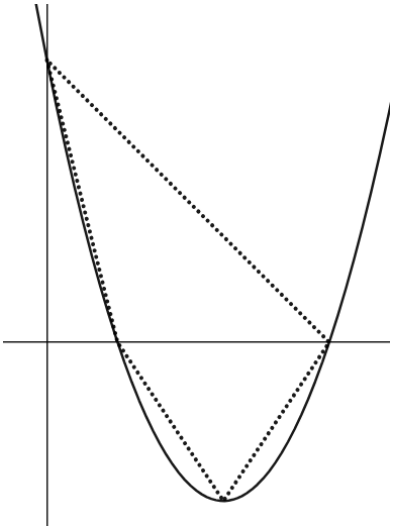
	Answer	Solution
1	2218	Evaluate: 20 + 200 + 2000 – 2 20 + 200 + 2000 – 2 = 2218
2	54 [inches]	How many inches are in 4.5 feet? 4.5 * 12 = 54
3	540 [miles]	A car drives at an average rate of 60 miles per hour for 9 hours. How far in miles does the car drive? 60 * 9 = 540
4	[x =] 17	Solve for x: 22x + 50 = 424 22x + 50 = 424 → 22x = 374 → x = 17
5	8 [squares]	How many shaded squares are in the figure, which is composed of unit squares? There is one shaded square in each of the 8 columns, so 8 shaded squares total. 
6	4	Four friends take turns rolling a pair of dice. Ayush rolls first. Benjamin does not roll last. Chenrui rolls before Benjamin and Dhriti. Who rolls last? Answer as an integer only: 1 = Ayush, 2 = Benjamin, 3 = Chenrui, or 4 = Dhriti. The order is Ayush, Chenrui, Benjamin, Dhriti, so the answer is 4
7	12 [meters]	A rectangle has an area of 336 square meters and a length of 28 meters. In meters, what is the width of the rectangle? 336/28 = 12
8	989	A palindrome is a number that reads the same forwards as backwards, like 1221. What is the largest 3-digit palindrome in which the tens digit is different from the ones digit? The largest 3-digit palindrome is 999, but all the digits are the same. The next largest one is 989.
9	85 [%]	What percent of 20 is 17? 17/20 = x/100 → x = 85
10	213	The first three terms of an arithmetic sequence are 15, 37, 59, . . . What is the tenth term in the sequence? The common difference is 22 so the tenth term will be 15 + 9(22) = 213

11	[A + B =] 6	During each 5-day work week, Tina works from home four days and drives to the office on the fifth day. Assuming the day she drives to work is chosen at random, the probability during any given week that she will drive on Tuesday is a reduced common fraction A/B. What is A + B? The probability that he will drive on Tuesday is 1/5, and 1 + 5 = 6.
12	2	What is the median of the data set? 22/7, 256, $\frac{1+\sqrt{4}}{2}$, 2, 1 When written in order the list is 1, $\frac{1+\sqrt{4}}{2}$ = 1.5, 2, 22/7 ≈ 3.14, 256, and the median is the middle number, which is 2.
13	1980	Evaluate: $4 \cdot 11 \cdot 5 \cdot 9$ $4 \cdot 11 \cdot 5 \cdot 9 = 1980$
14	[x =] 21	Solve for x: $9 = \frac{2x}{3} - 5$ $9 = \frac{2x}{3} - 5 \rightarrow 14 = \frac{2x}{3} \rightarrow 42 = 2x \rightarrow x = 21$
15	36 [horses]	The ratio of donkeys to horses on a farm is 3:4. If the total number of donkeys plus the total number of horses on the farm equals 63, how many horses are on the farm? If donkeys:horses = 3:4, then horses:total = 4/7, so 4/7 = x/63, and x = 36.
16	10 [minutes]	If it takes ten people ten minutes to build ten card houses, each working at the same rate, how long in minutes does it take one person to build one card house? 10 people : 10 minutes : 10 card houses is equivalent to 10 people : 1 minute : 1 card house is equivalent to 1 person : 10 minutes : 1 card house, so the answer is 10
17	[A + B =] 17	The mean of all positive one-digit perfect squares is a reduced common fraction A/B. What is A + B? (1 + 4 + 9)/3 = 14/3 and 14 + 3 = 17
18	[a =] 67 [°]	What is the value of angle a in degrees? 180 – 90 – 23 = 67 
19	[A + B =] 27	As a reduced common fraction, the probability of drawing a red 8 from a standard deck of cards is A/B. What is A + B? There are 2 red eights in a standard deck so the probability of drawing a red 8 is 2/52 = 1/26, and 1 + 26 = 27
20	30	What number must be added to the data set to make the mean of the new data set equal to 50? 30, 100, 40 50*4 = 200 and 200 – 30 – 40 – 100 = 30
21	[P + Q =] 10	A circular spinner is divided into 6 congruent sectors, each of which is labeled as shown. As a reduced common fraction, the probability of spinning two vowels in a row is P/Q. What is the value of P + Q? 2/6 * 2/6 = 4/36 = 1/9, and 1 + 9 = 10 

22	46 [times]	A book has pages 5 through 267 and every page is numbered. How many times does the digit 8 appear in the page numbers of this book? When 8 is in the tens place it occurs 20 times, 80, 81, . . . , 88, 89, and 180, 181, . . . , 188, 189. When 8 is in the ones place it occurs 26 times, 8, 18, 28, . . . , 238, 248, 258. The answer is 20 + 26 = 46
23	5	Evaluate the expression: $\frac{(15-9)-11^2}{55-13\cdot 6}$ = ? $\frac{(15-9)-11^2}{55-13\cdot 6}$ = -115/-23 = 5
24	-104	What is the sum of a + b + c in the following equation? $(7x - 15)(5x + 8) = ax^2 + bx + c$ $(7x - 15)(5x + 8) = 35x^2 - 19x - 120$, and 35 + -19 + -120 = -104
25	[x =] 21 [in]	What is the value of x in the diagram? The triangles must be similar since 26.57 + 63.43 = 90, so 9/18 = 10.5/x, and x = 21. 
26	[B – A =] 27	Let A = 35% of 90 and let B = the number that is 35% less than 90. What is the value of B – A? A = 35% of 90 is 31.5. B = 35% less than 90, which is 65% of 90, which is 58.5. B – A = 58.5 – 31.5 = 27
27	39 [plums]	Six friends each picked a different integral number of plums from a tree. Ben picked 12 more than Juana, who picked the fewest. Fang picked 12 fewer than Mohamed, who picked the most. Clarissa picked half as many as Adeniyi, who picked more than Fang. Clarissa also picked fewer than Ben, who picked 30 plums. What is the lowest possible number of plums that Mohamed could have picked? Juana – 18, Ben – 30, Clarissa – 19, Adeniyi – 38, Mohamed – 39, Fang – 27.
28	72	If A = 30/x, then what is the sum of the positive integer values of x, such that A is an integer? 30 is evenly divisible by 1, 2, 3, 5, 6, 10, 15, and 30, so the answer is 1 + 2 + 3 + 5 + 6 + 10 + 15 + 30 = 72
29	43560 [ft^2]	A unit of land equal to 4840 square yards is called an acre. How many square feet are in 1 acre? One square yard = 9 square feet, so 4840 · 9 = 43560.
30	[A + B =] 13	The first three terms of a geometric sequence are 12500/288, 2500/144, 500/72, . . . In this sequence, the first term whose value is less than 1, as a reduced common fraction is A/B. What is the value of A + B? The common ratio in the sequence is 2/5, so the sequence continues as follows: 12500/288, 2500/144, 500/72, 100/36, 20/18, 4/9, so the answer is 4 + 9 = 13
31	[x =] 7	Solve for x: $\frac{1}{1+\frac{1}{1+\frac{1}{x}}} = \frac{8}{15}$ $\frac{1}{1+\frac{1}{1+\frac{1}{x}}} = \frac{8}{15} \rightarrow \frac{1}{1+\frac{1}{\frac{x}{x}+\frac{1}{x}}} = \frac{8}{15} \rightarrow \frac{1}{1+\frac{1}{\frac{x+1}{x}}} = \frac{8}{15} \rightarrow \frac{1}{1+\frac{x}{x+1}} = \frac{8}{15} \rightarrow \frac{1}{\frac{x+1}{x+1}+\frac{x}{x+1}} = \frac{8}{15} \rightarrow \frac{1}{\frac{2x+1}{x+1}} = \frac{8}{15} \rightarrow \frac{x+1}{2x+1} = \frac{8}{15} \rightarrow 15x + 15 = 16x + 8 \rightarrow x = 7$

<table>
<tr>
<td>32</td>
<td>[A + B =]
17</td>
<td>In a survey of 90 people, each person indicates whether they like or do not like hot dogs and whether they like or do not like hamburgers. The results include that 54 people like hot dogs, 63 people like hamburgers, and 39 people like both hot dogs and hamburgers. As a reduced common fraction, the probability that a person in the survey likes neither hot dogs nor hamburgers is A/B. What is A + B?
If you organize the information into a two-way table it looks like this:
<table>
<tr><td></td><td>Likes Hot Dogs</td><td>Doesn't like Hot Dogs</td><td>Total</td></tr>
<tr><td>Likes Hamburgers</td><td>39</td><td>24</td><td>63</td></tr>
<tr><td>Doesn't like Hamburgers</td><td>15</td><td>12</td><td>27</td></tr>
<tr><td>Total</td><td>54</td><td>36</td><td>90</td></tr>
</table>
So, the probability that a person likes neither is 12/90 = 2/15, and 2 + 15 = 17</td>
</tr>
<tr>
<td>33</td>
<td>119</td>
<td>There are three distinct ordered pairs of integers (a_1, b_1), (a_2, b_2) and (a_3, b_3) that are solutions to the following equation. What is the combined sum of $a_1 + a_2 + a_3 + b_1 + b_2 + b_3$?
$\sqrt{320} = a\sqrt{b}$
$\sqrt{320} = \sqrt{4 \cdot 80} = 2\sqrt{80}$
$\sqrt{320} = \sqrt{16 \cdot 20} = 4\sqrt{20}$
$\sqrt{320} = \sqrt{64 \cdot 5} = 8\sqrt{5}$
2 + 4 + 8 + 80 + 20 + 5 = 119</td>
</tr>
<tr>
<td>34</td>
<td>60
[scoops]</td>
<td>A cylindrical container of ice cream has a radius of 12 centimeters and a height of 15 centimeters. How many spherical scoops of ice cream can be formed from the contents of the container if the radius of each sphere is 3 centimeters? The ice cream can be 'smushed' and reformed into spheres.
Volume of the container = 144π*15 = 2160π
Volume of each scoop = $4\pi 3^3/3 = 36\pi$
2160π/36π = 60</td>
</tr>
<tr>
<td>35</td>
<td>[P + Q =]
2099</td>
<td>Let A equal the product of a positive single-digit prime number, a two-digit integer that is equal to a power of 3, and a two-digit composite number. As a reduced common fraction, the quotient of the largest possible value of A divided by the smallest possible value of A equals P/Q. What is P + Q?
The largest single-digit prime number is 7, the largest two-digit integer equal to a power of 3 is 81, and the largest two-digit composite number is 99.
The smallest single-digit prime number is 2, the smallest two-digit integer equal to a power of 3 is 27, and the smallest two-digit composite number is 10
The quotient of the largest value of A divided by the smallest value of A = (7·81·99)/(2·27·10) = (7·3·99)/20 = 2079/20, and 2079 + 20 = 2099</td>
</tr>
</table>

36	240 [ways]	How many ways are there to arrange the letters in the word COOKIE, if no two adjacent letters are the same? If there were no restrictions, the number of arrangements would be 6!/2! = 360. Determine the number of arrangements in which the Os are together and subtract these from 360. When the Os are together the remaining 4 letters can be arranged 4! = 24 ways. There are 5 ways the Os can be together: OO_ _ _ _ _ OO _ _ _ _ _ OO _ _ _ _ _ OO _ _ _ _ _ OO So, there are 5 · 24 = 120 arrangements with the Os together and the final answer is 360 – 120 = 240.
37	2 [mph]	Sienna rides her bike on the Burke-Gilman trail for 12 miles from the University of Washington to Bothell with the wind and back again against the wind in a total of 2.5 hours. Her average speed when there is no wind is 10 miles per hour. Assuming the wind speed remains constant throughout the ride, and in miles per hour, what is the wind speed? For the UW to Bothell part, $12 = (10 + w)t_1 \rightarrow t_1 = 12/(10 + w)$, with w = wind speed in mph and t_1 = time from UW to Bothell in hours. For the Bothell to UW part, $12 = (10 – w)t_2 \rightarrow t_2 = 12/(10 – w)$, with t_2 = time from Bothel to UW in hours. Then $t_1 + t_2 = 2.5$ or $12/(10 + w) + 12/(10 – w) = 2.5 \rightarrow 12(10 – w) + 12(10 + w) = 2.5(10 – w)(10 + w) \rightarrow 120 – 12w + 120 + 12w = 2.5(100 – w^2) \rightarrow 240 = 250 – 2.5w^2 \rightarrow 2.5w^2 = 10 \rightarrow w^2 = 4 \rightarrow w = 2$
38	[P + Q =] 105	Three 6-member data sets are represented by A, B, and C. The mean of data set A, the median of data set B, and the unique mode of data set C are each 25. The 18 members of the three sets consist of two 25s in set C and an additional 16 distinct positive one- or two-digit integer values. As a reduced common fraction, the largest possible mean of these 16 distinct integers is P/Q. What is P + Q? In set A, the six integers must add up to 150 and the six integers can be large, small, or medium, as long as they add up to 150. A possible set A could be: 18, 19, 20, 30, 31, 32 In set B, three integers need to be smaller than 25 and three need to be larger than 25. The largest possibility for these integers would be for set B to be: 22, 23, 24, 26, 98, 99 In set C there are two 30s and the other four are any four integers that are not the same as each other or any of the integers in sets A or B. So, set C could be: 25, 25, 94, 95, 96, 97 So, the largest mean of the remaining 16 integers is (150 + 22 + 23 + 24 + 26 + 94 + 95 + 96 + 97 + 98 + 99)/16 = 824/16 = 206/4 = 103/2, and 103 + 2 = 105

39	[A + B =] 107	The vertex, the two x-intercepts and the y-intercept of the parabola whose equation is $f(x) = 2x^2 - 5x + 2$ are connected with segments to form a quadrilateral, as shown below. As a reduced common fraction, the area of this quadrilateral is A/B units^2. What is A + B? $0 = 2x^2 - 5x + 2 = (2x - 1)(x - 2)$, so the x-intercepts are (1/2, 0) and (2, 0). The x-value of the vertex is (1/2 + 2)/2 = 5/4, and the y-value of the vertex is $f(5/4) = 2(5/4)^2 - 5(5/4) + 2 = 50/16 - 100/16 + 32/16 = -18/16 = -9/8$. The y-intercept is (0, 2). If you divide the quadrilateral into two triangles, the area of the top triangle is 1.5(2)/2 = 1.5 and the area of the bottom triangle is 1.5(9/8)/2 = (3/4)(9/8) = 27/32 and 1.5 + 27/32 = 48/32 + 27/32 = 75/32 and 75 + 32 = 107. 
40	1330 [points]	Find the number of integer grid points (x, y) in the region between the graphs of $y = x^2 + 1$ and y = 100, including those lying on $y = x^2 + 1$ and y = 100. For example, (2, 12) is one integer grid point that lies between the two graphs. From: (1, 2) ➔ (1, 100) = 99 points (2, 5) ➔ (2, 100) = 96 points (3, 10) ➔ (3, 100) = 91 points (4, 17) ➔ (4, 100) = 84 points (5, 26) ➔ (5, 100) = 75 points (6, 37) ➔ (6, 100) = 64 points (7, 50) ➔ (7, 100) = 51 points (8, 65) ➔ (8, 100) = 36 points (9, 82) ➔ (9, 100) = 19 points sum = 615 Multiply by 2 to account for negative side. On the y-axis: (0, 1) ➔ (0, 100) = 100 points 615x2 + 100 = 1330

"Math is Cool" Championships -- 2023-24

7th/8th Grade

Multiple Choice Solutions

<table>
<tr><th></th><th>Answer</th><th>Solution</th></tr>
<tr><td colspan="3">USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #1 THROUGH #3.

In a survey of 10826 adults in the United States given in September 2023, people responded to the following question: "Within the next 10 years, how likely do you think it is that artificial intelligence (AI) will enable people to have conversations with other species of animals?"

The five answer choices are shown in the table below. The percent of the whole survey population along with the percent of 4 different age groups to respond with each answer choice are shown as well.

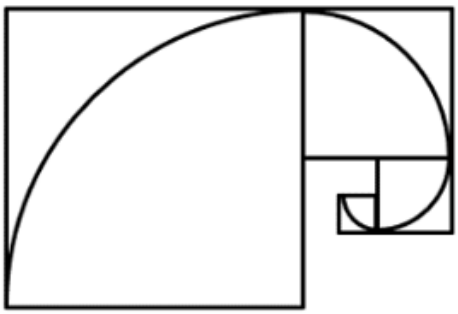
<table>
<tr><td></td><td>All</td><td>18 – 29 years old</td><td>30 – 44 years old</td><td>45 – 64 years old</td><td>65+ years old</td></tr>
<tr><td>Very likely</td><td>13%</td><td>22%</td><td>19%</td><td>7%</td><td>4%</td></tr>
<tr><td>Somewhat likely</td><td>24%</td><td>33%</td><td>28%</td><td>20%</td><td>16%</td></tr>
<tr><td>Not very likely</td><td>24%</td><td>21%</td><td>20%</td><td>25%</td><td>30%</td></tr>
<tr><td>Not likely at all</td><td>20%</td><td>10%</td><td>17%</td><td>24%</td><td>28%</td></tr>
<tr><td>Not sure</td><td>19%</td><td>14%</td><td>16%</td><td>24%</td><td>22%</td></tr>
</table>
</td></tr>
<tr><td>1</td><td>B</td><td>Based on the survey, which age group is most likely to think that AI will make it possible to have conversations with animals of other species?

A) All B) 18 – 29 C) 30 – 44 D) 45 – 64 E) 65+
55% of the 18 – 29 age group said very likely or somewhat likely, more than any other group.</td></tr>
<tr><td>2</td><td>C</td><td>Based on the survey, about how many persons responded that it is "Very likely" that AI will make it possible to have conversations with animals of other species?
A) 141 B) 1400 C) 1407 D) 14000 E) 14074
$0.13 \cdot 10826 = 13 \cdot 108.26 = 10 \cdot 108.26 + 3 \cdot 108.26 = 1082.6 + 324.78 = 1407.38 \approx 1407$</td></tr>
</table>

3	D	If 50% of the respondents were in the 18 – 29 age group, then based on the survey what is the approximate difference in the number of respondents who responded with "Very likely" or "Somewhat likely" versus those who responded with "Not very likely" or "Not likely at all"? A) 130 B) 1191 C) 1245 D) 1299 E) 2598 $(0.22 + 0.33) \cdot (1/2) \cdot 10826 - (0.21 + 0.10) \cdot (1/2) \cdot 10826 = (0.55 - 0.31) \cdot 5413 = 0.24 \cdot 5413 = 24 \cdot 54.13 = 20 \cdot 54.13 + 4 \cdot 54.13 = 1082.6 + 216.52 = 1299.12 \approx 1299$
USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #4 THROUGH #6. A sock drawer has 32 socks in it that all feel identical and are randomly mixed together. Altogether, there are 3 pairs of white socks, 5 pairs of brown socks, 4 pairs of black socks, 2 pairs of green socks, 1 pair of polka dot socks and 1 pair of striped socks.		
4	B	If you close your eyes and pull socks one by one from the drawer, how many socks must you pull to ensure that you have at least one pair that match? A) 6 B) 7 C) 8 D) 10 E) 15 Since there are 6 sock types, it's possible to get one of each on the first 6 socks taken from the drawer. In that scenario, the 7^{th} sock has to match one of the first 6.
5	A	If two socks are randomly pulled from the drawer without replacement, what is the probability that they are both brown socks? A) 45/496 B) 45/248 C) 45/124 D) 5/248 E) 5/124 $10/32 \cdot 9/31 = 45/496$
6	C	If two socks are randomly pulled from the drawer without replacement, what is the probability that they will not be a matching pair? A) 47/248 B) 95/496 C) 25/31 D) 87/496 E) 49/248 P(not a match) = 1 – P(match) $P(\text{match}) = 10/32 \cdot 9/31 + 8/32 \cdot 7/31 + 6/32 \cdot 5/31 + 4/32 \cdot 3/31 + 2/32 \cdot 1/31 + 2/32 \cdot 1/31 = (90 + 56 + 30 + 12 + 2 + 2)/(32 \cdot 31) = 192/992 = 48/248 = 6/31$ and $1 - 6/31 = 25/31$

USE THE FOLLOWING INFORMATION TO SOLVE PROBLEMS #7 THROUGH #10.

In the figure shown there are four squares. The perimeter of the smallest square is 8 centimeters and is half of the perimeter of the next smallest square, whose perimeter is half the perimeter of the next smallest square, and so on. Each square also has a quarter-circle inscribed in it.



7	E	What is the perimeter of the second-largest square? A) 12 cm B) 16 cm C) 20 cm D) 24 cm E) 32 cm $8 \cdot 2 \cdot 2 = 32$
8	D	What is the circumference of the quarter-circle inscribed in the smallest square? A) π/4 cm B) π/2 cm C) 2 cm D) π cm E) 3π/2 cm The perimeter of the smallest square is 8 cm, so the side length is 2 cm and this is the radius of the quarter-circle. Therefore, the circumference of the whole circle is 4π, making the length of the quarter-circle π.
9	B	Let A equal the combined area of the four quarter-circles in the diagram and let B equal the combined area of the region inside the squares but not including the four quarter-circles. What is the ratio A/B? A) 4 B) π/(4 – π) C) π/(2 – π) D) 1/340 E) Answer not given $A = \pi + 4\pi + 16\pi + 64\pi = 85\pi$ $B = (4 + 16 + 64 + 256) - 85\pi = 340 - 85\pi$ $A/B = 85\pi/(340 - 85\pi) = \pi/(4 - \pi)$

10 D

If the figure above consisted of only the smallest square, the square has a perimeter of 8 centimeters. If the figure consisted of only the two smallest squares, then the outer perimeter of the two squares (solid lines) can be determined in centimeters.

If the figure given above is expanded, following the same pattern, to include a total of 8 squares, what will the outer perimeter of the diagram be?

A) 576 cm B) 586 cm C) 764 cm D) 1532 cm
E) 3068 cm

Begin by investigating the pattern of outer perimeters, based on how many squares are included in the figure.

# squares	Perimeter (cm)	Difference between two terms
1	8	
		$+12 = 12x2^0$
2	20	
		$+24 = 12x2^1$
3	44	
		$+48 = 12x2^2$
4	92	
		$+96 = 12x2^3$
5	188	
		+192
6	380	
		+384
7	576	
		+768
8	1344	

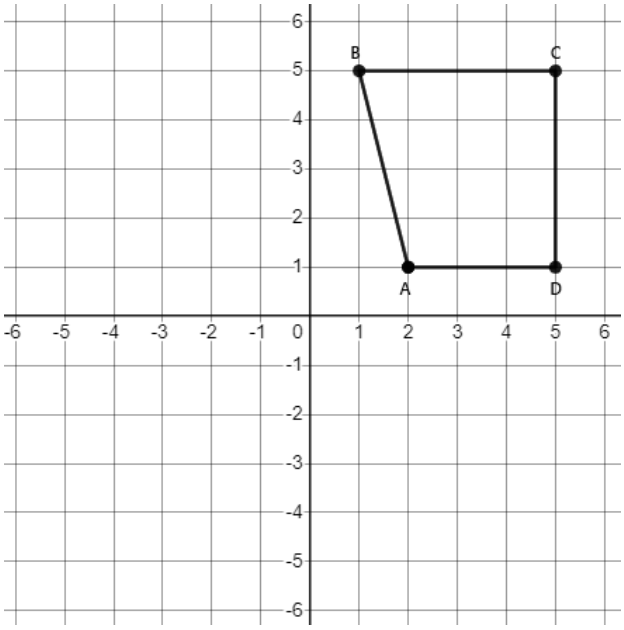
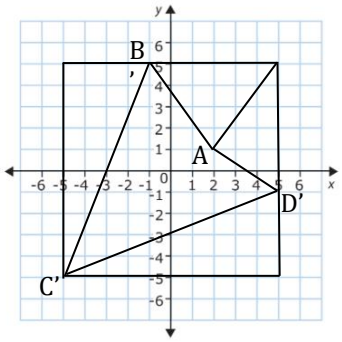
A pattern emerges, which can be used to calculate the perimeter of the figure when the number of squares equals 8.
The answer is D) 1344 cm.

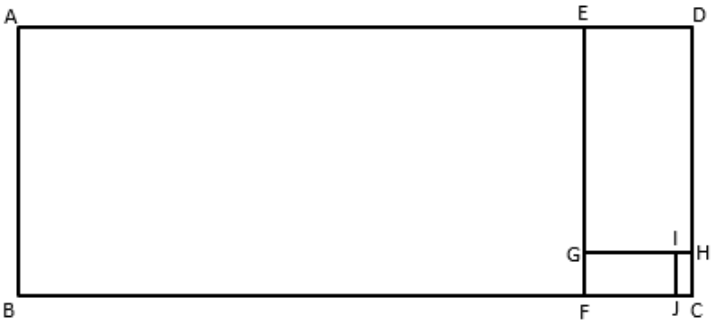
"Math is Cool" Championships -- 2023-24

7th/8th Grade

Team Test Solutions

	Answer	Solution
1	13	What is the largest factor of 91, other than 91? Factors of 91: 1, 7, 13, 91
2	[B =] 7	Let A, B, and C represent three distinct single-digit integers. If A > C > B, then what is the largest possible value of B? A = 9, C = 8, B = 7
3	[A + B =] 10	When rolling two standard 6-sided dice, as a reduced common fraction, the probability of rolling two numbers whose product is 12 is A/B. What is A + B? (2, 6), (3, 4), (4, 3), (6, 2) are the four outcomes with a product of 12, so P(12) = 4/36 = 1/9, and 1 + 9 = 10
4	9 [values]	Tula has four pennies, three nickels, two dimes, and one quarter. How many different cent-values can any two of these coins have? For example, two pennies have a cent-value of 2 cents. 2 pennies = 2 cents, 2 nickels = 10 cents, 2 dimes = 20 cents, 1 penny & 1 nickel = 6 cents, 1 penny & 1 dime = 11 cents, 1 penny & 1 quarter = 26 cents, 1 nickel & 1 dime = 15 cents, 1 nickel & 1 quarter = 30 cents, 1 dime & 1 quarter = 35 cents, for a total of 9 cent-values
5	161	Athena bowls a score of 132, Blaze bowls a score of 164, and Cody bowls a score of 143. What score does Delaney need to bowl in order for the group to have a mean score of 150? 4(150) – 132 – 164 – 143 = 600 – 439 = 161
6	30 [minutes]	Armelle can wax a pair of skis in 10 minutes. Ansel takes 50% longer to wax the same-size pair of skis. How many minutes would it take them working together to wax 5 of the same-size pairs of skis? Armelle can wax 3 pairs in 30 minutes, while Ansel can wax 2 pairs in 30 minutes, so together they can wax 5 pairs in 30 minutes.
7	120 [%]	A concert ticket is purchased at 90% of its advertised price. It is then resold at 108% of its advertised price. What percent of the purchase price is the resale price? If x = advertised price, p = purchase price, and r = resale price, then p = 0.9x, r = 1.08x, and r/p = 1.08x/0.9x = 6/5 = 120%.

8	39 [units^2]	On the graph shown below, reflect B over the y-axis to get B', reflect D over the x-axis to get D', and rotate C 180° about the origin to get C'. In square units, what is the area of quadrilateral AB'C'D'?  The area of the surrounding rectangle is $10 \cdot 10 = 100$ and the areas of the four triangles that are inside the surrounding rectangle and outside of quadrilateral A B'C'D' are $(10 \cdot 4)/2 = 20$, $(6 \cdot 4)/2 = 12$, $(6 \cdot 3)/2 = 9$, and $(10 \cdot 4)/2 = 20$. The area of quadrilateral AB'C'D' is $100 - 20 - 12 - 9 - 20 = 39$. 

<table>
<tr>
<td>9</td>
<td>105 [cm]</td>
<td>In the figure below rectangles ABCD, DEFC, GFCH, and HIJC are all similar, HI = 32 mm, and IJ = 80 mm. What is the length of AE in centimeters?
 

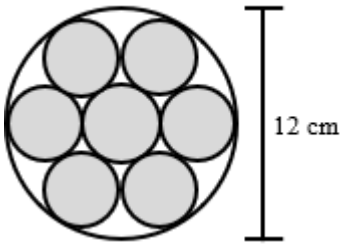
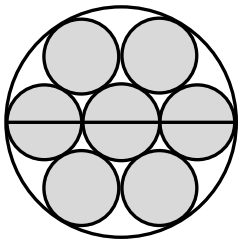
Since the rectangles are similar, HI/HC = HC/FC = FC/DC = DC/AD = 2/5. FC then equals $5/2 \cdot 80 = 200$, DC = $5/2 \cdot 200 = 500$, and AD = $5/2 \cdot 500 = 1250$. AE = AD – FC = 1250 – 200 = 1050 mm = 105 cm.</td>
</tr>
<tr>
<td>10</td>
<td>16192</td>
<td>A sequence consists of 2024 terms. Each term after the first term is 1 greater than the previous term. The sum of the 2024 terms is 33,396. Determine the sum of the terms in the odd-numbered positions in the sequence. For example, starting with the first term plus the 3rd term and so on, and ending with the second-to-last term.
The sequence can be thought of like this:
<table>
<tr><td>n</td><td>a_n</td></tr>
<tr><td>1</td><td>a_1</td></tr>
<tr><td>2</td><td>$a_2 = a_1 + 1$</td></tr>
<tr><td>3</td><td>$a_3 = a_1 + 2$</td></tr>
<tr><td>…</td><td>…</td></tr>
<tr><td>2023</td><td>$a_{2023} = a_1 + 2022$</td></tr>
<tr><td>2024</td><td>$a_{2024} = a_1 + 2023$</td></tr>
</table>
Therefore, the sum of the sequence is: $2024 \cdot a_1 + (1 + 2 + \ldots + 2022 + 2023)$
$1 + 2 + \ldots + 2022 + 2023 = (2023)(2024)/2 = 2{,}047{,}276$
Therefore, $2024 \cdot a_1 + 2{,}047{,}276 = 33{,}396$
$2024 \cdot a_1 = -2{,}031{,}880$
$\cdot a_1 = -995$, is the first term in the sequence of all 2024 numbers
What we want is just the sequence of odd numbered terms:
<table>
<tr><td>n</td><td>a_n</td></tr>
<tr><td>1</td><td>-995</td></tr>
<tr><td>2</td><td>-993</td></tr>
<tr><td>3</td><td>-991</td></tr>
<tr><td>…</td><td>…</td></tr>
<tr><td>1012</td><td>$= a_1 + 2022 =$ 1027</td></tr>
</table>
Sum = $1012(-995 + 1027)/2 =$ 16,192</td>
</tr>
</table>

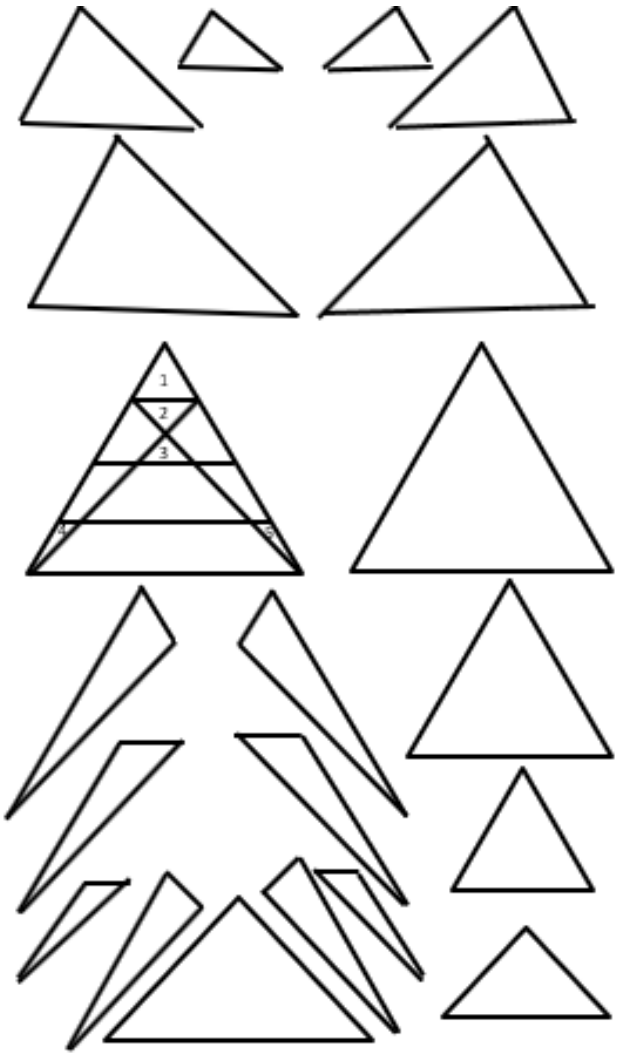
"Math is Cool" Championships -- 2023-24

7th/8th Grade

Linda Moore Triple Jump Solutions

	Answer	Solution
1	7500 [lbs]	One ton is equal to 2000 pounds. How many pounds are in $3\frac{3}{4}$ tons? $3\frac{3}{4} = \frac{15}{4}$ and $\frac{15}{4} \cdot 2000 = 7500$
2	11 [sandcastles]	Josias takes 10 minutes to build one sandcastle. Ezra can build 6 sandcastles in 72 minutes. Working together, how many sandcastles can Josias and Ezra build in one hour? Josias can build 6 in one hour and Ezra can build 5 in one hour, so they can build a total of 11 when working together.
3	540	What is the sum of the terms in the arithmetic sequence: 12, 18, 24, . . . , 66, 72, 78? The series has 11 terms, so the sum is $12 \cdot (12 + 78)/2 = 540$
4	6	Evaluate the expression if x = -8 and y = 4: $\frac{3x^2 + 2xy - 5y^2}{-x}$ $\frac{3(-8)^2+2(-8)(4)-5(4)^2}{-(-8)} = \frac{192-64-80}{8} = \frac{48}{8} = 6$
5	936	What is the largest 3-digit number such that all of the digits are different and the number is divisible by each of the digits? Start with a 9 in the hundreds place. Therefore the number needs to be divisible by 9, so the sum of the other 2 digits must be 9. Possible pairs are: 8 and 1, 7 and 2, 6 and 3, 5 and 4. Use trial and error to discover that 981, 972, 963, 954, and 945 are not divisible by all 3 digits. 936 is the first one that is divisible by all 3 digits.

6	5 [integers]	Kenji erases some integers from the following list: 5, 9, 2, 3, 7, 4, 6, 5, 6, 4, 10, 7, 2, 5. When he is done, the range of the remaining list is 3. What is the least possible number of integers that Kenji could have erased? Put the numbers in order: 2, 2, 3, 4, 4, 5, 5, 6, 6, 7, 7, 9, 10. Look for the subset of integers with a range of 3 that has the most members. 2 through 5 has 7 integers – 6 erased 3 through 6 has 7 integers – 6 erased 4 through 7 has 8 integers – 5 erased 6 through 9 has 5 integers – 8 erased 7 through 10 has 4 integers – 9 erased So, the answer is 5.
7	51 [%]	Jiya and Gordon are away at college. They visit home on random weekends. The probability that Jiya will visit on any given weekend is 15%. The probability that Gordon will visit is 40%. As a percent, what is the probability on any given weekend that neither will visit? P(Jiya won't visit) = 0.85 and P(Gordon won't visit) = 0.60, so P(neither will visit) = $0.85 \cdot 0.60 = 0.51 = 51\%$
8	[C + D =] 8	The figure consists of one large circle with diameter 12 centimeters and 7 smaller congruent circles that are externally tangent to each other and internally tangent to the large circle. Let the area of the unshaded region inside the large circle, but not including any of the 7 smaller circles be A. Let the area of the seven shaded circles be B. As a decimal, the ratio of B/A equals C.D, where C and D are single digit integers. What is C + D? If you draw in the diameter, it becomes apparent that the radius of one of the small circles, 2 cm, is 1/3 the radius of the large circle, 6 cm. The area of the large circle is 36π and the area of the 7 small circles is $7 \cdot 4\pi = 28\pi$. So, $B/A = 28\pi/(36\pi - 28\pi) = 28\pi/8\pi = 7/2 = 3.5$ and $3 + 5 = 8$.  

9	24 [triangles]	How many triangles of any size are in the figure below? There are 5 numbered on the original figure plus 19 more that are shown here, for a total of 24. 

10	15 [$/kg]	Isa is making and selling trail mix for a fund raiser. He makes three different mixtures, each containing cashews, chocolate chips, and raisins, each of which costs a certain number of dollars per kilogram. Each of the three mixtures is sold at a price of $18 per kilogram. If the cashews, chocolate chips and raisins are combined in a ratio of 1:1:1 by weight, Isa makes a dollar profit of 20% over the cost. If the cashews, chocolate chips and raisins are combined in a ratio of 3:2:1 respectively by weight, the profit is 8%. If the cashews, chocolate chips and raisins are combined in a ratio of 1:4:2 by weight, the profit is 26%. What is the cost of the chocolate chips in dollars per kilogram? C = cost of cashews, $/kg H = cost of chocolate chips, $/kg A = cost of raisins, $/kg Total price = total kg * 18 $/kg 20% profit: Cost = Price/1.2 8% profit: Cost = Price/1.08 26% profit: Cost = Price/1.26 Mix 1: C + H + A = 45 Mix 2: 3C + 2H + A = 100 Mix 3: C + 4H + 2A = 100 Solve system of equations for: H = 15 $/kg

Mix #	Cash (kg)	Choc (kg)	Raisins (kg)	Total price ($)	Total cost ($)
1	1	1	1	54	45
2	3	2	1	108	100
3	1	4	2	126	100

"Math is Cool" Championships -- 2023-24

7th/8th Grade

College Bowl Round #1 Solutions

	Answer	Solution
1	4864	What is the product of seventy-six times sixty-four? $76 \cdot 64 = (70 + 6)(70 - 6) = 4900 - 36 = 4864$
2	[X – Y =] 30	Twenty is X percent of twenty-five and Y percent of forty. What is X minus Y? 20 is 80% of 25 and 50% of 40, and 80 – 50 = 30
3	36	Twin primes are two prime numbers that differ by two. What is the sum of the two prime numbers in the smallest twin primes greater than twelve? 17 + 19 = 36
4	[P + Q =] 7	The letters A, B, C, D, E, and F are randomly assigned to make a six-letter arrangement. As a reduced common fraction, the probability that the last letter in the arrangement is D is P over Q. What is P plus Q? There are 5! = 120 ways that the last letter can be D and 6! = 720 total arrangements, so the probability is 120/720 = 1/6, and 1 + 6 = 7
5	4 [years old]	Charlene is currently three times Sameer's age. In four years, she will be twice Sameers's age. In years, how old is Sameer now? $J = 3K$ and $J + 4 = 2(K + 4) \rightarrow 3K + 4 = 2K + 8 \rightarrow K = 4$
6	2160	The first two terms of a series are four-fifths and fifteen. Beginning with the third term, each successive term is the product of the two previous terms. What is the fifth term? The first five terms are: 4/5, 15, 12, 180, 2160.
7	90 [minutes]	Anaya runs an eighteen-mile race at an average rate of nine miles per hour. Brenda runs the same race averaging ten miles per hour but starts ten minutes after Anaya. How many minutes will it take Brenda to catch Anaya? In ten minutes, which is 1/6 of an hour, Anaya has run 9/6 = 1.5 miles. Since Brenda is running 1 mile per hour faster than Anaya, it will take her 1.5 hours or 90 minutes to catch Anaya.
8	4 [products]	When you multiply any two distinct digits one through five together, how many different two-digit products are possible? $5 \cdot 4$, $5 \cdot 3$, $5 \cdot 2$, and $4 \cdot 3$
9	250 [mm]	How many millimeters are equivalent to one-fourth of a meter? 1 meter = 1000 millimeters, so 1/4 meter = 250 millimeters

10	196 [cm^2]	In square centimeters, what is the area of a square whose perimeter is fifty-six centimeters? $56/4 = 14$ and $14^2 = 196$

"Math is Cool" Championships -- 2023-24

7th/8th Grade

College Bowl Round #2 Solutions

	Answer	Solution
1	232 [blueberries]	One cantaloupe weighs the same as eight apples. One apple weighs the same as twenty-nine blueberries. How many blueberries weigh the same as one cantaloupe? $1c = 8a \rightarrow 1a = 29b \rightarrow 1c = 8 \cdot 29b = 232b$
2	[x =] 21	Solve the following equation for X: negative nineteen plus five X equals eighty-six $-19 + 5x = 86 \rightarrow 5x = 105 \rightarrow x = 21$
3	112 [squares]	How many three-inch-by-three-inch squares can a twenty-four-inch-by-forty-two-inch rectangle be divided into? $24 = 3 \cdot 8$ and $42 = 3 \cdot 14$, so $8 \cdot 14 = 112$
4	74	The mean of three different positive integers is twenty-six. What is the largest possible positive difference between the largest and the smallest integers? $26 \cdot 3 = 78$, so the integers could be 1, 2, and 75, and $75 - 1 = 74$
5	[A + B =] 49	An eight-sided die with sides numbered one through eight and a twelve-sided die with sides numbered one through twelve are rolled. As a reduced common fraction, the probability that the sum of the numbers showing is nineteen is A over B. What is A plus B? The numbers showing need to be 8 and 11, or 7 and 12, and there are $8 \cdot 12 = 96$ possible pairs of numbers showing, so the probability is $2/96 = 1/48$, and $1 + 48 = 49$
6	642	My number is a positive, even three-digit integer. The sum of the ones and tens digits equals the hundreds digit. The hundreds digit is three times the ones digit. What is my number? The hundreds digit must be 6 since it's the only single digit that is equal to 3 times an even digit. The tens and ones must be 4 and 2, so the number is 642.
7	15	A sequence of fractions begins with one half. Each successive term is a new fraction in which the numerator and denominator of the previous term both increase by one. What is the sum of the numerator and denominator of the seventh term? The first 7 terms of the series are 1/2, 2/3, 3/4, 4/5, 5/6, 6/7, 7/8 and $7 + 8 = 15$
8	945	What is the product of the first five positive odd integers? $1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 = 945$

9	898	What is the largest even three-digit palindrome? Any 3-digit palindrome greater than 898 will be odd.
10	72 [sandwiches]	Ken can make twenty-four sub sandwiches in an hour. Barbie can make sub sandwiches at one and a half times the rate of Ken. How many sandwiches can Barbie make in two hours? Barbie's rate is $24 \cdot 1.5 = 36$ per hour and $36 \cdot 2 = 72$

"Math is Cool" Championships -- 2023-24

7th/8th Grade

College Bowl Round #3 Solutions

	Answer	Solution
1	[A + B =] 43	A jar has thirteen red marbles and fifteen white marbles. As a reduced common fraction, the probability of drawing a white marble from the jar is A over B. What is A plus B? P(white) = 15/28 and 15 + 28 = 43
2	108	The first three terms of a geometric sequence are one, three, and nine. What is the sum of the next two terms? The series is 1, 3, 9, 27, 81, . . . and 27 + 81 = 108
3	54	What is the median of all positive two-digit multiples of eighteen? The median of 18, 36, 54, 72, 90 is 54
4	16 [hours]	Aman takes twelve hours to read a book. Aziz reads at a rate that is three-fourths the rate of Aman. How many hours does it take Aziz to read the same book? Let r = Aman's reading rate, and t = the time it takes for Aziz to read the book, then $12r = (3/4)rt \rightarrow 12 = (3/4)t \rightarrow t = (4/3)12 = 16$
5	20	Two hundred percent of P equals ten percent of Q. What is Q divided by P? $2P = 0.1Q \rightarrow 20P = Q \rightarrow 20 = Q/P$
6	[A + B =] 10	In the interval from ten to twenty-nine inclusive, as a reduced common fraction, the ratio of prime numbers to composite numbers is A over B. What is A plus B? There are 6 primes: 11, 13, 17, 19, 23, 29 There are 14 composites: 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28 The ratio is 6/14 = 3/7 and 3 + 7 = 10
7	[X =] 3	Consider circle A of radius R. Circle B has an area that is three times the area of circle A. The radius of circle B is R times the square root of X. What is the value of X? The area of circle A is $R^2\pi$, so $3R^2\pi = (R\sqrt{X})^2\pi \rightarrow 3R^2\pi = X \cdot R^2\pi \rightarrow X = 3$
8	162 [inches]	How many inches are in thirteen point five feet? $13.5 \cdot 12 = 162$

9	2	In a set of three distinct positive integers, A, B, and C, the largest integer is A, which is a two-digit integer, and the smallest integer is B, which is a one-digit integer. What is the smallest possible value of C? If B is 1, then the smallest possible value of C is 2.
10	288	What is the sum of the three largest even two-digit integers? 94 + 96 + 98 = 288

"Math is Cool" Championships -- 2023-24

7th/8th Grade

College Bowl Round #4 Solutions

	Answer	Solution
1	2 [mph]	Blanca takes ten minutes to walk from home to school. If school is one-third of a mile from home, what is her average walking speed in miles per hour? 1/3 mile in 10 min = 2 miles in 60 min = 2 mph
2	12 [blue marbles]	A jar has two green marbles and some blue marbles in it. As a reduced common fraction, the probability of drawing a green marble is one over seven. How many blue marbles are in the jar? 1/7 = 2/(2 + g), so g must be 12
3	4 [years]	Amad is three years older than Barry and seven years older than Cristina. How many years older than Cristina is Barry? A = B + 3 and A = C + 7 → B + 3 = C + 7. B = C + 4, so the answer is 4.
4	216	What is the sum of the positive integer factors of one hundred and eighty-seven? Factors of 187 are 1, 11, 17, and 187 and 1 + 11 + 17 + 187 = 216
5	2 [times]	Brooklyn takes forty-five minutes to mow the lawn. Cooper takes one and a half hours to mow the same lawn. As an integer, how many times faster is Brooklyn's lawn-mowing rate compared to Coopers's lawn-mowing rate? If she does it in half the time, her rate is twice as fast. Therefore, 2 times faster.
6	9 [multiples]	How many positive three-digit multiples of ninety-five are there? 190, 285, 380, 475, 570, 665, 760, 855, 950, makes 9 multiples
7	70	A set of three distinct integers have a range of forty-five. The largest integer is ten times the smallest integer. The smallest integer is one-third the middle integer. What is the sum of the three integers? The ratio of the three numbers is 1:3:10. Check multiples of this ratio until finding one with a range of 45: 1:3:10 – range = 9 2:6:20 – range = 18 3:9:30 – range = 27 4:12:40 – range = 36 5:15:50 – range = 45, sum = 5 + 15 + 50 = 70

8	[A + B =] 10	As a reduced common fraction, one-half of six-sevenths is A over B. What is A plus B? 1/2 of 6/7 = 3/7 and 3 + 7 = 10
9	196 [°]	An angle measures eighty-two degrees. In degrees, what is the measure of twice its supplementary angle? Supplementary angle = 180 – 82 = 98 98x2 = 196
10	[N =] 7	If five N plus ten D equals one hundred and seventy-five and D equals two N, what is the value of N? 5N + 10D = 175 and D = 2N → 5N + 20N = 175 → 25N = 175 → N = 7

"Math is Cool" Championships -- 2023-24

7th/8th Grade

College Bowl Round #5 Solutions

	Answer	Solution
1	21	As a reduced common fraction, the probability of rolling an eleven on a standard twenty-sided die with sides numbered one through twenty is A over B. What is A plus B? P(11) = 1/20 and 1 + 20 = 21
2	438 [cents]	A two-gallon container of fertilizer costs thirty-five dollars and four cents. In cents, what is the price per quart? $35.04/2 = $17.52/gallon 17.52/4 = $4.38/quart = 438 cents/quart
3	9	What is the mean of the counting numbers six through twelve, inclusive? The mean of a consecutive set of numbers is the same as the average of the smallest and largest numbers, so (6 + 12)/2 = 9
4	2 [sandcastles]	Penelope can build five sandcastles in three hours. How many sandcastles can she build in one hour and twelve minutes? 5 c in 3 h = 5 c in 180 m = 1 c in 36 min = 2 c in 72 min = 2 c in 1h 12m
5	290 [pounds]	At the county fair people can win a prize by guessing the weight of a pig. No one who guessed was correct, but the three closest guesses were two hundred seventy, two hundred eighty, and two hundred ninety-five pounds. One of the guesses was off by five, one was off by ten, and one was off by twenty. In pounds, what does the pig weigh? The 3 guesses are 270, 280, and 295. Since one is off by 5 and another is off by 10, the correct weight must be either 285 or 290. Since the third guess is off by 20, the answer must be 290.
6	72 [ways]	Three boys and three girls form a line, with the first person being closest to the doorway. How many different ways can they line up relative to the doorway if they must alternate boy-girl, boy-girl, boy-girl, or girl-boy, girl-boy, girl-boy? There are 3! = 6 ways for the boys to arrange and 6 ways for the girls to arrange, so there are 6 · 6 = 36 ways when a boy is at the head of the line, and also 36 ways when a girl is at the head of the line. The total is 36 + 36 = 72
7	91809	What is one hundred and one times nine hundred and nine? 101 · 909 = 101 · 900 + 101 · 9 = 90900 + 909 = 91809

8	80	What is the least common multiple of sixteen and forty? $16 = 2 \cdot 8$ and $40 = 5 \cdot 8$, so $\text{LCM}(16, 40) = 2 \cdot 5 \cdot 8 = 80$
9	36	A series of numbers starts with one, three, six, ten, and so on. What is sum of the next two numbers in the series? The first 6 numbers are 1, 3, 6, 10, 15, 21, and $15 + 21 = 36$
10	3 [miles]	Raj rides his bike at an average rate of ten miles per hour. How many miles does he ride in eighteen minutes? $18/60 = x/10 \rightarrow x = 3$

"Math is Cool" Championships -- 2023-24

7th/8th Grade

College Bowl Round #6 Solutions

	Answer	Solution
1	5	What is one thousand divided by ten, divided by twenty? 1000/10 = 100 and 100/20 = 5
2	[x =] 19	Solve the following equation for X: five times the quantity X plus nine equals one hundred and forty $5(x + 9) = 140 \rightarrow 5x + 45 = 140 \rightarrow 5x = 95 \rightarrow x = 19$
3	225 [cents]	Seventy-two pencils cost nine dollars. How many cents do eighteen pencils cost? $72p = 900c \rightarrow 18p = 225c$
4	16	A data set consists of fifteen distinct integers and the largest number in the set is twenty-three. What is the largest possible median of the set? One data set having the largest possible median is: 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23. The median is 16.
5	56	What is the ninth term in the arithmetic sequence whose first term is eight and whose third term is twenty? 20 – 8 = 12, so the common difference is 6 and the 9^{th} term is 8 + 6(8) = 56
6	1078	Let A equal a two-digit integer and B equal a three-digit integer. The greatest common factor of A and B is eleven. What is the largest possible value of A plus B? If $A = 99 = 11 \cdot 9$ and $B = 979 = 11 \cdot 89$, then GCF(99, 979) = 11 and 99 + 979 = 1078
7	[A + B =] 8	Two cubes have a combined volume of one hundred and fifty-two cubic centimeters, and the dimensions of both cubes are integers. As a reduced common fraction, the ratio of the smaller cube's side length to the larger cube's side length is A over B. What is A plus B? The volumes must be from the list of perfect cubes, 1, 8, 27, 64, 125, . . . and the only pair of these that add up to 152 is 27 and 125, so the ratio of side lengths is 3/5 and 3 + 5 = 8
8	3	What is the median of the five numbers in the following data set? zero point five, ten, negative one, three, and eight The set {-1, 0.5, 3, 8, 10} has a median of 3

9	[A + B =] 13	A bag of M and Ms has six red, three green, five brown, two yellow, and four blue M and Ms. As a reduced common fraction, the probability of randomly drawing a red M and M from the bag is A over B. What is A plus B? There are 6R, 3G, 5Br, 2Y, and 4Bl in the bag, so P(red) = 6/20 = 3/10 and 3 + 10 = 13
10	3 [ways]	In how many ways can two positive integers be multiplied together to make thirty-two? The order in which the integers are multiplied does not matter. $1 \cdot 32, 2 \cdot 16, 4 \cdot 8$